COORDINATE TRANSFORMATIONS

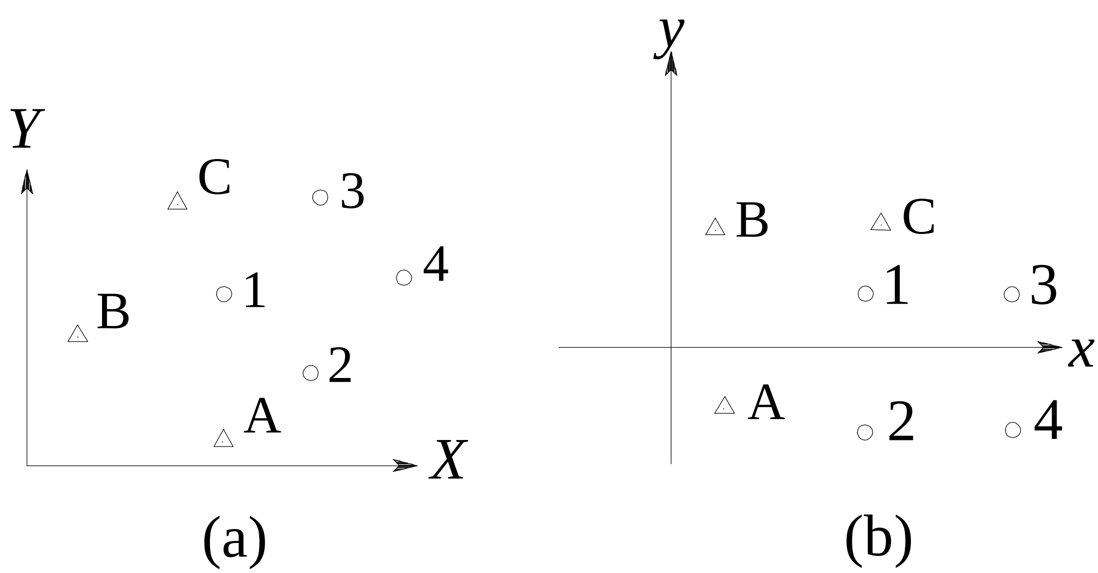
TWO DIMENSIONAL TRANSFORMATIONS

The *two dimensional conformal coordinate transformation* is also known as the *four parameter similarity transformation* since it maintains scale relationships between the two coordinate systems.

PARAMETERS

1. Scaling
2. Rotation
3. Translation in X and Y

2D CONFORMAL TRANSFORMATIONS



Steps in transforming coordinates measured in the coordinate system shown in (b) into that shown in (a).

2D CONFORMAL TRANSFORMATIONS

Step 1: *SCALING*. To make the length between A and B in (b) equal to the length between A and B in (a).

$$S = \frac{AB_a}{AB_b}$$

and

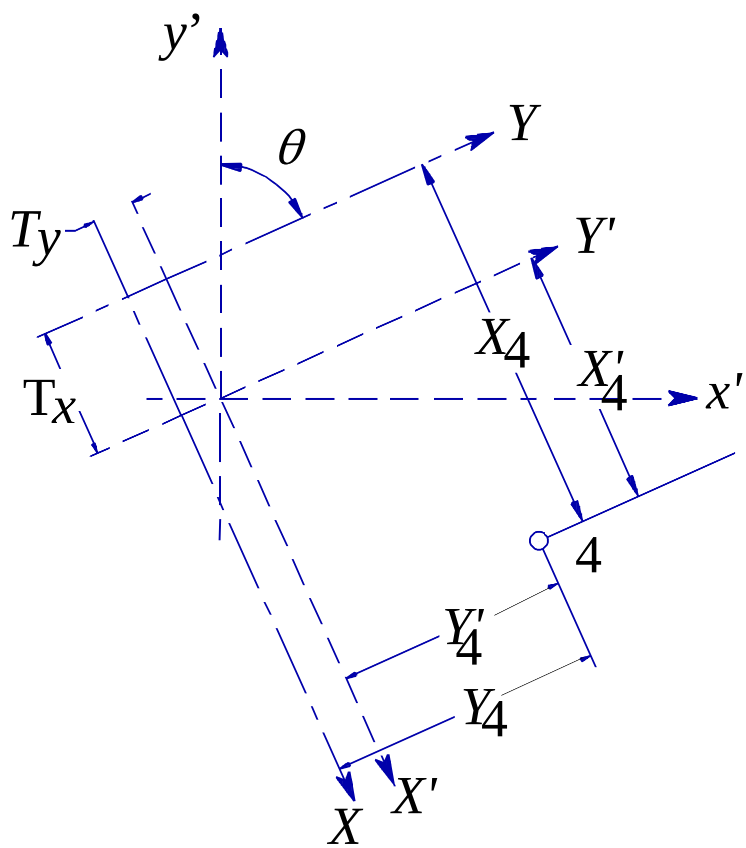
$$x' = S\,x$$

$$y' = S\,y$$

Call this scaled system (b').

2D CONFORMAL TRANSFORMATIONS

Step 2: *ROTATION*. Rotate coordinate system in scaled system (b') so that points in (b') coincide with points in (X', Y') system.



$$X' = x' \cos\theta - y' \sin\theta$$

$$Y' = x' \sin\theta + y' \cos\theta$$

2D CONFORMAL TRANSFORMATIONS

STEP 3: *TRANSLATIONS*

Use coordinates of common point in scaled-rotated system (X', Y') to compute T_x and T_y.

$$T_x = X - X'$$

$$T_y = Y - Y'$$

OBSERVATION EQUATIONS

Combining equations for scale, rotation, and translation yields:

$$X = (S \text{ Cos } \theta)x \ - \ (S \text{ Sin } \theta)y \ + T_X$$

$$Y = (S \text{ Sin } \theta)x \ + \ (S \text{ Cos } \theta)y \ + T_Y$$

Let S Cos $\theta = a$, S Sin $\theta = b$, $T_X = c$, and $T_Y = d$

Add residuals to develop observation equation.

$$ax - by + c = X + v_X$$

$$ay + bx + d = Y + v_Y$$

NOTE:

$$\theta = Tan^{-1}\left(\frac{b}{a}\right)$$

$$S = \frac{a}{Cos \ \theta}$$

LEAST SQUARES EXAMPLE

Transform points in (*x*, *y*) system into (E,N).

Point	E	N	*x*	*y*
A	1,049,422.40	51,089.20	121.622	-128.066
B	1,049,413.95	49,659.30	141.228	187.718
C	1,049,244.95	49,884.95	175.802	135.728
1			174.148	-120.262
2			513.520	-192.130
3			754.444	- 67.706
4			972.788	120.994

Develop observation equations in form, AX = L + V

where

$$A = \begin{bmatrix} x_a & -y_a & 1 & 0 \\ y_a & x_a & 0 & 1 \\ x_b & -y_b & 1 & 0 \\ y_b & x_b & 0 & 1 \\ x_c & -y_c & 1 & 0 \\ y_c & x_c & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad L = \begin{bmatrix} X_A \\ Y_A \\ X_B \\ Y_B \\ X_C \\ Y_C \end{bmatrix} \quad V = \begin{bmatrix} v_{X_A} \\ v_{Y_A} \\ v_{X_B} \\ v_{Y_B} \\ v_{X_C} \\ v_{Y_C} \end{bmatrix}$$

LEAST SQUARES EXAMPLE

$$A = \begin{bmatrix} 121.622 & 128.066 & 1.000 & 0.000 \\ -128.066 & 121.622 & 0.000 & 1.000 \\ 141.228 & -187.718 & 1.000 & 0.000 \\ 187.718 & 141.228 & 0.000 & 1.000 \\ 175.802 & -135.728 & 1.000 & 0.000 \\ 135.728 & 175.802 & 0.000 & 1.000 \end{bmatrix} \quad L = \begin{bmatrix} 1049422.40 \\ 51089.20 \\ 1049413.95 \\ 49659.30 \\ 1049244.95 \\ 49884.95 \end{bmatrix}$$

Solve system using unweighted least squares method.

$$X = (A^T A)^{-1} A^T L$$

$$X = \begin{bmatrix} -4.51249 \\ -0.25371 \\ 1050003.715 \\ 50542.131 \end{bmatrix}$$

SO a = -4.51249,
b = -0.25371,
T_x = 1,050,003.715, and
T_y = 50,542.131

USING OBSERVATION EQUATIONS TRANSFORM REMAINING POINTS

TABULATE RESULTS

Transformed Control Points

POINT	X	Y	VX	VY
A	1,049,422.40	51,089.20	-0.004	0.029
B	1,049,413.95	49,659.30	-0.101	0.077
C	1,049,244.95	49,884.95	0.105	-0.106

Transformation Parameters and estimated errors

a	=	-4.51249	±	0.00058
b	=	-0.25371	±	0.00058
Tx	=	1,050,003.715	±	0.123
Ty	=	50,542.131	±	0.123

Transformed Points

POINT	X	Y	±Sx	±Sy
1	1,049,187.361	51,040.629	0.173	0.173
2	1,047,637.713	51,278.829	0.339	0.339
3	1,046,582.113	50,656.241	0.453	0.453
4	1,045,644.713	49,749.336	0.578	0.578

Rotation = 183° 13' 05.0"
Scale = 4.51962

Adjustment's Reference Variance = 0.0195

2D AFFINE TRANSFORMATION

The Six Parameter Transformation

OBSERVATION EQUATIONS

$$ax + by + c = X + V_X$$

$$dx + ey + f = Y + V_Y$$

Each axis has a different scale factor.

EXAMPLE

PT	X	Y	*x*	*y*	σ*x*	σ*y*
1	-113.000	0.003	0.764	5.960	0.026	0.028
3	0.001	112.993	5.062	10.541	0.024	0.030
5	112.998	0.003	9.663	6.243	0.028	0.022
7	0.001	-112.999	5.350	1.654	0.024	0.026
306			1.746	9.354		
307			5.329	9.463		

Determine the *most probable values* for the 2D affine transformation parameters for the data above. Transform points 306 and 307 into the (X, Y) system.

2D AFFINE TRANSFORMATION

Observation equations

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \\ x_4 & y_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_4 & y_4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{bmatrix} + \begin{bmatrix} v_{X_1} \\ v_{Y_1} \\ v_{X_2} \\ v_{Y_2} \\ v_{X_3} \\ v_{Y_3} \\ v_{X_4} \\ v_{Y_4} \end{bmatrix}$$

2D AFFINE TRANSFORMATION

$$A = \begin{bmatrix} 0.764 & 5.960 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.764 & 5.960 & 1.000 \\ 5.062 & 10.541 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 5.062 & 10.541 & 1.000 \\ 9.663 & 6.243 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 9.663 & 6.243 & 1.000 \\ 5.350 & 1.654 & 1.000 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 5.350 & 1.654 & 1.000 \end{bmatrix} \quad L = \begin{bmatrix} -113.000 \\ -113.000 \\ 0.001 \\ 0.001 \\ 112.998 \\ 112.998 \\ 0.001 \\ 0.001 \end{bmatrix}$$

Solution: $X = (A^T A)^{-1} A^T L$

$$X = \begin{bmatrix} 25.37152 \\ 0.82220 \\ -137.183 \\ -0.80994 \\ 25.40166 \\ -150.723 \end{bmatrix}$$

TABULATE RESULTS

Transformed Control Points

POINT	X	Y	VX	VY
1	-113.000	0.003	0.101	0.049
3	0.001	112.993	-0.086	-0.057
5	112.998	0.003	0.117	0.030
7	0.001	-112.999	-0.086	-0.043

Transformation Parameters:

a = 25.37152 ± 0.02532
b = 0.82220 ± 0.02256
c = -137.183 ± 0.203
d = -0.80994 ± 0.02335
e = 25.40166 ± 0.02622
f = -150.723 ± 0.216

Adjustment's Reference Variance = 34.9248

Transformed Points

POINT	X	Y	±σx	±σy
1	-112.899	0.052	0.244	0.267
3	-0.085	112.936	0.338	0.370
5	113.115	0.033	0.348	0.353
7	-0.085	-113.042	0.247	0.254
306	-85.193	85.470	0.296	0.330
307	5.803	85.337	0.324	0.352

2D PROJECTIVE TRANSFORMATION

(*The Eight Parameter Transformation*)

OBSERVATION EQUATIONS

$$X = \frac{a_1 x + b_1 y + c_1}{a_3 x + b_3 y + 1}$$

$$Y = \frac{a_2 x + b_2 y + c_2}{a_3 x + b_3 y + 1}$$

Note that these equations are non-linear.

Use exact solution to compute initial values for unknown parameters.

LINEARIZED EQUATIONS

For every point, the matrix for is:

$$\begin{bmatrix} \left(\frac{\partial X}{\partial a_1}\right)_o & \left(\frac{\partial X}{\partial b_1}\right)_o & \left(\frac{\partial X}{\partial c_1}\right)_o & 0 & 0 & 0 & \left(\frac{\partial X}{\partial a_3}\right)_o & \left(\frac{\partial X}{\partial b_3}\right)_o \\ 0 & 0 & 0 & \left(\frac{\partial Y}{\partial a_2}\right)_o & \left(\frac{\partial Y}{\partial b_2}\right)_o & \left(\frac{\partial Y}{\partial c_2}\right)_o & \left(\frac{\partial Y}{\partial a_3}\right)_o & \left(\frac{\partial Y}{\partial b_3}\right)_o \end{bmatrix} \begin{bmatrix} da_1 \\ db_1 \\ dc_1 \\ da_2 \\ db_2 \\ dc_2 \\ da_3 \\ db_3 \end{bmatrix} = \begin{bmatrix} X - X_o \\ Y - Y_o \end{bmatrix}$$

where

$$\frac{\partial f}{\partial a_1} = \frac{x}{a_3 x + b_3 y + 1} \qquad \frac{\partial f}{\partial b_1} = \frac{y}{a_3 x + b_3 y + 1}$$

$$\frac{\partial f}{\partial c_1} = \frac{1}{a_3 x + b_3 y + 1} \qquad \frac{\partial g}{\partial a_2} = \frac{x}{a_3 x + b_3 y + 1}$$

$$\frac{\partial g}{\partial b_2} = \frac{y}{a_3 x + b_3 y + 1} \qquad \frac{\partial g}{\partial c_2} = \frac{1}{a_3 x + b_3 y + 1}$$

$$\frac{\partial f}{\partial a_3} = -\frac{a_1 x + b_1 y + c_1}{(a_3 x + b_3 y + 1)^2} x \qquad \frac{\partial f}{\partial b_3} = -\frac{a_1 x + b_1 y + c_1}{(a_3 x + b_3 y + 1)^2} y$$

$$\frac{\partial f}{\partial a_3} = -\frac{a_2 x + b_2 y + c_2}{(a_3 x + b_3 y + 1)^2} y \qquad \frac{\partial f}{\partial a_3} = -\frac{a_2 x + b_2 y + c_2}{(a_3 x + b_3 y + 1)^2} y$$

EXAMPLE

Compute the transformation parameters for the following data using a 2D projective transformation.

Pt	*X*	*Y*	*x*	*y*	σx	σy
1	1420.407	895.362	90.0	90.0	0.3	0.3
2	895.887	351.398	50.0	40.0	0.3	0.3
3	-944.926	641.434	-30.0	20.0	0.3	0.3
4	968.084	-1384.138	50.0	-40.0	0.3	0.3
5	1993.262	-2367.511	110.0	-80.0	0.3	0.3
6	-3382.284	3487.762	-100.0	80.0	0.3	0.3
7			-60.0	20.0	0.3	0.3
8			-100.0	-100.0	0.3	0.3

Initial parameter values solved by using only first four points.

a1 =	25.00505	b1 =	0.79067
c1 =	-135.788	a2 =	-8.00698
b2 =	24.97183	c2 =	-148.987
a3 =	0.00398	b3 =	0.00201

EXAMPLE

ITERATION 1

$$J = \begin{bmatrix} 58.445 & 58.445 & 0.649 & 0.000 & 0.000 & 0.000 & -83007.064 & -83007.064 \\ 0.000 & 0.000 & 0.000 & 58.445 & 58.445 & 0.649 & -52334.927 & -52334.927 \\ 39.064 & 31.251 & 0.781 & 0.000 & 0.000 & 0.000 & -35012.162 & -28009.729 \\ 0.000 & 0.000 & 0.000 & 39.064 & 31.251 & 0.781 & -13719.422 & -10975.538 \\ -32.608 & 21.739 & 1.087 & 0.000 & 0.000 & 0.000 & -30791.646 & 20527.764 \\ 0.000 & 0.000 & 0.000 & -32.608 & 21.739 & 1.087 & 20924.176 & -13949.451 \\ 44.644 & -35.715 & 0.893 & 0.000 & 0.000 & 0.000 & -43186.235 & 34548.988 \\ 0.000 & 0.000 & 0.000 & 44.644 & -35.715 & 0.893 & 61787.575 & -49430.060 \\ 85.941 & -62.502 & 0.781 & 0.000 & 0.000 & 0.000 & -171318.968 & 124595.613 \\ 0.000 & 0.000 & 0.000 & 85.941 & -62.502 & 0.781 & 203472.866 & -147980.266 \\ -131.572 & 105.258 & 1.316 & 0.000 & 0.000 & 0.000 & -445062.971 & 356050.376 \\ 0.000 & 0.000 & 0.000 & -131.572 & 105.258 & 1.316 & 458870.304 & -367096.243 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.158 \\ -0.088 \\ -0.388 \\ 0.195 \\ -0.636 \\ -0.250 \\ 0.729 \\ -0.120 \\ -0.198 \\ 0.090 \\ 0.362 \\ 0.175 \end{bmatrix} \qquad X = \begin{bmatrix} 0.0027039323 \\ 0.0035789799 \\ -0.0156917169 \\ 0.0014774310 \\ 0.0031559533 \\ 0.0018094254 \\ 0.0000022203 \\ 0.0000030887 \end{bmatrix}$$

TABULATE RESULTS

Transformation Parameters:

a1 =	25.00274 ± 0.01538	b1 =	0.80064 ± 0.01896
a2 =	-8.00771 ± 0.00954	b2 =	24.99811 ± 0.01350
c1 =	-134.715 ± 0.377	c2 =	-149.815 ± 0.398
a3 =	0.00400 ± 0.00001	b3 =	0.00200 ± 0.00001

Adjustment's Reference Variance = 3.8888

Number of Iterations = 2

Transformed Control Points

POINT	X	Y	VX	VY
1	1,420.165	895.444	-0.242	0.082
2	896.316	351.296	0.429	-0.102
3	-944.323	641.710	0.603	0.276
4	967.345	-1,384.079	-0.739	0.059
5	1,993.461	-2,367.676	0.199	-0.165
6	-3,382.534	3,487.612	-0.250	-0.150

Transformed Points

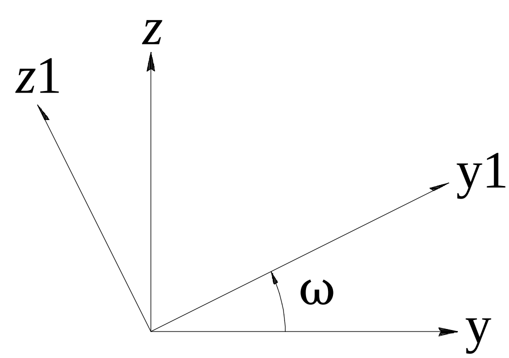
POINT	X	Y	$\pm\sigma x$	$\pm\sigma y$
1	1,420.165	895.444	2.082	1.375
2	896.316	351.296	1.053	0.679
3	-944.323	641.710	0.890	0.681
4	967.345	-1,384.079	1.231	1.227
5	1,993.461	-2,367.676	3.165	3.323
6	-3,382.534	3,487.612	7.646	7.561
7	-2,023.678	1,038.310	2.273	1.329
8	-6,794.740	-4,626.976	31.288	21.315

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

Similar to two-dimensional conformal transformation with 3 rotational parameters.

Rotation about *x* axis, ω rotation.

$$X_1 = M_1 X'$$



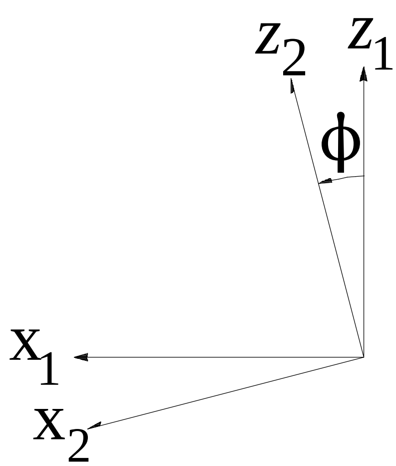
where

$$X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cos(\omega) & Sin(\omega) \\ 0 & -Sin(\omega) & Cos(\omega) \end{bmatrix} \quad and \quad X' = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

Rotation about Y_1 axis, ϕ rotation.

$$X_2 = M_2 X_1$$



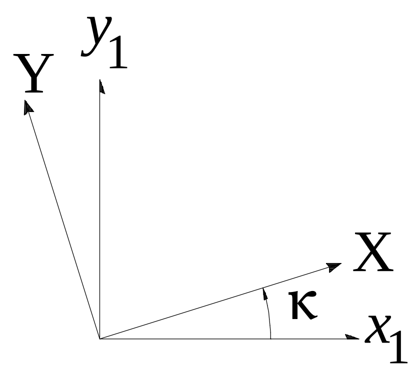
where

$$X_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \ and \ M_2 = \begin{bmatrix} Cos(\phi) & 0 & -Sin(\phi) \\ 0 & 1 & 0 \\ Sin(\phi) & 0 & Cos(\phi) \end{bmatrix}$$

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

Rotation about Z_2 axis, κ rotation.

$$\bar{x} = M_3 X_2$$



where

$$\overline{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}, \; and \; M_3 = \begin{bmatrix} Cos(\kappa) & Sin(\kappa) & 0 \\ -Sin(\kappa) & Cos(\kappa) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

Final combined expression:

$$\bar{x} = M_3\, M_2\, M_1\, X' = M\, X'$$

where M is $\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$

where:

$m_{11} = \mathrm{Cos}(\phi)\ \mathrm{Cos}(\kappa)$

$m_{12} = \mathrm{Sin}(\omega)\ \mathrm{Sin}(\phi)\ \mathrm{Cos}(\kappa) + \mathrm{Cos}(\omega)\ \mathrm{Sin}(\kappa)$

$m_{13} = -\mathrm{Cos}(\omega)\ \mathrm{Sin}(\phi)\ \mathrm{Cos}(\kappa) + \mathrm{Sin}(\omega)\ \mathrm{Sin}(\kappa)$

$m_{21} = -\mathrm{Cos}(\phi)\ \mathrm{Sin}(\kappa)$

$m_{22} = -\mathrm{Sin}(\omega)\ \mathrm{Sin}(\phi)\ \mathrm{Sin}(\kappa) + \mathrm{Cos}(\omega)\ \mathrm{Cos}(\kappa)$

$m_{23} = \mathrm{Cos}(\omega)\ \mathrm{Sin}(\phi)\ \mathrm{Sin}(\kappa) + \mathrm{Sin}(\omega)\ \mathrm{Cos}(\kappa)$

$m_{31} = \mathrm{Sin}(\phi)$

$m_{32} = -\mathrm{Sin}(\omega)\ \mathrm{Cos}(\phi)$

$m_{33} = \mathrm{Cos}(\omega)\ \mathrm{Cos}(\phi)$

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

OBSERVATION EQUATIONS:

$$X = S(m_{11}\, x + m_{12}\, y + m_{13}\, z) + Tx$$
$$Y = S(m_{21}\, x + m_{22}\, y + m_{23}\, z) + Ty$$
$$Z = S(m_{31}\, x + m_{32}\, y + m_{33}\, z) + Tz$$

Linearized Observation Equations for a single point.

$$\begin{bmatrix} \left(\frac{\partial X}{\partial S}\right)_o & 0 & \left(\frac{\partial X}{\partial \phi}\right)_o & \left(\frac{\partial X}{\partial \kappa}\right)_o & 1 & 0 & 0 \\ \left(\frac{\partial Y}{\partial S}\right)_o & \left(\frac{\partial Y}{\partial \omega}\right)_o & \left(\frac{\partial Y}{\partial \phi}\right)_o & \left(\frac{\partial Y}{\partial \kappa}\right)_o & 0 & 1 & 0 \\ \left(\frac{\partial Z}{\partial S}\right)_o & \left(\frac{\partial Z}{\partial \omega}\right)_o & \left(\frac{\partial Z}{\partial \phi}\right)_o & \left(\frac{\partial Z}{\partial \kappa}\right)_o & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} dS \\ d\omega \\ d\phi \\ d\kappa \\ dTx \\ dTy \\ dTz \end{bmatrix} = \begin{bmatrix} X - X_o \\ Y - Y_o \\ Z - Z_o \end{bmatrix}$$

THREE DIMENSIONAL CONFORMAL COORDINATE TRANSFORMATION

where

$$\frac{\partial X}{\partial S} = m_{11}x + m_{12}y + m_{13}z$$

$$\frac{\partial Y}{\partial S} = m_{21}x + m_{22}y + m_{23}z$$

$$\frac{\partial Z}{\partial S} = m_{31}x + m_{32}y + m_{33}z$$

$$\frac{\partial Y}{\partial \omega} = -S\ [m_{13}x + m_{23}y + m_{33}z]$$

$$\frac{\partial Z}{\partial \omega} = S\ [m_{12}x + m_{22}y + m_{32}z]$$

$$\frac{\partial X}{\partial \phi} = S\ [-Sin(\phi)\ Cos(\kappa)\ x + Sin(\phi) Sin(\kappa)\ y + Cos(\phi)\ z]$$

$$\frac{\partial Y}{\partial \phi} = S\ [Sin(\omega) Cos(\phi) Cos(\kappa) x - Sin(\omega) Cos(\phi) Sin(\kappa) y + Sin(\omega) Sin(\phi) z]$$

$$\frac{\partial Z}{\partial \phi} = S\ [m_{12}x + m_{22}y + m_{32}z]$$

$$\frac{\partial X}{\partial \kappa} = S\ [m_{21}x - m_{11}y]$$

$$\frac{\partial Y}{\partial \kappa} = S\ [m_{22}x - m_{12}y]$$

$$\frac{\partial Z}{\partial \kappa} = S\ [m_{23}x - m_{13}y]$$

EXAMPLE

Pt	X	Y	Z	$x \pm S_x$	$y \pm S_y$	$z \pm S_z$
1	10037.81	5262.09	772.04	1094.883±0.007	820.085±0.008	109.821±0.005
2	10956.68	5128.17	783.00	503.891±0.011	1598.698±0.008	117.685±0.009
3	8780.08	4840.29	782.62	2349.343±0.006	207.658±0.005	151.387±0.007
4	10185.80	4700.21	851.32	1395.320±0.005	1348.853±0.008	215.261±0.009
5				265.346±0.005	1003.470±0.007	78.609±0.003
6				784.081±0.006	512.683±0.008	139.551±0.008

What are the most probable values for the 3D transformation parameters?

```
                              J matrix                                                  K matrix
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~     ~~~~~~~~~~
     0.000    102.452   1284.788      1.000      0.000      0.000   -206.164        -0.000
   -51.103     -7.815   -195.197      0.000      1.000      0.000  -1355.718         0.000
 -1287.912    195.697      4.553      0.000      0.000      1.000     53.794         0.000
     0.000    118.747   1418.158      1.000      0.000      0.000    761.082        -0.000
   -62.063     28.850    723.004      0.000      1.000      0.000  -1496.689        -0.000
 -1421.832   -722.441     42.501      0.000      0.000      1.000     65.331        -0.000
     0.000    129.863   1706.020      1.000      0.000      0.000  -1530.174         0.060
   -61.683    -58.003  -1451.826      0.000      1.000      0.000  -1799.945         0.209
 -1709.922   1452.485    -41.580      0.000      0.000      1.000     64.931         0.000
     0.000    204.044   1842.981      1.000      0.000      0.000    -50.417         0.033
  -130.341     -1.911    -46.604      0.000      1.000      0.000  -1947.124        -0.053
 -1849.740     47.857     15.851      0.000      0.000      1.000    137.203         0.043

X matrix
~~~~~~~~
-0.0000347107
-0.0000103312
-0.0001056763
 0.1953458986
-0.0209088384
-0.0400969773
-0.0000257795

Measured Points
-----------------------------------------------------------------------------
      NAME             x             y             z       Sx       Sy       Sz
-----------------------------------------------------------------------------
         1      1094.883       820.085       109.821    0.007    0.008    0.005
         2       503.891      1598.698       117.685    0.011    0.008    0.009
         3      2349.343       207.658       151.387    0.006    0.005    0.007
         4      1395.320      1348.853       215.261    0.005    0.008    0.009

CONTROL POINTS
-----------------------------------------------------------------------------
      NAME             X       VX             Y       VY              Z       VZ
-----------------------------------------------------------------------------
         1     10037.810    0.064      5262.090    0.037        772.040    0.001
         2     10956.680    0.025      5128.170   -0.057        783.000    0.011
         3      8780.080   -0.007      4840.290   -0.028        782.620    0.007
         4     10185.800   -0.033      4700.210    0.091        851.320   -0.024
```

EXAMPLE

Transformation Coefficients

Scale	=	0.94996	± 0.00004
Omega	=	2° 17' 05.3"	± 0° 00' 30.1"
Phi	=	-0° 33' 02.8"	± 0° 00' 09.7"
Kappa	=	224° 32' 10.9"	± 0° 00' 06.9"
Tx	=	10233.858	± 0.065
Ty	=	6549.981	± 0.071
Tz	=	720.897	± 0.213

Reference Standard Deviation: 8.663
Degrees of Freedom: 5
Iterations: 2

Transformed Coordinates

NAME	X	Sx	Y	Sy	Z	Sz
1	10037.874	0.078	5262.127	0.087	772.041	0.284
2	10956.705	0.085	5128.113	0.093	783.011	0.299
3	8780.073	0.103	4840.262	0.109	782.627	0.335
4	10185.767	0.090	4700.301	0.102	851.296	0.343
5	10722.020	0.073	5691.221	0.080	766.068	0.248
6	10043.246	0.072	5675.898	0.080	816.867	0.248

STATISTICALLY VALID PARAMETERS

The adjusted parameters divided by their standard deviation represents a *t statistic*. Thus the parameter checked for statistical significance.

That is: $$t = \frac{|\,parameter\,|}{S}$$

EXAMPLE

Assume results of two dimensional projective transformation with 2 degrees of freedom are:

Parameter	*S*	*t-value*
a = 25.37152	± 0.02532	1002
b = 0.82220	± 0.02256	36.4
c = -137.183	± 0.203	675.8
d = -0.80994	± 0.02335	34.7
e = 25.40166	± 0.02622	968.8
f = -150.723	± 0.216	697.8

Are these parameters statistically different from 0 at a 5% level of significance?

H_o: parameter = 0
H_a: parameter ≠ 0

Rejection Region is when *t-value* > $t_{\alpha/2, v}$

$$t\text{-}value > 4.303 = t_{\alpha/2, v} = t_{0.025, 2}$$

Yes, all parameters are statistically different from 0 since their *t-values* are greater than 4.303