TRAVERSE AND NETWORK
ADJUSTMENTS

Observation equations consists of same linearized observation equations used for distances, azimuths, and angles. Method of creating $J$ matrix same as previously shown. Only difference is that weighted adjustment should always be used due to difference between angle and distance observations.

Distances:
\[
\left( \frac{x_i - x_j}{IJ} \right)_o \ dx_i + \left( \frac{y_i - y_j}{IJ} \right)_o \ dy_i + \left( \frac{x_j - x_i}{IJ} \right)_o \ dx_j + \left( \frac{y_j - y_i}{IJ} \right)_o \ dy_j = k_{ij} + v_{ij}
\]

Angles:
\[
\left( \frac{y_i - y_b}{IB^2} \right)_o \ dx_b + \left( \frac{x_b - x_i}{IB^2} \right)_o \ dy_b + \left( \frac{y_b - y_i}{IB^2} \ - \frac{y_j - y_i}{IF^2} \right)_o \ dx_i + \left( \frac{y_j - y_i}{IF^2} \right)_o \ dy_i = k_{\theta_{bf}} + v_{\theta_{bf}}
\]

Directions:
\[
\left( \frac{y_i - y_j}{IJ_o^2} \right)_o \ dx_i + \left( \frac{x_j - x_i}{IJ_o^2} \right)_o \ dy_i + \left( \frac{y_j - y_i}{IJ_o^2} \right)_o \ dx_j + \left( \frac{x_i - x_j}{IJ_o^2} \right)_o \ dy_j = k_{Az_{ij}} + v_{Az_{ij}}
\]
NUMERICAL EXAMPLE

\[ x = 1223.00 \quad y = 1186.50 \]
\[ x = 1400.00 \quad y = 1186.50 \]

OBSERVATIONS

Distances
- RU = 200.00 ±0.05
- US = 100.00 ±0.08

Angles
- \( \theta_1 = 240^\circ 00' \pm 30'' \)
- \( \theta_2 = 150^\circ 00' \pm 30'' \)
- \( \theta_3 = 240^\circ 01' \pm 30'' \)
NUMERICAL EXAMPLE

1. Compute values for coordinates of Station $U$.

\[
x_{u_o} = 1000 + 200 \sin(60^\circ) = 1173.20
\]

\[
y_{u_o} = 1000 + 200 \cos(60^\circ) = 1100.00
\]

2. Compute observations from coordinates.

\[
RU = 200.00 \quad \quad \quad US = 99.91
\]

\[
\angle QRU = 240^\circ 00' 00"
\]

\[
\angle RUS = 149^\circ 55' 51"
\]

\[
\angle UST = 240^\circ 04' 12"
\]
NUMERICAL EXAMPLE

3. Formulate matrices:

<table>
<thead>
<tr>
<th>Obs.</th>
<th>Subscript subs</th>
</tr>
</thead>
<tbody>
<tr>
<td>RU</td>
<td>$R = i, U = j$</td>
</tr>
<tr>
<td>US</td>
<td>$U = i, S = j$</td>
</tr>
<tr>
<td>QRU</td>
<td>$Q = b, R = i, U = f$</td>
</tr>
<tr>
<td>RUS</td>
<td>$R = b, U = i, S = f$</td>
</tr>
<tr>
<td>UST</td>
<td>$U = b, S = i, T = f$</td>
</tr>
</tbody>
</table>

$$K = \begin{bmatrix}
    k_{l_{ru}} & 200.00 \text{ ft} - 200.00 \text{ ft} \\
    k_{l_{us}} & 100.00 \text{ ft} - 99.91 \text{ ft} \\
    k_{\theta_1} & 240^{\circ}00'00'' - 240^{\circ}00'00'' \\
    k_{\theta_2} & 150^{\circ}00'00'' - 149^{\circ}55'51'' \\
    k_{\theta_3} & 240^{\circ}01'00'' - 240^{\circ}04'12''
\end{bmatrix} = \begin{bmatrix}
    0.00 \text{ ft} \\
    0.19 \text{ ft} \\
    0'' \\
    249'' \\
    -192''
\end{bmatrix}$$
NUMERICAL EXAMPLE

\[
J = \begin{bmatrix}
\left(\frac{1173.20-1000.00}{200.00}\right) & \left(\frac{1100.00-1000.00}{200.00}\right) \\
\left(\frac{1173.20-1223.00}{99.81}\right) & \left(\frac{1100.00-1186.50}{99.81}\right) \\
\left(\frac{1100.00-1000.00}{(200.00)^2}\right) & \left(\frac{1000.00-1173.20}{(200.00)^2}\right) \\
\left(\frac{1000.00-1100.00 - 1186.50-1100.00}{(200.00)^2 - (99.81)^2}\right) & \left(\frac{1173.20-1000.00}{(200.00)^2 - (99.81)^2}\right) \\
\left(\frac{1186.50-1100.00}{(99.81)^2}\right) & \left(\frac{1173.20-1223.00}{(99.81)^2}\right)
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
0.866 & 0.500 \\
-0.499 & -0.867 \\
515.7 & -893.2 \\
-2306.6 & 1924.2 \\
1790.9 & -1031.1
\end{bmatrix}
\]
NUMERICAL EXAMPLE

4. Set the stochastic model, (determine the weights)

**distances:** \( w_{l_{ij}} = \frac{1}{S_{l_{ij}}^2} \)

**angles:** \( w_{\theta_{bof}} = \frac{1}{S_{\theta_{bof}}^2} \)

\[
W = \begin{bmatrix}
\frac{1}{(0.05)^2} & & & \\
& \frac{1}{(0.08)^2} & & \\
& & \frac{1}{(30)^2} & \\
& & & \frac{1}{(30)^2}
\end{bmatrix}
= \begin{bmatrix}
400.00 & & \\
& 156.2 & & \\
& & 0.0011 & \\
& & & 0.0011
\end{bmatrix}
\]
NUMERICAL EXAMPLE

5. Solve for most probable solution using weighted least squares adjustment.

\[ X = (J^T W J)^{-1} J^T W K \]

\[ X = \begin{bmatrix} -0.11 \\ -0.01 \end{bmatrix} \]

SECOND ITERATION:

1. Update coordinates.

\[ x_u = 1173.20 - 0.11 = 1173.09 \]
\[ y_u = 1100.00 - 0.01 = 1099.99 \]

This iteration produced negligible corrections.
# TABULATE POST-ADJUSTMENT STATISTICS

## Final Iteration

<table>
<thead>
<tr>
<th>J</th>
<th>K</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.86591</td>
<td>0.50020</td>
<td>0.10723</td>
</tr>
<tr>
<td>-0.49972</td>
<td>-0.86619</td>
<td>0.12203</td>
</tr>
<tr>
<td>516.14929</td>
<td>-893.51028</td>
<td>48.62499</td>
</tr>
<tr>
<td>-2304.96717</td>
<td>1925.52297</td>
<td>17.26820</td>
</tr>
<tr>
<td>1788.81788</td>
<td>-1032.01269</td>
<td>-5.89319</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>Qxx</th>
</tr>
</thead>
<tbody>
<tr>
<td>10093.552</td>
<td>-7254.153057</td>
</tr>
<tr>
<td>-7254.153</td>
<td>6407.367420</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>J</th>
<th>Qxx J(^T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001130</td>
<td>-0.001195</td>
</tr>
<tr>
<td>-0.001195</td>
<td>0.001282</td>
</tr>
<tr>
<td>-0.447052</td>
<td>0.510776</td>
</tr>
<tr>
<td>0.055151</td>
<td>-0.161934</td>
</tr>
<tr>
<td>0.391901</td>
<td>-0.348843</td>
</tr>
</tbody>
</table>

PLATE 15-8
# TABULATE POST-ADJUSTMENT STATISTICS

## Adjusted Stations

<table>
<thead>
<tr>
<th>Station</th>
<th>X</th>
<th>Y</th>
<th>Sx</th>
<th>Sy</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1,173.09</td>
<td>1,099.99</td>
<td>0.042</td>
<td>0.053</td>
</tr>
</tbody>
</table>

## Adjusted Distance Observations

<table>
<thead>
<tr>
<th>Station Occupied</th>
<th>Station Sighted</th>
<th>Distance</th>
<th>V</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>U</td>
<td>199.89</td>
<td>-0.11</td>
<td>0.061</td>
</tr>
<tr>
<td>U</td>
<td>S</td>
<td>99.88</td>
<td>-0.12</td>
<td>0.065</td>
</tr>
</tbody>
</table>

## Adjusted Angle Observations

<table>
<thead>
<tr>
<th>Station Backsight</th>
<th>Station Occupied</th>
<th>Station Foresight</th>
<th>Angle</th>
<th>V</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>R</td>
<td>U</td>
<td>239° 59' 11&quot;</td>
<td>-49&quot;</td>
<td>29.0&quot;</td>
</tr>
<tr>
<td>R</td>
<td>U</td>
<td>S</td>
<td>149° 59' 43&quot;</td>
<td>-17&quot;</td>
<td>44.1&quot;</td>
</tr>
<tr>
<td>U</td>
<td>S</td>
<td>T</td>
<td>240° 01' 06&quot;</td>
<td>6&quot;</td>
<td>35.0&quot;</td>
</tr>
</tbody>
</table>

Reference Standard Deviation = ±1.8
MINIMUM AMOUNT OF CONTROL

Every horizontal survey must have:

- One point fixed in \((x, y)\) coordinates to positionally fix the network in space.

- One course fixed in azimuth to rotationally fix the network in space.

- More than this amount results in an over-constrained adjustment. That is there is more control than is necessary to do the adjustment.
PREPARING DATA FOR AN ADJUSTMENT

A. COMPUTE INITIAL STATION COORDS AND REDUCE DATA TO SPCS GRID

B. CHECK DATA FOR LARGE ERRORS
   1. Large errors in data can be caused by:
      a) measurement,
      b) recording,
      c) entry
   2. Large errors in data will prevent adjustment from converging.
GOODNESS OF FIT TEST

At the completion of a weighted least squares adjustment, where weighing is determined by:

\[ w_i = \frac{S_o^2}{S_i^2} \]

A \( \chi^2 \) can be done on the final reference variance of the adjustment. Since the final value is an estimate for the \( a \ priori \) (preceding the adjustment) value.

That is, if the weights for the distance and angle observations are:

\[ w_{dist} = \frac{1}{S_{dist}^2} \]
\[ w_\perp = \frac{1}{S_\perp^2} \]

Then the a priori value for the reference variance is 1 and its computed value is \( S_o^2 \)
GOODNESS OF FIT TEST

Hypotheses:

\[ H_0: \ S^2 = 1 \]
\[ H_a: \ S^2 \neq 1 \]

Test Statistic:

\[ \chi^2 = \frac{v \ S^2}{\sigma^2} \]

Rejection Region:

\[ \chi^2 > \chi^2_{a/2,v} \]
GOODNESS OF FIT TEST

Traverse Example:

\[ S_o^2 = 3.31 \text{ with } v = 3 \text{ using an } \alpha = 0.05 \]

Hypothesis:

\[ \begin{align*}
\text{H}_0 & : S^2 = 1 \\
\text{H}_a & : S^2 \neq 1
\end{align*} \]

Test Statistic:

\[ \chi^2 = \frac{3 \times 3.31}{1} = 9.93 \]

Rejection Region:

\[ \chi^2 = 9.93 > 9.35 = \chi^2_{0.025,3} \]

Rejection region satisfied so there is reason to believe that either data contains a blunder, stochastic model is wrong, systematic errors are present in data, or control is in error at a 95% level of confidence.