

# Logistic Growth Demonstration

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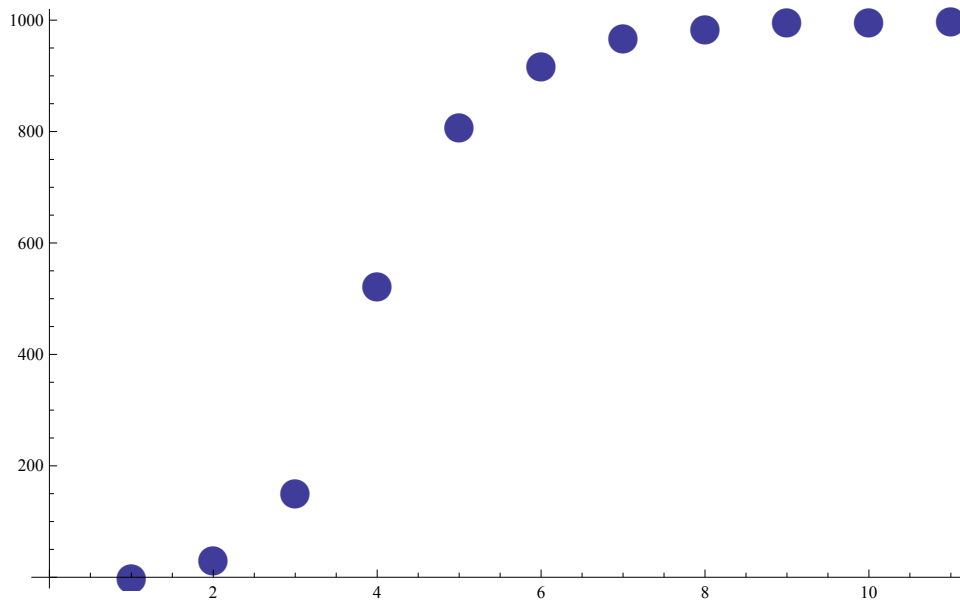
## Simulating an epidemic

```
Clear[n, population, L, uninfected, infected, epidemic];
epidemic[n_, p_, n0_: 1] := Module[{population, L, uninfected},
  population = Join[Table[1, {n0}], Table[0, {n - n0}]];
  L = {};
  While[Total[population] < Length[population],
    L = Append[L, Total[population]];
    uninfected = Position[population, 0];
    (*infected=Complement[Range[25], uninfected];*)
    For[i = 1, i ≤ Total[population] (*Length[infected]*), i++,
      For[j = 1, j ≤ Length[uninfected], j++,
        If[Random[] ≤ p,
          population[[uninfected[[j]]]] = 1;
        ];
      ];
    ];
  L = Append[L, Total[population]];
  L
]

P = 1000;

data = epidemic[P, 0.001, 1];
```

```
ListPlot[data, ImageSize → 500, PlotMarkers → {Automatic, Large}]
```



## Qualitative fit of Logistic Growth model

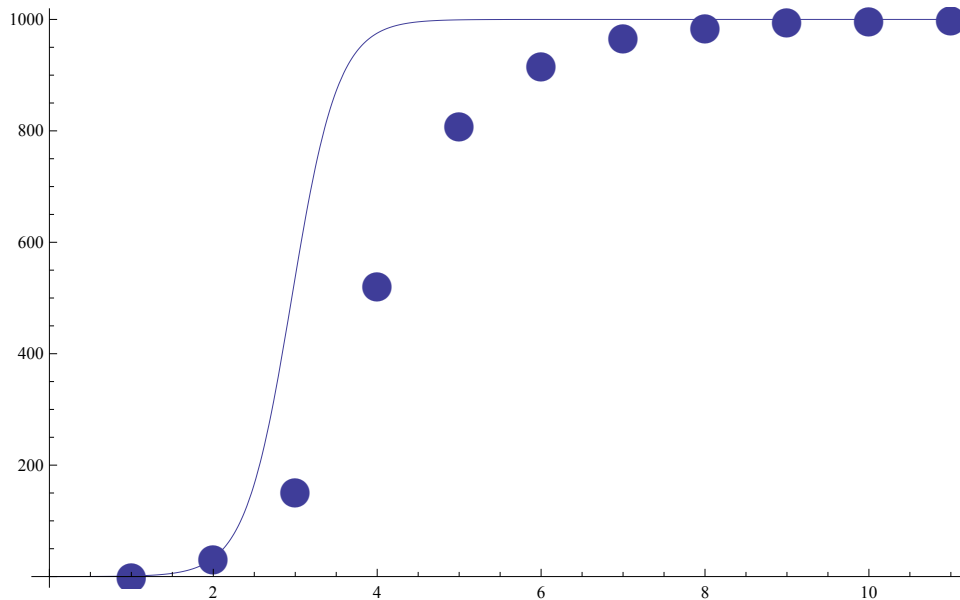
```
Manipulate[Show[ListPlot[data, ImageSize → 500, PlotMarkers → {Automatic, Large}],
  Plot[ $\frac{P}{1 + b e^{-k t}}$ , {t, 0, Length[data]}]], {{b, P}, 0, P}, {{k, 1}, 0, 2}]
```

$$f[t_] := \frac{P}{1 + b e^{-k t}};$$

```
params = NSolve[{f[1] == data[[1]], f[2] == data[[2]]}, {b, k}, Reals][[1]]
```

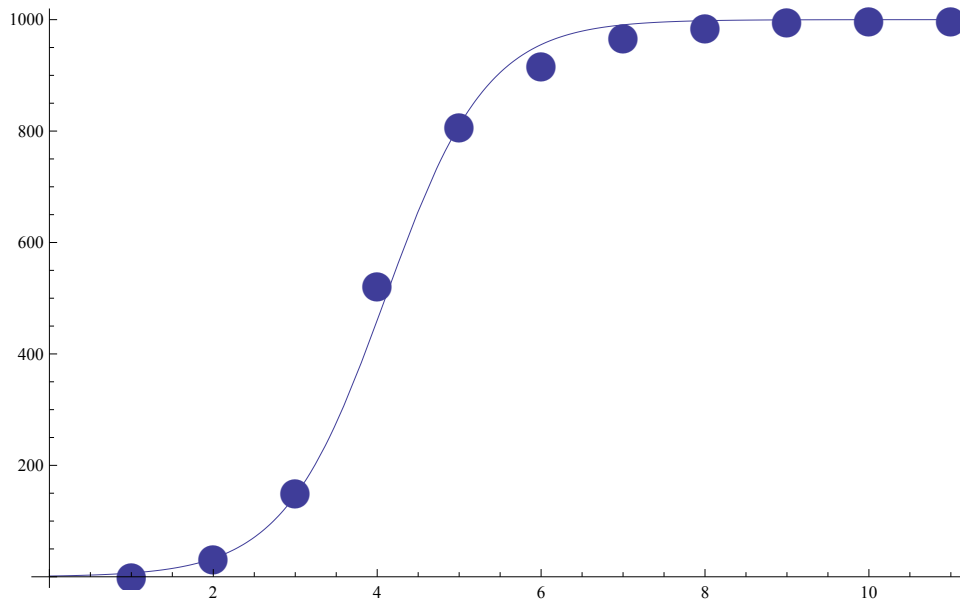
```
{b → 34 057.9, k → 3.52906}
```

```
Show[ListPlot[data, ImageSize -> 500, PlotMarkers -> {Automatic, Large}],
Plot[f[t] /. params, {t, 0, Length[data]}]]
```



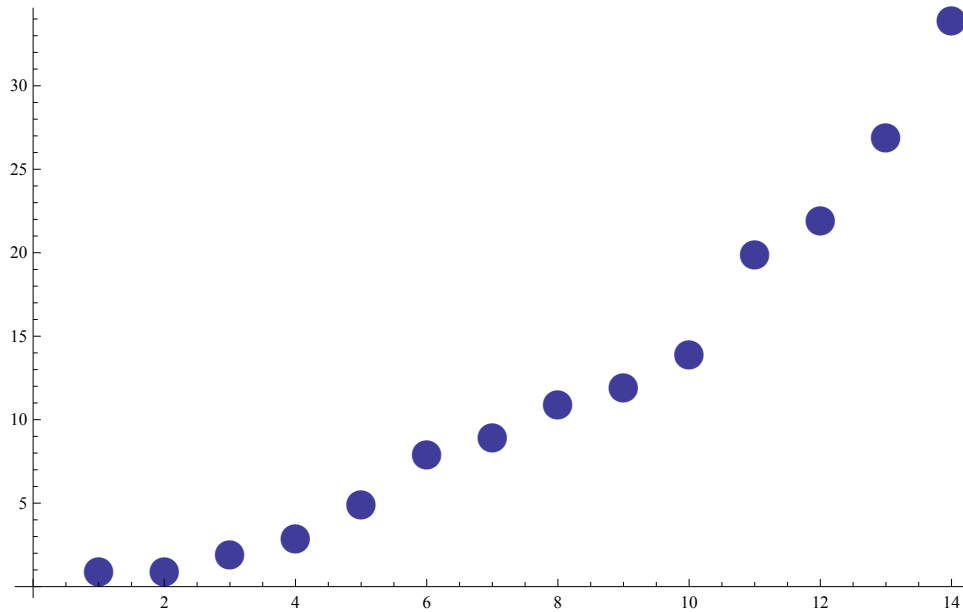
```
params = NSolve[{f[2] == data[[2]], f[5] == data[[5]]}, {b, k}, Reals][[1]]
{b -> 732.277, k -> 1.60923}
```

```
Show[ListPlot[data, ImageSize -> 500, PlotMarkers -> {Automatic, Large}],
Plot[f[t] /. params, {t, 0, Length[data]}]]
```



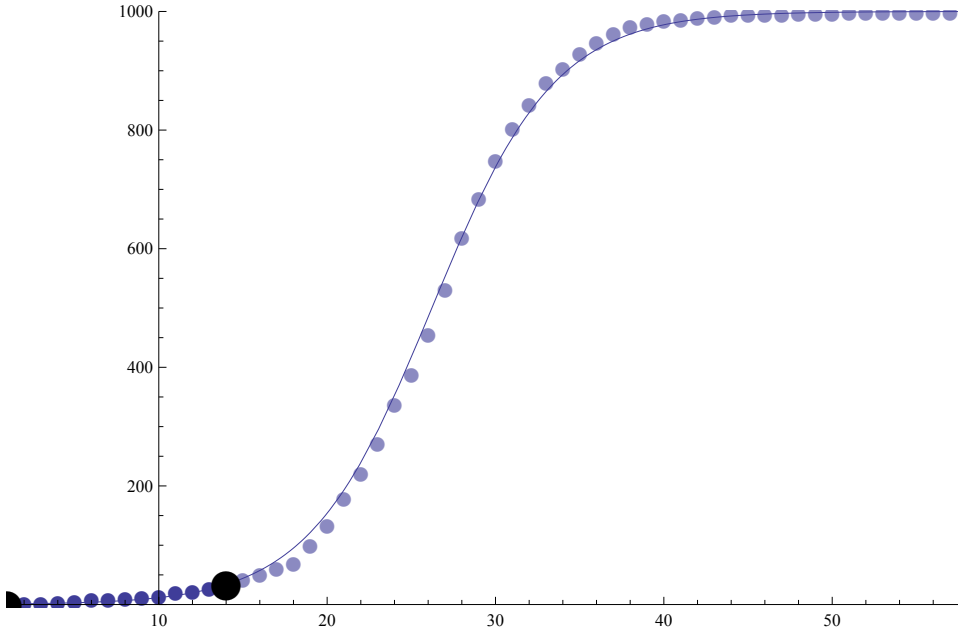
## Forecasting Growth

```
newdata = epidemic[P, 0.00025, 1];  
T = Floor[Length[newdata] / 4];  
ListPlot[newdata[[1 ;; T]], ImageSize -> 500, PlotMarkers -> {Automatic, Large}]
```



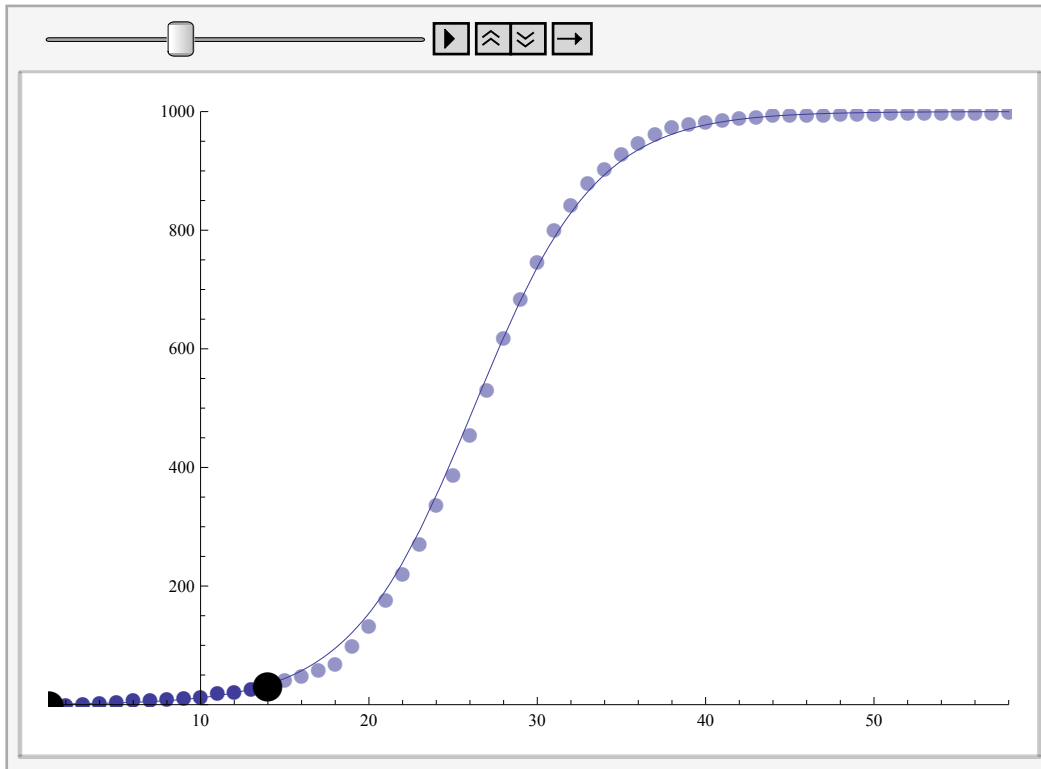
```
params = NSolve[{f[1] == newdata[[1]], f[T] == newdata[[T]]}, {b, k}, Reals]  
{b -> 1313.69, k -> 0.273842}
```

```
Show[With[
  {params = NSolve[{f[1] == newdata[[1]], f[T] == newdata[[T]]}, {b, k}, Reals][[1]]},
  Plot[f[t] /. params, {t, 1, Length[newdata]},
    PlotRange -> {{1, Length[newdata]}, {0, P}}, ImageSize -> 500]],
  ListPlot[Table[{i, newdata[[i]]}, {i, 2, T - 1}], PlotMarkers -> {Automatic, Medium}],
  ListPlot[Table[{i, newdata[[i]]}, {i, T + 1, Length[newdata]}],
    PlotStyle -> Directive[Opacity[0.55]], PlotMarkers -> {Automatic, Medium}],
  ListPlot[{{1, newdata[[1]]}, {T, newdata[[T]]}], PlotStyle -> Black,
    PlotMarkers -> {Automatic, Large}], ImageSize -> 500]
```



```
ani = Table[
  Show[With[{params =
    NSolve[{f[1] == newdata[[1]], f[T] == newdata[[T]]}, {b, k}, Reals][[1]]},
    Plot[f[t] /. params, {t, 1, Length[newdata]}, PlotRange ->
      {{1, Length[newdata]}, {0, P}}, ImageSize -> 500]],
    ListPlot[Table[{i, newdata[[i]]}, {i, 2, Max[2, T - 1]}],
      PlotMarkers -> {Automatic, Medium}],
    ListPlot[Table[{i, newdata[[i]]}, {i, T + 1, Length[newdata]}],
      PlotStyle -> Directive[Opacity[0.55]], PlotMarkers -> {Automatic, Medium}],
    ListPlot[{{1, newdata[[1]]}, {T, newdata[[T]]}], PlotStyle -> Black,
      PlotMarkers -> {Automatic, Large}], ImageSize -> 500],
  {T, 2, Floor[2 Length[newdata] / 3]}] // Quiet;
```

ListAnimate[ani]



## Exercises

1. Your marketing company starts a social media advertising campaign directed towards UF's student body. They kick off the campaign on July 1st, 2014, and hire 80 "seed" reviewers to share the advertisement and get things going. One week later, your market research department determines that only 200 students, including the seed reviewers, have shared the advertisement in total. Your boss wants to know how long it will be until 75% of the student body knows about your product. What do you tell him?
  - Note that this question may require you to look up additional information not provided in the question. You may also have to engage in some guesswork to make your numbers reasonable. Be sure to include justification for your each of your estimates in your report, as if I was really your boss. Think about what you would do if you were really given this task in a professional environment.
2. Suppose that after the advertising campaign is finished, you find that your prediction was *completely* off-base. What are some reasons why the logistic growth model would fail in this situation? Name at least three, and discuss how you might adapt your forecasting technique to compensate for these problems in the next advertising campaign.