

Logistic Growth (and other basics of applied differential equations):

We have approximately two "good" ways of modeling populations:

1. "limited growth" $\frac{dy}{dt} = k(N-y)$

2. "logistic growth" $\frac{dy}{dt} = k \left(1 - \frac{y}{N}\right) y$

In both equations, $y(t)$ is the population as a function of time t , N is the maximum value of the population, which we call the "carrying capacity", and k is a constant

P1. Solve both differential equations for $y(t)$.

P2. Graph each solution using your calculator, or Wolfram Alpha. When the book solves the logistic growth equation, it finds it useful to define $b = \frac{N-y_0}{y_0}$, where y_0 is the initial

size of a population.

P3. Interpret b . What does it mean if b is low? High? What's the highest and lowest b can be?

P4. What is the point of inflection on the logistic curve? (Show that this is true, don't just give the answer.)

P5: What does this inflection point mean, in English words?

P6. How does b affect this inflection point? Does that make sense intuitively?

Suppose that you get some data from your company comparing worker salaries to the percentage of their time that they are actually doing something useful.

P7. Which differential equation would you use to model the data? Why? Write the equation and your rationale.

P8. In general, when would Logistic growth be a more natural model for a real world situation than Limited Growth?

The book says that Logistic growth is related, in some way, to the equation $q(t) = \frac{m}{1 + (\frac{m}{q_0})e^{-kmt}}$

and "its derivatives."

P9. What are that function's derivatives? Determine an expression for the n^{th} derivative of the logistic growth function.

Here is a short list of real world situations:

1. the spread of word of mouth advertising over a large population
2. the spread of word of mouth advertising over a small population
3. a worker saving his paychecks in a bank
4. investors jumping onto a hot new stock
5. a student learning calculus, gaining math skills

P10. In each situation, what would be the best choice of differential equation to make a mathematical model, and why?

P11. Name three situations that arise in business, finance, economics, accounting, etc. that could be modeled using limited growth.

P12: Same question as P11, for logistic growth.



Additional book problems

• for mechanical practice:

new book: p.141 2, 3, 5, 8, 11, 14, 16, 18, 21, 22, 29, 32, 33

old book: p.617 12, 21 (and solve it),

p.623 1, 3, 5, 8, 11, 12, 13, 14, 15

19, 20, 23

Here are some more practice problems

from the book. I recommend doing them

after you finish P1-P10

of mine, as mine will

help you understand theirs better. As usual, I recommend

doing all of them, but

I've circled particularly

important ones in red

pen. This isn't to say

you should only do those -

just don't skip them.

• for interpretive practice:

new book: 35, 36, 40 (as far as you can

get with your equipment), 47 (same), 53

old book: p.632 3, 4, 6 (show the

"it can be shown" too), 10, 11,

15, 21

One last thing - the old book has a much better introduction chapter for Diffy Qs, in some ways, but it also includes a lot more irrelevant content, intended for engineers or life science people (or masochists). (Nobody cares about the salinity levels in cylindrical tanks of water in this class.) To get the best out of the old book, you should

- read p. 612 fully
- skip to "solutions of diffy Q's" on 613, read + work the examples on 614 and 615
- engrain the graphs of Figure 5 (p. 615) and the explore and discuss (p. 616) deeply into your mind ←
- read 619 and example 1
- fully understand example 3 (p. 621-622) ←
- understand the terminology "unrestricted growth" (p. 625) "restricted growth" (p. 626) and logistic growth (from earlier in this homework)

I would also recommend the users of the new book take a look at the two bullets above that have red arrows next to them. (I have scanned the corresponding pages into Resources.)