Optional Homework #2

May 31, 2014

Note: For these problems, you'll need to use the Internet to look up certain physical quantities. While doing so, make sure you stick to one consistent set of units, or your numbers won't turn out right. Personally, I chose pounds, miles, and hours. In these units, the Gravitational constant is

$$G = 9.41 \times 10^{-14} \frac{\text{mi}^3}{\text{lb} \cdot \text{hr}^2}.$$

Just search for stuff like "weight of the Earth in pounds" on Google and it will tell you.

Escape Velocity. The gravitational force exerted between two objects is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the universal gravitational constant, m_1 and m_2 are the masses of the objects, and r is the distance between them. In this problem, we will be considering the case where m_1 is the mass of some object and m_2 is the mass of the Earth. We'll write r_0 for the distance from the center of the Earth to its surface.

(a) When an object is lifted away from the Earth's surface, the work done (i.e. the *energy required* to move the object that much) is given by integrating the force of gravity with respect to the distance moved.

Write an improper integral to calculate the work required to move an object weighing m_1 units from the Earth's surface to a point completely out of reach of (or *"very far away"* from) Earth's gravity.

(b) I recently weighed in at 190lb. (Still 10 away from heavyweight, sigh.) How much energy would be required to launch me into space?

(c) The *kinetic energy* of a moving object is given by $\frac{1}{2}mv^2$, where m is the mass of the object and v is its velocity. To escape the Earth's gravitational pull, an object must be traveling at a sufficient velocity when it leaves the Earth's surface so that its kinetic energy is at least equal to the energy in part (a). What is the expression for that velocity?

(d) So how fast would I have to be going to leave this planet?

(e) Does it matter how much the object weighs in part (c)?

(f) Could Aroldis Chapman, pitcher for the Cincinnati Reds, throw a fastball into space from the surface of the moon, never to return?

(g) What is my escape velocity? (The velocity you'd need to escape my gravitational field.)

(h) A black hole is an object with an escape velocity that exceeds the speed of light. How much weight would I have to gain to become a black hole?

(i) Is it true that black holes have to weigh a lot? How small would I have to be squished if I wanted to be a black hole?

(j) **Financial escape velocity.** Can you think of a situation in finance, economics, or business that is analagous to escape velocity? How would you formulate that mathematically?