MAC2234: Quiz 4 Solutions

June 11, 2014

P1	P2	Q4

P1. Evaluate

$$\int_0^3 \int_{\frac{1}{3}x}^1 \frac{1}{y^2+3} dydx$$

$$\begin{aligned}
\int_0^3 \int_{\frac{1}{3}x}^1 \frac{1}{y^2+3} dydx &= \int_0^1 \int_0^{3y} \frac{1}{y^2+3} dxdy \\
&= \int_0^1 \int_0^{3y} \frac{1}{y^2+3} dxdy \\
&= \int_0^1 \left. \frac{x}{y^2+3} \right|_0^{3y} dy \\
&= \int_0^1 \frac{3y}{y^2+3} - \frac{0}{y^2+3} dy \\
&= 3 \int_0^1 \frac{y}{y^2+3} dy \\
&= \frac{3}{2} \int \frac{1}{u} du \\
&= \frac{3}{2} \ln(u) \\
&= \frac{3}{2} \left. \ln(y^2+3) \right|_0^1 \\
&= \frac{3}{2} (\ln(4) - \ln(3)) \\
&= \frac{3}{2} \ln\left(\frac{4}{3}\right)
\end{aligned}$$

P2. The manufacturing cost of a conical canvas tent can be approximated by its surface area,

$$S(r,h) = \pi r\left(r + \sqrt{r^2+h^2}\right)$$

where r is the radius of the base of the tent and h is its height. Approximate the cost to manufacture a 4.083 foot tall tent that has a base radius of 2.833.

First let's make our total differential.

$$\begin{aligned}
S_r(r,h) &= \pi\left(r+\sqrt{r^2+h^2}\right) + \pi r\left(1+\frac{r}{\sqrt{r^2+h^2}}\right) \\
&= \frac{\pi\left(\sqrt{r^2+h^2}+r\right)^2}{\sqrt{r^2+h^2}} \\
S_h(r,h) &= \frac{\pi r h}{\sqrt{r^2+h^2}} \\
dS &= S_r(r,h)\,dr + S_h(r,h)\,dh \\
&= \frac{\pi\left(\sqrt{r^2+h^2}+r\right)^2}{\sqrt{r^2+h^2}}\,dr + \frac{\pi r h}{\sqrt{r^2+h^2}}\,dh
\end{aligned}$$

Now, you should notice is that 4.083 is pretty close to 4 and 2.833 is pretty close to 3. (In fact, I chose these numbers because .083 feet is very close to 1/12 of a foot, otherwise known as one inch.) These are the most natural numbers to choose for the approximation. That gives us $dr = -0.167$ and $dh = 0.083$. We have

$$S(2.833, 4.083) \approx S(3,4) + dS$$

Now we calculate $S(3,4)$.

$$\begin{aligned}
S(3,4) &= \pi\times 3\times\left(3+\sqrt{3^2+4^2}\right) \\
&= \pi\times 3\times(3+5) \\
&= 24\pi \\
&\approx 75.3982
\end{aligned}$$

And finally we calculate

$$\begin{aligned}
dS &= \frac{\pi\left(\sqrt{r^2+h^2}+r\right)^2}{\sqrt{r^2+h^2}}\,dr + \frac{\pi r h}{\sqrt{r^2+h^2}}\,dh \\
&= \frac{\pi\left(\sqrt{3^2+4^2}+3\right)^2}{\sqrt{3^2+4^2}}\times(-0.167) + \frac{\pi\times 3\times 4}{\sqrt{3^2+4^2}}\times(0.083) \\
&= \frac{\pi\,(5+3)^2}{5}\times(-0.167) + \frac{12\pi}{5}\times(0.083) \\
&\approx 75.398 - 6.090 \\
&\approx 69.308
\end{aligned}$$