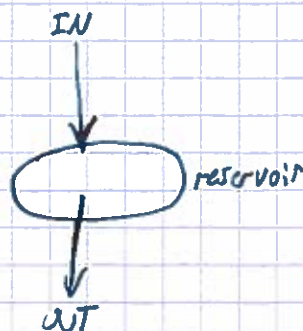


Concentration Problems

The math behind the salt concentration examples (such as Ex. 4 on p. 212 of the new book, or problems 18-24 on p. 215 of the new book) is, actually, very important to business and finance. However, I couldn't think of a less interesting example than calculating salt concentrations, or one that appears less relevant to our class. Let's make it more "business-y." The basic idea is this:

- you have a rate of "stuff" going into a reservoir and a rate of "stuff" leaving the reservoir.

You want to calculate the amount of "stuff" inside the reservoir at any given time.



- The catch is, the "in" and "out" rates can depend not only on time, but on the amount of "stuff" in the reservoir as well.

My favorite idea is to think of the reservoir as a bank account, the "in" rate as an income stream, and the "out" rate as a rate of expenses. Here's an example:

EX. To "reinvest money continuously at a rate r " means, mathematically, that

$$\frac{dW}{dt} = rW$$

where $W(t)$ is the amount of wealth in the company account at a time t . Suppose that all we do is reinvest, and watch an initial balance grow as it accrues interest. If the initial balance is B , we have

$$\begin{cases} \frac{dW}{dt} = rW \\ W(0) = B \end{cases} \longrightarrow \int \frac{dW}{W} = \int r dt \longrightarrow \ln W = rt + C \longrightarrow e^{\ln W} = e^{rt} e^C \longrightarrow W(t) = Ae^{rt}$$

and since $W(0) = B$, thus $W(t) = Be^{rt}$

PROBLEM. Suppose that we earn a revenue of $R(t)$ and reinvest our money continuously at a rate r at the same time. Derive a formula for $W(t)$ in a similar way to the above example. Assume that $W(0) = B$, if needed.

(Hint: your income stream will be the sum of $R(t)$ with the money earned by interest from reinvesting. You will need to use the integrating factor method.)

Does your solution look familiar?