

Final Exam

MAC2234: Survey of Calculus II

Thursday, August 7th, 2014.

Score			
P1	/ 5	P6	/ 5
P2	/ 5	P7	/ 5
P3	/ 5	P8	/ 5
P4	/ 5	P9	/ 5
P5	/ 5	P10	/ 5
		EX3	/ 50

Question 1. (*Techniques of Single Variable Integration*)

In 2010, Transocean's Deepwater Horizon oil rig exploded fifty miles off the coast of Louisiana, leaking hundreds of thousands of gallons of crude oil into the surrounding waters. According to BP's corporate memos, the Horizon was expelling oil into the water at a rate of 210 000 gallons per day on the morning of April 28th. One week later, a containment dome was lowered over the wreckage, and the leak was subsequently measured at 208 329 gallons of oil per day.

We will model the spill by representing the rate of pollution with the function

$$r'(t) = ae^{-bt}.$$

Use the data above to find the constants a and b . Then, write an improper integral to compute the total amount of oil that will enter the Gulf of Mexico as a result of the explosion.

Question 2. (*Numerical Integration*)

Approximate $\int_{-1}^1 f(x)dx$ to the nearest 0.1, where

$$f(x) = e^{-e^{-x^2}}.$$

You may use that $|f^{(4)}(x)| \leq 1.8$ on $[-1, 1]$.

Question 3. (*Multivariable Optimization*)

An aluminum can must hold 125cm^3 of yellow asparagus. Aluminum costs roughly 5 cents per square centimeter. The asparagus itself costs 55 cents per cubic centimeter. What is the minimum possible cost to produce one unit of canned yellow asparagus?

Question 4. (*Total Differentials and Double Integrals*)

A forensic economist is analyzing the productivity of the military sector in a foreign regime. He uses a Cobb-Douglas model, given by

$$\mathcal{P}(x, y) = 6x^{0.35}y^{0.65}$$

where x represents the amount of capital (in millions of dollars) and y represents the amount of labor (in thousands of active duty soldiers).

- (a) The economist believes that the amount of capital being spent is 44 million dollars, and he knows that this is true to within an maximum possible error of one million dollars. His contacts tell him that there are around 20 thousand active duty soldiers in the military, and they are sure that this figure can't be off by more than 20%. What is the maximum possible error in the economist's productivity estimate?
- (b) Find the average level of production if the capital varies between 40 and 50 million dollars and the labor varies between 10 and 25 thousand active duty soldiers. Recall that the average of a multivariable function $f(x, y)$ over a region \mathcal{R} with area A is given by

$$\frac{1}{A} \iint_{\mathcal{R}} f(x, y) \, dx \, dy.$$

Question 5. (*Elementary Differential Equations*)

Find the general solution to the following differential equation.

$$y(t) = \frac{1-t}{t} \frac{dy}{dt} - t^2$$

Question 6. (*Numerical Differential Equations*)

Using 6 subintervals, adapt Euler's method to approximate $y(t)$ on the interval $[-3, 3]$, given that

$$\frac{dy}{dt} = \frac{1}{9 + (y - t)^2} \quad \text{and} \quad y(0) = 0.$$

Keep a working precision of 0.01. Include a table of values, and sketch the graph.

Question 7. (*Probability*)

Let

$$f(x) = \frac{k e^{2x}}{(e^{2x} + 1)^2}.$$

- (a) For which value of k is $f(x)$ a PDF on the interval $[-\infty, \infty]$?
- (b) Using the value of k found in part (a), find the expected value of $f(x)$ on $[0, \infty]$.
- (c) Approximate the variance of $f(x)$ on the interval $[-2, 2]$ using Simpson's rule with 4 subintervals.

Question 8. (*Matrices and Systems of Linear Equations*)

Which \bar{x} satisfies the following equation?

$$\begin{pmatrix} 0 \\ 2 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} -4 \\ 2 \\ -4 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix} \bar{x} = \begin{pmatrix} 1 \\ 3 \\ 2 \\ 1 \end{pmatrix}$$

Question 9. (*Inverse Matrices and Matrix Algebra*)

One way to encrypt a secret message is to use a *matrix cypher*. It works like this: first, you change the characters in your message into a list of numbers (1 for A, 2 for B, etc., and 27 for a space). Then, split your list up into blocks of four numbers. Apply an invertible matrix A to each of the blocks, treating them like vectors. Finally, put all the numbers back in order, and send the message to your friend. When your friend receives the encrypted message, he does the process in reverse, using A^{-1} to recover the original vectors and read the secret message. You can find an example very similar to this on page 383 of the new book, though they use blocks of three letters instead of four.

I have an important message for you! It's

78, -21, -42, 81, 104, 59, -98, 109, 97, 25, -82, 96.

I encoded it using the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & -2 & 0 & 1 \\ -2 & 0 & -2 & 0 \\ 0 & 1 & 2 & 2 \end{pmatrix}$$

What does my message say? Decode it, then write me something back.

Question 10. (*Linear Programming*)

World champion bodybuilder Boris Potapov feels small. After weighing in at only 250lbs, he decides to go on a bulk. To maximize his gains, he must eat daily totals of at least 1.5g of protein per pound of bodyweight (so, 375g) and 2.5g of carbs per pound of bodyweight (so, 625g). He will only eat two foods:

	Protein	Carbs	Price
Haggis	39.9	30.3	23
Beets and Goat Cheese	18.3	33.6	17

Boris isn't a rich man, and he wants to minimize the cost of what he eats. How much of each food should he eat each day? *Note: you may use either the graphical method or the simplex method here, depending on which you are more comfortable with.*