

Midterm Exam 2

MAC2234: Survey of Calculus II

Tuesday, July 15th, 2014.

Score			
P1	/ 5		
P2	/ 5	P5	/ 5
P3	/ 5	P6	/ 5
P4	/ 5	P7	/ 5
		EX2	/ 35

Question 1. (*Techniques of Single Variable Integration*)

Evaluate

$$\int_e^{e^2} \frac{\ln(\ln x) \ln x}{x} dx$$

Question 2. (*Numerical Integration*)

The standard normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

Give a numerical approximation of $\mathcal{P}(-2 \leq X \leq 2)$. Use 8 subintervals.

Question 3. (*Multivariable Optimization*)

You're building a custom made yurt shaped like a rectangular prism. The floor of the yurt must be made of oak, which costs \$2 per square foot. The frame (i.e. the remaining edges of the prism) must be made of bamboo poles, which are \$7 per foot. The walls and ceiling of the yurt must be made of canvas tarp, which costs 50 cents per square foot. The customer gives you a lump sum of \$750 and asks you to build the most spacious yurt you can. What dimensions will yield the yurt with the greatest volume?

Question 4. (*Total Differentials and Double Integrals*)

Kanye West has decided that when he dies he wishes to be buried in a solid gold, cone-shaped mausoleum, which is to be built in Giza and referred to as "the Greatest Pyramid." He insists that the building be 148 feet meters tall (one meter taller than the Great Pyramid) with a base radius of 231 meters (one meter wider than the base of the Great Pyramid). The contractor he hires measures each of these lengths with a maximum error of five centimeters. Estimate the maximum error in calculating the surface area of Kanye West's tomb.

Note: Because I am a nice guy, here is the formula for the surface area of a cone.

$$SA(r, h) = \pi r \left(r + \sqrt{h^2 + r^2} \right)$$

Question 5. (*Elementary Differential Equations*)

The Gompertz equation is a population growth model similar to logistic growth. A group of statisticians famously used its solution in the early 2000s to forecast the rapid expansion of the cell phone industry, in a study whose predictions remain highly accurate to this day. The Gompertz equation is also used in medicine to analyze the growth of tumors.

Anyway, here it is:

$$\frac{dy}{dt} = ky \ln\left(\frac{M}{y}\right)$$

Find the general solution to the Gompertz equation and discuss its long term behavior.

Question 6. (*Numerical Differential Equations*)

Question 7. (*Probability*)

GE aviation has determined that their airplane turbines have a probability of failure given by

$$f(t) = ae^{-at} \quad , \quad \text{where } a = 0.0588235$$

and t is in years. What is the expected lifespan of an airplane turbine? What is the median lifespan?