MAC2234: Exam 1 Review

June 06, 2014

Topics	N1	C1	C2	N2

These problems are intended to be significantly more difficult than the book homework. You will find most (though not all) to be resistant to blind application of the techniques you have learned. They are designed to test your understanding of the concepts and ability to adapt them to unfamiliar situations.

Topic N1: Single-Variable Integration Techniques

P1. Here are a few increasingly masochistic problems for those of you who need feel you need more practice on repeated integration by parts.

(a) $\int x^6 e^x \, dx$

(b) $\int x^6 e^{-x} \, dx$

(c) $\int x^6 e^{kx} \, dx$

P2. Evaluate the following improper integral.

$$\int_{-\infty}^{\infty} x^7 e^{-x^{10}} \, dx$$

P3. Evaluate the following definite integral.

$$\int_0^2 (x \quad 1) \log jx \quad 1j \, dx$$

Topic C1: Numerical Integration

P1. Approximate the value of $\ln(5)$ to within 0.003. Show that your error is that low.

P2.

(a) Approximate the value of $\int_0^2 1000^{-x} \, dx$ using Simpson's rule to sufficient accuracy (by which I mean, "accurate enough for practical purposes"). Try to do this ***without*** computing M.

(b) Approximate the area between the curves

$$f(x) = e^{-x} \ln\left(x^2 + 1\right) \qquad \text{and} \qquad g(x) = \frac{e^x}{100x}$$

on the interval $[e^{-2}, e^{3/2}]$ to sufficient accuracy.

P3. What is the difference between the Trapezoid rule and the composite Trapezoid rule?

P4. Draw a picture showing how the derivation of the composite Trapezoid rule works.

P5. Derive the formula for the composite trapezoid rule from your picture in **P4**.

P6. Draw a picture showing how the derivation of the composite Simpson's rule works.

P7. Approximate $\int_{-2}^{2} f(x^2)\,dx$ using Simpson's rule, given that $f(0) = 2.55$, $f(1) = 1.20$, and $f(4) = 0.31$.

P8. Suppose that I ask you to approximate

$$\int_{1.75}^{4.5} f(x)\,dx$$

and give you the following table of values.

x	$f(x)$
1.75	0.6567
2.00	1.3936
2.25	0.9178
2.50	0.3765
2.75	0.3940
3.00	1.3190
3.25	1.2264
3.50	0.1464
3.75	0.1325
4.00	0.6145
4.25	0.0501
4.50	0.0130

Note that the table contains an even number of points (and thus an *odd number* of subintervals).

(a) Must you resort to the trapezoid rule to approximate the value of the integral? How could you improve your accuracy? *Hint: Think about what we did when deriving each integration formula.*

(b) Approximate the value of the integral.

P9. Suppose you are given the following table of values.

x	$f(x)$
0.00	2.5619
0.25	2.1043
0.50	2.0551
0.75	2.1454
1.00	2.3524
1.25	2.4077
1.50	2.3665
2.29	2.0618
2.75	1.4865
2.90	1.4111
3.00	1.3892
3.50	1.6035
4.00	2.0482
4.50	2.1463
4.84	1.8904
5.00	1.7091

How could you best adapt the numerical integration techniques that you know to approximate $\int_0^5 f(x)\,dx$?

Topic C2: Multivariate Optimization

P1. Pringles are described by the function

$$P(x, y) = x^2 \quad y^2$$

within the closed unit disc (the region $x^2+y^2 \quad 1$). How tall is a pringle? (To phrase this less esoterically: what is the difference between the maximum and minimum value of the pringle function?)

P2. UF needs to build a new power plant, which will send electricity to transformer hubs in three buildings: Little Hall (LIT), Southwest Recreation Center (SRC), and the Aquatic Pathobiology Laboratories (APL).

From the intersection of Gale Lemerand and Museum road, the coordinates of these buildings are:

	x	y
LIT	2.02	1.38
SRC	3.41	1.17
APL	0.50	2.06

The cost per hour to send power to a hub goes quadratically with distance (that is, to send power to a hub located at a distance d from the power plant, it will cost d^2 dollars per hour). Where should UF build the power plant so that its operating costs are minimized?

P3. Garrett is an amazing gourmet porcorn store in downtown Chicago, about five blocks from where I lived in my early 20s. They sell popcorn tins with three types of popcorn: butter, pecan caramel, and smoked cheese. The tin is a cylinder (made of tin) with cardboard dividers separating each type of popcorn into equal portions.

According to wholesale commodity websites, the cost of cardboard is \$0.42 per square foot, and tin is \$3.68 per square foot. It costs \$2.19 per cubic foot to make buttered popcorn, \$5.19 per cubic foot to make cheese popcorn, and \$7.51 per cubic foot to make caramel popcorn.

Given this information, what is the minimum production cost of a two gallon popcorn tin (0.267 cubic feet)? Garrett sells this product for \$77.00 - how much profit are they making per sale?

P4. In this exercise, we're just going to practice building Lagrangians. There is no need to solve them - determine the function, the constraint, and move on.

(a) A ventilation company is designing an air duct that has the overall shape of a hollow rectangular box with one square end. It will be made of a rectangular outer casing, a rectangular inner casing, and two caps on each side enclosing the volume between the two casings. Write the Lagrangian to maximize the enclosed volume (the volume between the casings) subject to a constrained surface area A.

(b) A file drawer must enclose a volume V. The drawer is a rectangular box with two faces removed, one in the z direction and one in the x direction, and contains two separators (parallel to the yz-plane). Minimize its surface area.

(c) I unfold a wire coat hanger and smooth it out until it is straight, with length ℓ. I then bend part of the coat hanger into a square shape, and the rest of it into a circle shape. What's the smallest amount of area I could enclose this way? *Note: You can actually solve this one if you want.*

P5. This exercise is a three-part generalization of **P2**.

(a) First, we generalize by allowing the three locations to have variable coordinates. Instead of the three locations given in **P2**, suppose that you are given three points, P_1, P_2, and P_3, with coordinates (x_1, y_1), (x_2, y_2), and (x_3, y_3), respectively. What is the point which minimizes the total square distance to all three points?

(b) Next, we generalize the number of points given. Rather than three points, suppose that you are given an arbitrary number of points, $P_1, P_2, \ldots, P_n$, with coordinates (x_1, y_1), (x_2, y_2), $\ldots$, (x_n, y_n), respectively. What is the point which minimizes the total square distance to all n points?

(c) The final challenge is to generalize the dimension of the space. This one is for now just for fun and definitely won't be on this exam (although it will help prepare you for the linear algebra topic later in the class). Suppose that you are given an arbitrary number of points in an m-dimensional space: $\overline{P_1} = (x_{1,1}, x_{2,1}, \dots, x_{m,1})$, $\overline{P_2} = (x_{1,2}, x_{2,2}, \dots, x_{m,2})$, ..., $\overline{P_n} = (x_{1,n}, x_{2,n}, \dots, x_{m,n})$. What is the point which minimizes the total square distance to all n points? *Note: the distance function for an m-dimensional space is* $d(\overline{x}, \overline{y}) = \sqrt{(x_1 \quad y_1)^2 + (x_2 \quad y_2)^2 + \quad + (x_n \quad y_n)^2}$.

P6. Kanye West has decided that when he dies he wishes to be buried in a solid gold, cone-shaped mausoleum, which is to be built in Giza and referred to as "the Greatest Pyramid." He insists that the building should be 148 feet meters tall (one meter taller than the Great Pyramid) with a base radius of 231 meters (one meter wider than the base of the Great Pyramid). The contractor he hires measures each of these lengths with a maximum error of five centimeters. Estimate the maximum error in calculating the surface area of Kanye West's tomb.