MAC2234: Quiz 3 *(core)* *May 28, 2014*

Name: Answers

Score	P1	P2	Q3

P1. Grandma Mildred's Fabulous Metallic Products, LLC, produces an steel-uranium alloy that is then used to make guitar strings for melodic death metal bands. Their expected profit can be approximated by

$$P(s,u) = -s^3 - 27u^3 + 108su$$

where s is the amount of steel and u is the amount of uranium used in production. Find the combination of steel and uranium that maximizes the expected profit, and compute what that profit is.

$P_s(s,u) = -3s^2 + 108u = 0 \longrightarrow s = \pm 6\sqrt{u}$

$P_u(s,u) = -81u^2 + 108s = 0 \longrightarrow -81u^2 + 108(\pm 6\sqrt{u}) = 0$

$\longrightarrow u = 0$ or 4, so $(s,u) = (0,0)$ or $(12,4)$

$P_{ss}(s,u) = -6s$

$P_{uu}(s,u) = -162u$ $\rightarrow$

$P_{su}(s,u) = 108$

$\Delta(s,u) = P_{ss}(s,u)P_{uu}(s,u) - (P_{su}(s,u))^2 = -11664 + 972su$

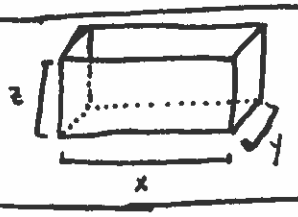
$\Delta(0,0) = -11664$ so $(0,0)$ is a saddle point

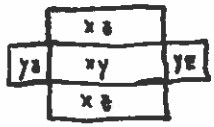
$\Delta(12,4) = 34992$ and $P_{ss}(12,4) = -72$ so $(12,4)$ is a local maximum. The profit is then $P(12,4) = 1728$.

P2. You just got a job working for Giant Fish Tanks for Aquariums Incorporated, a company which constructs giant fish tanks for aquariums. Your boss wants you to determine the least amount of money that can be spent to construct a rectangular manatee aquarium, open on top, with a volume of 343 square meters. Glass costs \$35 per square meter. What is the minimum cost? *Hint: Find the dimensions that will minimize the surface area (and hence the cost of glass.)*

constraint: $g(x,y,z) = xyz - 343$

minimize: $f(x,y,z) = xy + 2xz + 2yz$





$F(x,y,z,\lambda) = xy + 2xz + 2yz - \lambda(xyz - 343)$

$F_x(x,y,z,\lambda) = y + 2z - \lambda yz = 0 \rightarrow \lambda = \frac{y+2z}{yz}$

$F_y(x,y,z,\lambda) = x + 2z - \lambda xz = 0 \rightarrow \lambda = \frac{x+2z}{xz}$

$F_z(x,y,z,\lambda) = 2x + 2y - \lambda xy = 0 \rightarrow \lambda = \frac{2x+2y}{xy}$

$F_\lambda(x,y,z,\lambda) = xyz - 343 = 0$

$\frac{y+2z}{yz} = \frac{x+2z}{xz} \rightarrow x = y$

$\frac{x+2z}{xz} = \frac{2x+2y}{xy} \rightarrow y = 2z$

So, $x = y = 2z \rightarrow 2z \cdot 2z \cdot z - 343 = 0 \rightarrow z = \frac{7}{\sqrt[3]{4}}, \; x = y = \frac{14}{\sqrt[3]{4}}$

so $f(x,y,z) \times 35 = 147 \cdot 2^{2/3} \approx 233.388$