MAC2234: Quiz 2 *(core)* *May 28, 2014*

Name: Solutions

Score	P1	P2	Q2

P1. Find a numerical approximation of

$$\int_{-2}^{2} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx$$

call this integrand $f(x)$

to within 0.01 of the exact value. Show that you are this accurate using the appropriate error formula.

too much work

Trapezoid Rule: $E_T = \frac{M \cdot 4^3}{12 n^2} = 0.01$ given: $|f^{(2)}(x)| \le \sqrt{\frac{2e^3}{\pi}}$ $\rightarrow n = \sqrt{\frac{4^3}{12 \cdot 0.01}} \cdot \sqrt{\frac{2e^3}{\pi}} \approx 44$

better!

Simpson's Rule: $E_S = \frac{M \cdot 4^5}{180 n^4} = 0.01$ given: $|f^{(4)}(x)| \le \frac{3}{\sqrt{2\pi}}$ $\rightarrow n = \sqrt[4]{\frac{4^5}{180 \cdot 0.01} \cdot \frac{3}{\sqrt{2\pi}}} \approx 5.1$

Since Simpson's rule requires an even number of sub-intervals, we round up to the next highest even integer: $n = 6$.

Then, we have $\Delta x = \frac{2-(-2)}{6} = \frac{2}{3}$.

i	x_i	$f(x_i)$
0	-2	0.054
1	-4/3	0.164
2	-2/3	0.319
3	0	0.399
4	2/3	0.319
5	4/3	0.164
6	2	0.054

So the Simpson's rule approximation is as follows:

$\frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots\right)$

$= \frac{2}{9}\left(0.054 + 4 \cdot 0.164 + 2 \cdot 0.319 + \cdots\right)$

$= 0.954$ (actual: 0.95545...)

P2. Approximate the area bounded by the curves $f(x) = \sqrt{x^2+1}$ and $g(x) = \frac{2}{\sqrt{x^2+1}}$ on the interval $[0, 2]$.

The area between curves is given by $\int_a^b |f(x) - g(x)|\,dx$. (You may or may not recognize it with absolute value signs, but when we "find which function is bigger", this is what we're doing.) So, let $h(x) = |f(x) - g(x)|$, find $\int_0^2 h(x)\,dx$.

I think that Simpson's rule on eight subintervals will give sufficient accuracy.

$n = 8$

$\Delta x = \frac{1}{4}$

Simpson's Rule

i	x_i	$\lvert f(x_i) - g(x_i)\rvert$
0	0	1.000
1	1/4	0.910
2	1/2	0.671
3	3/4	0.350
4	1	0
5	5/4	0.351
6	3/2	0.693
7	7/4	1.023
8	2	1.342

So the Simpson's rule approximation is as follows:

$\frac{\Delta x}{3}\left(|f(x_0) - g(x_0)| + 4|f(x_1) - g(x_1)| + 2|f(x_2) - g(x_2)| + \cdots\right)$

$= \frac{1}{12}\left(1 + 4 \cdot 0.91 + 2 \cdot 0.671 + 4 \cdot 0.35 + \cdots\right)$

$= 1.3005$ (actual: 1.30052...)