MAC1140 Review and Extra Credit

Due Sept. 19th, 2013

Problem 1. Using only words, answer the following questions.

(a) What is the implied domain of an expression?

(b) Under what conditions do we say that two expressions are equal?

(c) Under what conditions do we say that two expressions are not equal?

(d) Relate your statement in (b) to the definition of an equation. What does it mean to solve an equation?

(e) What is the difference between an equation and an identity?

(f) What is an extraneous solution? Does this relate to the domain concept in some way?

(g) What is the relationship between factoring polynomials and solving polynomial equations?

Problem 2. In each problem, find an integer c such that the polynomial can be factored. (In general, there can be more than one correct answers.)

(a) $6x^2 + 11x + c$

(b) $5x^2 + cx - 12$

(c) $cx^2 + x + 6$

Problem 3.

(a) Simplify $(\sqrt[n]{x})^n$.

(b) Simplify $\sqrt[n]{x^n}$. *be careful! don't let me trick you. what makes this one harder than the last one?*

(c) *(A riddle)* What do you get when you drink root beer out of a square cup?

Problem 4. When using the quadratic formula, it's can save you time to compute $b^2 - 4ac$ before anything else. Explain the following three things.

(a) If $b^2 - 4ac < 0$, the equation has no solutions. Why?

(b) If $b^2 - 4ac = 0$, the equation has one solution. Why?

(c) If $b^2 - 4ac > 0$, the equation has two solutions. Why?

Try it out:

(a) Solve $2x^2 + 3x + 1 = 0$.

(b) Solve $9x^2 + 7x + 3 = 0$.

(c) Solve $x^2 - 6x + 9 = 0$.

(d) Solve $x^2 + 7x + 12 = 0$.

Challenge: Now go back to problem 2 and write down exactly the conditions that c must meet for each polynomial to have (at least one) root.

Problem 5. Explain in words:

(a) Why do we exchange $<$ with $>$ (or $\leq$ with $\geq$) when we multiply both sides of an inequality by a negative number? Should we also do this when dividing?

(b) Analogously, what do we do when multiplying a double inequality by a negative number?

Problem 6. Give examples of

(a) An absolute value inequality which is an identity.

(b) An absolute value inequality for which only one number is a solution.

Problem 7.

(a) Write $|x - a| < 10$ as a double inequality.

(b) Can you write $|x - a| > 10$ as a double inequality? If not, how else could you write it? *(Hint: if you get stuck, try drawing this on the number line.)*

(c) In words, which sets of numbers do the above inequalities represent?

(d) How would you express "x is 7 units or more away from 4" using an absolute value inequality?

(e) How would you express "x is less than 13 units away from -11" using an absolute value inequality?

(f) Now write the absolute value expressions in (a), (b), (d), and (e) using intervals (like (a, b) or $(-\infty, a] \cup [b, \infty)$, for example).

Problem 8. Find the domain of

(a) $\sqrt[8]{2x - 7}$

(b) $x^{-1/2}$

(c) $\sqrt{3 - x}$

(c) $\sqrt{2x - 10} + \sqrt{3 - x}$

(e) $\sqrt[4]{x^2 - 81}$

(f) $\sqrt[4]{81 - x^2}$

Problem 9. Solve

(a) $\frac{1}{3x+1} - \frac{1}{x-6} = \frac{1}{3x^2-17x-6}$

(b) $\frac{1}{x} \leq 6$ *be careful! don't let me trick you. what makes this one harder than the last one?*