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Probabilistic Goods: A Creative Way of Selling Products and Services

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This paper defines a unique type of product or service offering, termed *probabilistic goods*, and analyzes a novel selling strategy, termed *probabilistic selling* (PS). A probabilistic good is not a concrete product or service but an offer involving a probability of getting any one of a set of multiple distinct items. Under the probabilistic selling strategy, a multi-item seller creates probabilistic goods using the existing distinct products or services and offers such probabilistic goods as additional purchase choices. The probabilistic selling strategy allows sellers to benefit from introducing a new type of buyer uncertainty, i.e., uncertainty in product assignments. First, introducing such uncertainty enables sellers to create a "virtual" product or service (i.e., probabilistic good), which opens up a creative way to segment a market. We find that the probabilistic selling strategy is a general marketing tool that has the potential to benefit sellers in many different industries. Second, this paper shows that creating buyer uncertainty in product assignments is a new way for sellers to deal with their own market uncertainty. We illustrate two such benefits: (a) offering probabilistic goods can reduce the seller's information disadvantage and lessen the negative effect of demand uncertainty on profit, and (b) offering probabilistic goods can solve the mismatch between capacity and demand and enhance efficiency. Emerging technology is creating exciting (previously unfeasible) opportunities to implement PS and to obtain these many advantages.

Key words: probabilistic selling; probabilistic goods; opaque goods; pricing; product differentiation; e-commerce; yield management; inventory management; product line; price discrimination
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1. Introduction

In this paper, we define a unique type of product or service offering, termed *probabilistic goods*, and analyze a novel selling strategy, termed *probabilistic selling*. To understand these concepts, consider the recently observed "opaque sales" offered by two online travel intermediaries, *Priceline* and *Hotwire*. For example, *Priceline* offers "opaque" hotel rooms in which a buyer specifies dates, city, and approximate quality (e.g., star rating), but the particular hotel property is not revealed until after payment has been made. *Priceline* requires buyers to bid for price (known as "name your own price" or NYOP). If the bid is accepted, the buyer's credit card is charged. A no-refund policy is strictly enforced.

Priceline's business model has attracted increasing attention from consumers, the media, and academia. Consumers share their experiences in specialized web discussion forums (e.g., BetterBidding.com). Many articles and books offer tips to help travelers take advantage of *Priceline*'s pricing strategy (e.g., "Priceline.com for Dummies" (Segan 2005)). Professional reviewers gather and disperse information about such "blind" sites. Marketing scholars have also shown interest in such new business models, and several recent studies have examined the NYOP mechanism from the perspectives of service providers, intermediaries, and consumers (Chernev 2003; Hann and Terwiesch 2003; Fay 2004, 2008; Spann et al. 2004; Ding et al. 2005; Spann and Tellis 2006; Wang et al. 2006). This research stream has provided insights on various important theoretical and empirical issues such as online bidding, channel coordination, and the impact of an opaque intermediary on traditional channels.

While the current academic attention has mainly focused on the various aspects of the business model of *Priceline*, our interest is to explore the fundamental product/market conditions required for the benefit of introducing uncertainty in product assignments by offering "probabilistic goods," which we define as a gamble involving a probability of getting any one of a set of multiple distinct items. We use *probabilistic selling* to denote the selling strategy under which the seller creates probabilistic goods using the seller's existing distinct products or services (referred to hereafter as *component goods*) and offers such probabilistic goods to potential buyers as additional purchase choices. For example, a retailer selling two different colors of sweaters, red and green, may offer

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an additional "probabilistic sweater," which can be either the red or green sweater. A theatre that offers two different shows on a given weekend can sell an additional probabilistic ticket, "Saturday or Sunday performance." We use the term *traditional selling* to denote the conventional selling strategy under which the seller only offers the component goods for sale.

It is important to note that *Priceline* operates in a complicated market environment with many unique industry characteristics: It is an online intermediary that depends on multiple suppliers (e.g., different airlines, hotel chains, and car rental firms) and also competes with these suppliers' direct channels; it is subject to special characteristics of the travel industry (e.g., a nonstorable, perishable good, capacity constraints, and demand uncertainty); and it sets prices via a complicated buyer bidding system (i.e., NYOP). It is unclear what are the key factors that motivate the seller to introduce uncertainty in product assignments and if such uncertainty can benefit firms that are not subject to these specific industry characteristics. Thus, our primary objective is to uncover the fundamental factors required for probabilistic selling to be advantageous to a seller. Specifically, we examine *why*, *when*, and *how* a seller can benefit from introducing a probabilistic good. We provide answers to several important questions: What are the conditions under which offering a probabilistic good improves profit? How does the degree of horizontal product differentiation between the component goods impact the profit advantage of probabilistic selling? Is probabilistic selling more or less profitable for sellers facing demand uncertainty? How do capacity constraints affect the profit advantage of probabilistic selling? We develop a formal model to address these issues.

Our results reveal that probabilistic selling can improve profit without requiring specific industry characteristics such as multiple suppliers, an intermediary market structure, selling perishable goods, and a buyer bidding system. First, we find that probabilistic goods have a fundamental advantage of allowing the seller to benefit from a special type of buyer heterogeneity—differentiation in the *strength* of buyers' preferences (e.g., some vacationers may have a strong preference but others may have a weak preference about the two different bus tours offered at a national park). A discounted probabilistic good, because it attracts consumers with weaker preferences while allowing the seller to obtain high margins on sales to those consumers with strong preferences, can enhance price discrimination and expand the total market. We illustrate that offering a probabilistic good can improve profit even if the seller achieves full market coverage under the traditional selling strategy and even if *all* consumers prefer the same product. Given that in almost all markets with multiple product offerings, consumers differ in the strength of these preferences, probabilistic selling can be a general marketing tool that potentially benefits many sellers.

Second, we find that when there is an advantage from introducing a probabilistic good, it is generally optimal to assign an equal probability to each component product as the probabilistic good. Such a symmetric assignment maximizes the profit under probabilistic selling even if the seller faces asymmetric demand (i.e., consumers on average prefer one product over the other). While this result may not be intuitive, as we will explain later, deviating from such an equal probability will diminish the two positive effects of probabilistic selling: price discrimination and market expansion.

Third, our analyses reveal an intriguing benefit of offering a probabilistic good: It can provide a buffer against a seller's own demand uncertainty. A seller offering multiple products often has uncertainty about which product will turn out to be the high-demand (or low-demand) product. For example, a toy manufacturer may not be sure which of the two new toys introduced for the holiday season will be "hot" at the time of product launching. Under traditional selling, such uncertainty reduces profit because the firm is unable to tailor prices to demand conditions (i.e., charging a higher price for the "hot" toy). However, we find that introducing a probabilistic good can reduce, and sometimes even eliminate, the need to make prices depend on the demand for each individual product, which reduces the negative effect of demand uncertainty on profit. As a result, demand uncertainty increases the profit advantage of probabilistic selling.

Fourth, our model reveals that probabilistic selling may be particularly beneficial in industries or markets subject to both demand uncertainty and capacity constraints, because probabilistic selling can increase capacity utilization and enhance efficiency via reducing the *mismatch* between capacity and demand. For many industries, mismatch between demand and capacity can occur because one product may turn out to be more popular than the other products (e.g., one of the two shows offered in the same week may have an ex post higher demand than the other one). Under the traditional selling strategy, the seller facing both demand uncertainty and capacity constraints can not use price to shift demand to match capacity because the seller is not certain which product will have binding capacity and which will have excess capacity.[1] However, we show that under probabilistic

[1] Using price to shift demand may also not be practical for other reasons, e.g., menu costs.

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selling, the seller can smooth demand across the products without knowing the direction of the mismatch between demand and capacity for each product. As a result, introducing a probabilistic good helps improve capacity usage rates and also ensures that capacity will be available to serve those consumers with strong preferences for the high demand product.

This paper contributes to marketing theory by proposing and illustrating the benefit of introducing a new type of buyer uncertainty, i.e., uncertainty in product assignments. We show that introducing such uncertainty enables sellers to create a "virtual" product or service (i.e., probabilistic good), which opens up a new dimension to segment a market. Sellers currently carrying any number of component products or services may benefit from adding probabilistic goods to their product lines. Such additions would be particularly valuable when adding additional new concrete products or services is too costly or logistically impossible. For example, a theatre already open every night of the week may not be able to introduce an additional performance, but it could offer a number of different probabilistic tickets using its current performances as the component goods (e.g., "Tuesday or Wednesday" and "Wednesday or Thursday").

These contributions complement extant research on buyer uncertainty. Recent papers (Shugan and Xie 2000, Xie and Shugan 2001) suggest that firms can benefit from creating buyer uncertainty by selling in advance, i.e., completing transactions with consumers before they learn their valuations. Our paper suggests that firms can profit from creating a different type of buyer uncertainty, i.e., offering a probabilistic good that can turn out to be any one out of a set of multiple items. The two strategies not only differ in the nature of buyer uncertainty created (i.e., uncertainty about one's own consumption state versus uncertainty about the specific product one will receive) but also differ in the fundamental sources of their profit advantages. In contrast to advance selling where the seller benefits by reducing consumer heterogeneity, i.e., by selling to consumers at a time before idiosyncratic differences are known to consumers, probabilistic selling, as shown in our analysis, instead capitalizes on idiosyncratic differences by selling the probabilistic product to consumers with weak preferences and selling the specified products to consumers with strong preferences.[2]

This work also augments the recent literature on strategies that can mitigate the negative effect of seller uncertainty on profit in industries with capacity constraints (e.g., travel related industries).[3] For example, Biyalogorsky and Gerstner (2004) propose that sellers facing demand uncertainty and capacity constraints can benefit from a contingent pricing strategy, i.e., canceling the sales to low-paying advance buyers if high valuation customers show up later. Biyalogorsky et al. (2005) illustrate that providers with multiclass services can increase capacity utilization by introducing "upgradeable tickets," which upgrade the ticket holders to a higher class service at the time of service delivery only if the reserved higher class capacity remain unsold. Recent research on revenue management (Gallego and Phillips 2004) suggests that sellers can overcome the difficulty of mismatch between demand and capacity by selling a *flexible product*, i.e., assigning one out of a set of alternative products to the prepaid buyer only after the seller has learned which product has excess capacity. Each of these innovative strategies addresses potential mismatches between capacity and demand by reserving seller flexibility. Our paper offers a different approach. In our model, the allocated product is confirmed immediately after the completion of each transaction, i.e., before the seller has acquired any additional information about demand. We show how, under certain conditions, probabilistic selling enables the seller to spread demand more evenly across the products and thus improves capacity utilization without delaying the confirmation of product assignments.

This research is particularly vital and urgent to practitioners because advances in new technology are making implementation of a probabilistic selling strategy much more efficient and practical. In the past, displaying and selling probabilistic goods could be very costly and inefficient. For example, a consumer may balk at having to carry two blouses up to the cash register in order for the random draw, say, by a coin flip to take place. However, the Internet is creating a more efficient shopping environment for selling probabilistic goods. In an online setting, the seller could easily create a split-screen, illustrating the two possible component goods from which the probabilistic good is drawn, using existing product descriptors. The random draw could be implemented

[2] Several other papers also provide important insights on the links between buyer uncertainty and marketing strategies. For example, Venkatesh and Mahajan (1993) show that bundling services may allow a seller to profit from buyer uncertainty about the availability of their future spare time. Xie and Gerstner (2007) illustrate that when buyers are uncertain if they will find alternative offers, the sellers can benefit by offering partial refunds for service cancellations. Guo (2006) shows that buyer uncertainty about their product valuation motivates them to reserve "consumption flexibility" by purchasing multiple items, which can create a "flexibility trap" and reduce profit.

[3] There is an extensive literature that considers some general strategies that can reduce a seller's uncertainty about demand uncertainty without capacity constraints, such as acquiring the ability to target individual customers (Chen et al. 2001) and sharing information with a retailer (Kulp et al. 2004).

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via integrated software and data communication networks. New retailing technologies, such as "Anyplace Kiosk" (recently developed by IBM) and radio frequency identification (RFID) technology, are also easing the implementation of probabilistic selling even in an offline setting. As the costs of these technologies fall and consumers become more familiar with and accepting of them, the probabilistic selling strategy will become increasingly feasible for a wider array of industries.

The remainder of this paper is organized as follows: In the next section, we present our basic model and offer several generalizations of this basic model. Section 3 focuses on demand uncertainty and capacity constraints. Finally, §4 discusses managerial implications and future research. Sketches of proofs of all propositions are in the appendix. The full proofs are contained in the Technical Appendix online at http://mktsci.journal.informs.org.

2. The Model

In this section, we start with a standard Hotelling model (referred to hereafter as the "basic model") to examine conditions required for the profit advantage of probabilistic selling. Our objective is to first use a simple model to illustrate the basic economic force behind the probabilistic selling strategy and offer key insights without introducing unnecessary mathematical complexity. We then examine the robustness of our findings and offer additional insights by relaxing several of the assumptions of the basic model and adopt some alternative demand structures (see §2.3).

2.1. Assumptions

Seller behavior. Consider a seller offering two component products, $j = 1, 2$, which have symmetric production costs: $c_1 = c_2 = c$. To ensure the seller can have a positive demand at a price above marginal cost, we assume $0 \leq c < 1$. In the basic model, we assume that the seller is aware of the demand and able to satisfy all demand (if it so desires). We extend the analysis in §3 by considering the case where the seller faces capacity constraints and demand uncertainty. The seller considers two strategies: *traditional selling* (TS) and *probabilistic selling* (PS). Under the traditional selling strategy, the seller sells each component product j at a price, P_j^{TS}. Under the probabilistic selling strategy, the seller sells each component product j at a price, P_j^{PS}, and also a probabilistic good, which has a probability to be either component product, at a price of P_o^{PS}. Let φ be the probability that product 1 is allocated as the probabilistic product, which is determined by the seller and is naturally restricted to the interval $[0, 1]$.

Buyer behavior. Let v_{ji} be the value of product j to consumer i. The basic model assumes that valuations for the two component products follow a Hotelling model in which the value for one's ideal product is normalized to one, the fit-cost-loss coefficient equals t where $0 < t \leq 1$, and the consumer's location on the Hotelling line is x_i. The valuations are given in Equation (1):

$$\begin{cases} v_{1i} = 1 - tx_i \\ v_{2i} = 1 - t(1 - x_i) \end{cases} \quad \text{where } x_i \sim U[0, 1]. \tag{1}$$

Each consumer buys at most one unit of one good, i.e., there is no value from consuming a second product. Each consumer chooses the product offering that maximizes her expected surplus. For example, when the seller adopts the probabilistic selling strategy, consumer i has four choices: (a) buy product 1, (b) buy product 2, (c) buy the probabilistic good, and (d) buy nothing. She chooses the option that leads to the highest expected surplus.

The consumer's surplus from buying the probabilistic good is the difference between her expected valuation and the price of the probabilistic good. It is important to notice that a consumer's valuation for the probabilistic good depends on her valuations for products 1 and 2 and her expectation about the probability that the probabilistic good will turn out to be product 1. We assume that consumers are rational and forward-looking, i.e., their expectations are confirmed in equilibrium. Thus, a consumer's valuation for the probabilistic good is $\varphi v_{1i} + (1 - \varphi) v_{2i}$.

2.2. Optimal Strategy

Let $D_j^{\mathrm{TS}}(P_1^{\mathrm{TS}}, P_2^{\mathrm{TS}})$ represent the demand for the component product j under the traditional selling strategy, and let $D_j^{\mathrm{PS}}(P_1^{\mathrm{PS}}, P_2^{\mathrm{PS}}, P_o^{\mathrm{PS}}, \varphi)$ and $D_o^{\mathrm{PS}}(P_1^{\mathrm{PS}}, P_2^{\mathrm{PS}}, P_o^{\mathrm{PS}}, \varphi)$ represent the demand for the component products and the probabilistic good, respectively, under the probabilistic selling strategy. The profit functions under the two strategies are given in Equation (2):

$$\begin{cases} \text{Traditional selling strategy:} \\ \quad \Pi^{\mathrm{TS}} = \sum_{j=1}^{2} (P_j^{\mathrm{TS}} - c) D_j^{\mathrm{TS}} \\ \text{Probabilistic selling strategy:} \\ \quad \Pi^{\mathrm{PS}} = \sum_{j=1}^{2} (P_j^{\mathrm{PS}} - c) D_j^{\mathrm{PS}} + (P_o^{\mathrm{PS}} - c) D_o^{\mathrm{PS}} \end{cases}. \tag{2}$$

The seller maximizes profit by choosing prices (P_j^{TS}) under the traditional selling strategy and both prices $(P_j^{\mathrm{PS}}, P_O^{\mathrm{PS}})$ and probability (φ) under the probabilistic selling strategy.

The detailed solutions to the seller's profit maximization problem are given in the appendix (see Table A.1). Lemma 1 reports the important relationships we derive from these solutions.

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Lemma 1 (Advantage of Probabilistic Selling). *The following relationships hold for the two selling strategies*:

Comparison	*Interpretation*
(a) $\Delta\Pi = \Pi^{\mathrm{PS}} - \Pi^{\mathrm{TS}} > 0$ *if* $0 < \varphi < 1$, $c < \bar{c}$	*Conditions required for the advantage of PS*
(b) $\begin{cases} \Delta p_j > 0, \quad \Delta D = 0 & \text{if } c < \bar{\bar{c}} \\ \Delta p_j \geq 0, \quad \Delta D > 0 & \text{if } \bar{\bar{c}} < c < \bar{c} \end{cases}$	*Sources of profit advantage of PS, where* $\Delta p_j = P_j^{\mathrm{PS}} - P_j^{\mathrm{TS}}$ *and* $\Delta D = (D_1^{\mathrm{PS}} + D_2^{\mathrm{PS}} + D_0^{\mathrm{PS}}) - (D_1^{\mathrm{TS}} + D_2^{\mathrm{TS}})$
(c) $\dfrac{\partial \Pi^{\mathrm{PS}}}{\partial \varphi} \begin{cases} \geq 0 & \text{if } \varphi \leq 1/2 \\ \leq 0 & \text{if } \varphi \geq 1/2 \end{cases}$	*The impact of probability that assigns a component product as the probabilistic good*
(d) $\dfrac{\partial \Delta p_j}{\partial t} > 0$, $\dfrac{\partial \Pi_o^{\mathrm{PS}}}{\partial t} < 0$, $\dfrac{\partial \Delta\Pi}{\partial t} \begin{cases} \geq 0 & \text{if } t \leq \bar{t} \\ < 0 & \text{otherwise}, \end{cases}$	*The impact of degree of horizontal product differentiation, where* $\Pi_o^{\mathrm{PS}} = (P_o^{\mathrm{PS}} - c)D_o^{\mathrm{PS}}$

where $\bar{c} = 1 - t/2$, $\bar{\bar{c}} = 1 - t$, *and* $\bar{t}$ *is defined in the appendix.*

Proposition 1 summarizes the key results in Lemma 1.

Proposition 1 (Advantage of Probabilistic Selling).

(a) *Adding a probabilistic good to the seller's product line strictly improves profit if marginal costs are sufficiently low* (*i.e.*, $c < \bar{c}$).

(b) *Such profit improvement comes from enhanced price discrimination alone if the cost is sufficiently low, but from both price discrimination and market expansion if cost is in a midrange.*

(c) *The profit advantage of probabilistic selling does not require assigning an equal probability to each component product as the probabilistic good, but reaches the maximum under such an equal probability* (*i.e.*, $\varphi^* = 1/2$).

(d) *The profit advantage of probabilistic selling is highest when the horizontal differentiation of the component products is at an intermediate level.*

First, Proposition 1 reveals that the profit advantage of probabilistic selling does not require specific characteristics of the travel industry such as capacity constraints and demand uncertainty, an intermediary channel structure, or particular features of some online travel agencies (e.g., Priceline.com) such as a NYOP pricing structure. As shown in Proposition 1, when buyers differ in the strength of their product preferences, offering a probabilistic good can improve profit as long as the marginal cost is sufficiently small. This result is important because it suggests that probabilistic selling may be a more general marketing tool to improve seller profit than currently thought.

Second, Proposition 1 shows that offering a probabilistic good can improve profit even if the market is already fully covered under the traditional selling strategy (i.e., $\Delta\Pi > 0$, $\Delta D = 0$ when $c < \bar{\bar{c}}$). In this case, probabilistic selling improves profit simply because it allows the seller to raise the prices of traditional goods (i.e., $\Delta p_j > 0$). Introducing the probabilistic good enables the seller to separate consumers with strong preferences (who buy the component goods) from consumers with weak preferences (who buy the probabilistic good), thus enhancing price discrimination. For sellers facing unfilled demand under traditional selling (i.e., $\bar{\bar{c}} < c < \bar{c}$), in addition to the discrimination effect, probabilistic selling also leads to market expansion ($\Delta D > 0$), i.e., allows the seller to sell the probabilistic good to low valuation consumers who would not have bought under the traditional selling strategy.

Third, it is interesting to learn that probabilistic selling can be profitable for ANY arrangement in which each component good has a positive probability of being assigned as the probabilistic good (i.e., $0 < \varphi < 1$). However, although an equal probability is not required for the profit advantage of probabilistic selling, it is in the seller's best interest to assign such an equal probability, i.e., $\varphi^* = 1/2$. This is because moving φ away from 1/2 negatively impacts profit in two ways. Consider an increase in φ above 1/2. Increasing φ makes the probabilistic good more attractive to consumers who prefer product 1 but less attractive to consumers who prefer product 2. The former implies a *cannibalization effect*: Some consumers who would have bought product 1 at a higher price under $\varphi = 1/2$ now buy the discounted probabilistic product under $\varphi > 1/2$. The latter implies a *demand reduction effect*: Some consumers who would have bought the probabilistic product under $\varphi = 1/2$ now do not buy anything under $\varphi > 1/2$. Can the seller reduce the two negative effects by adjusting the price of the probabilistic good? The answer is no because increasing the price of the probabilistic good would reduce the attractiveness of the probabilistic good to all consumers, which weakens the *cannibalization effect* but intensifies the *demand reduction effect*. As a result, it is optimal for the firm to assign an equal probability to each product as the probabilistic good.

When $\varphi = 1/2$, consumers have the same expected value for the probabilistic good. However, when $\varphi \neq 1/2$, consumers vary in the expected surplus that they receive from purchasing the probabilistic good. Thus, while identical valuations for the probabilistic good

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are *not* a necessary condition for probabilistic selling strategy to be advantageous, the seller will maximize profits by choosing φ such that consumers will have identical valuations for the probabilistic good. Interestingly, as we will show later, such symmetric assignments (i.e., equal probability) are optimal even if the products have asymmetric demand (see §2.3.1).

Finally, Proposition 1 reveals that the profit advantage of probabilistic selling is maximized at an intermediate value of t, which represents the degree of horizontal differentiation of the component products. Notice that the price discrimination effect is increasing in t (i.e., $\partial\Delta p_j/\partial t > 0$), but that the profit obtained from the probabilistic good is decreasing in t (i.e., $\partial\Pi_o^{PS}/\partial t < 0$) due to increasing expected fit-cost losses from consuming the probabilistic good. If product 1 and product 2 are near-perfect substitutes (t is close to zero), then the advantage of probabilistic selling is small because the price discrimination effect is small ($\Delta p_j \to 0$ as $t \to 0$). The seller is unable to extract a sizable premium for the component products by introducing a probabilistic good because no consumer views the probabilistic good as being very inferior to their most preferred product. On the other hand, if products 1 and 2 are extremely differentiated (t is large), then the expected value of the probabilistic good is very small, which implies that the price of the probabilistic good has to be very low to induce sales. Therefore, probabilistic selling is most advantageous when the products that comprise the probabilistic good have moderate differences—enough that some consumers will still be willing to pay a significant premium for their preferred product, but not too large that all consumers view the probabilistic good as being of very low value.

2.3. Model Robustness and Additional Insights

The basic model discussed above helps to understand the fundamental reasons *why* probabilistic selling can improve profit. In the rest of this section, we demonstrate that these benefits apply to more general settings and offer additional insights about the probabilistic selling strategy. In particular, each of the following two subsections considers one specific extension of the basic model. Subsection 2.3.1 relaxes the assumption of uniform distribution by assuming a general distribution of consumers and §2.3.2 increases the number of component products from 2 to N. Other assumptions remain the same as in the basic model.

2.3.1. A General Distribution of Consumers. The basic model assumes that consumers are uniformly-distributed along the Hotelling line. In this subsection, we relax the uniform-distribution assumption by allowing a general distribution $f(x)$, $0 \le x \le 1$. Proposition 2 summarizes our results in this more general setting.

Proposition 2 (Probabilistic Selling with a General Distribution). *Consider a general distribution of consumers* $f(x)$, $0 \le x \le 1$.

(a) *Adding a probabilistic good to the seller's product line strictly improves profit in the following situations*:

1. *The market is uncovered under the traditional selling strategy and costs are sufficiently low* (*i.e.*, $c < \bar{c}$, *where* $\bar{c}$ *is given in Lemma* 1).
2. *The market is covered under the traditional selling strategy and some customers are indifferent between the two component products* (*i.e.*, $0 < prob(x_i = 1/2) < 1$).

(b) *When probabilistic selling improves profit, the profit advantage of probabilistic selling reaches its maximum if the seller assigns an equal probability to each component product as the probabilistic good* (*i.e.*, $\varphi^* = 1/2$).

Corollary 1 (When All Consumers Prefer the Same Product). *Even if* all *consumers prefer the same product* (*e.g.*, $f(x) = 0$, $\forall x \ge 0.5$),

(a) *The seller can benefit from probabilistic selling.*

(b) *It is optimal to assign an equal probability* (*i.e.*, $\varphi^* = 1/2$).

Proposition 2 shows that the primary advantages of probabilistic selling do not require several features of the standard Hotelling model. In particular, there does not need to be a constant density of consumers along the Hotelling line nor is a symmetric distribution of customers required. In fact, from Proposition 2(a) we see that there does not need to be any consumer located exactly at the midpoint of the line. Any consumer that is not served under TS has weaker preferences between the two component goods than a consumer who purchases under TS. Thus, any heterogeneity in the strength of preferences that is strong enough to induce the seller to exclude some consumers under TS (i.e., uncovered market under TS) is also enough heterogeneity to allow PS to be advantageous as long as such market expansion can be obtained at a price that exceeds the marginal cost of production ($c < \bar{c}$). Furthermore, Proposition 2(a) verifies that the price discrimination effect will make PS advantageous if, under the TS strategy, the seller would choose to sell to weak-preferenced consumers (i.e., full market coverage under TS).

Equally important, Proposition 2(b) reveals that for any general distribution such that probabilistic selling is advantageous, it is still optimal for the seller to assign an equal probability to each component product as the probabilistic good. Corollary 1 further emphasizes that an equal probability is optimal even if *all* consumers prefer the same product. This result may not be intuitive because one would wonder if all consumers prefer product 1, would it not be more profitable if the seller announces and assigns a higher probability to product 1 as the probabilistic good? To

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understand why $\varphi^* = 1/2$ regardless of the distribution of consumer valuations, we need to notice that the probabilistic good targets those consumers with weak tastes, while the component products are targeted to those with stronger preferences. Deviating from $\varphi = 1/2$ reduces the seller's efficiency of achieving this segmentation. For example, suppose the firm skews assignments such that product 1 is more likely to be assigned as the probabilistic good. This deviation will increase the expected value of the probabilistic good for all consumers. However, the consumers that experience the greatest increase in expected value are those who have the strongest preferences for product 1. Thus, skewed assignments make it more difficult to attract a premium price for product 1.

The following numerical example helps to understand Corollary 1. Suppose a seller with two component products ($j = 1, 2$) has two potential customers (A and B) and has no marginal cost of production. The consumer valuations for products 1 and 2 are $A\{10, 0\}$, $B\{6, 4\}$. Clearly, in this case, all consumers prefer product 1. We first show $\Pi^{PS} > \Pi^{TS}$ when $\varphi = 1/2$. Under TS, the seller prices product 1 at \$10 and product 2 at \$4, making a profit of \$14 (by selling product 1 to A and product 2 to B). Under PS, the seller prices product 1 at \$10 and sells a probabilistic good (which is equally likely to be product 1 or product 2, i.e., $\varphi = 1/2$) for \$5. Customer A purchases product 1 and B purchases the probabilistic good, thus generating a profit of \$15, i.e., 7.1% higher than under TS. Next, let us see why $\varphi^* = 1/2$. Asymmetric assignments diminish the profit under PS. For example, if $\varphi = 0.4$, then B is only willing to pay \$4.80 for the probabilistic good and, thus, profit under PS would only be \$14.80. If $\varphi = 0.6$, then B's expected value for the probabilistic good is \$5.20 but A's expected value for the probabilistic good is even higher (\$6). So, for $P_O^{PS} = \$5.20$, P_1^{PS} must be no larger than \$9.20 in order to induce A to purchase product 1. Thus, profit under PS with $\varphi = 0.6$ is only \$14.40, which is lower than that with $\varphi = 0.5$. One might wonder if $\varphi^* = 1/2$ still holds if there are more type B consumers (because the seller gains from type B but loses from type A when increasing φ). It is easy to verify that PS is still advantageous and $\varphi^* = 1/2$ continues to hold for the case with two or three type B consumers. When the number of type B consumers is higher than three, PS can no longer outperform TS. For instance, consider the case with four type B consumers. Under TS, the seller would price product 1 at \$6 and make a profit of \$30. PS (for any level of φ) can not improve upon this. Hence, as stated in Proposition 2(b), when probabilistic selling improves profit, it is optimal to assign an equal probability $\varphi^* = 1/2$.

Notice from these examples that PS can be advantageous even though the market is covered under TS and no customer is completely indifferent between the two component products. Thus, the conditions in Proposition 2(a) are sufficient but not necessary to obtain an advantage under PS.

2.3.2. N **Component Goods (The Circle Model).** Thus far, we have assumed that the seller's initial product line consisted of two goods prior to the introduction of the probabilistic good. In practice, most sellers have more extensive product lines. In this subsection, we extend the basic model to allow for $N > 2$ component goods.

We assume a standard circular spatial market, a model which was first introduced by Salop (1979) and has been used and revised to fit many other research settings (e.g., Balasubramanian 1998, Dewan et al. 2003). Each consumer's ideal product, represented by a location x_i, is valued at one and these ideal points are distributed uniformly across a circle of unit circumference. Each consumer incurs a linear cost t per unit distance for consuming a product that differs from her ideal. The seller offers N component goods which are equally spaced along the circumference of this circle. In particular, product j ($j = 1$ to N) is located at j/N.

Under the traditional selling strategy, the seller chooses the component good prices P_j^{TS} in order to maximize the sum of profits across the N goods. Under the probabilistic selling strategy, the seller sets component good prices P_j^{PS} and also determines which probabilistic goods to offer (if any) and at what prices. Many different probabilistic goods are feasible by using two or more of the component goods and setting the probability that each will be assigned as the probabilistic good. In particular, the probability assigning each component good as the probabilistic good is defined as a vector $\{\varphi_1^k, \varphi_2^k, \ldots, \varphi_N^k\}$, where $\sum_{j=1}^N \varphi_j^K = 1$, $\varphi_j^k \geq 0$, $\varphi_j^k < 1$, and $\varphi_j^k > 0$ for at least two of the j's. Proposition 3 summarizes our results for this model.

Proposition 3 (Probabilistic Selling for Sellers with N Component Goods).

(a) *Adding probabilistic goods to the seller's product line strictly improves profit if costs are sufficiently low* (*i.e.*, $\Delta\Pi = \Pi^{PS} - \Pi^{TS} > 0$ *if* $c < \bar{c}$, *where* $\bar{c} = 1 - t/(2N)$).

(b) *The required cost condition for probabilistic selling to be advantageous is weaker as the number of component goods increases* (*i.e.*, $\partial\bar{c}/\partial N = t/(2N^2) > 0$).

(c) *The optimal probabilistic selling strategy is to offer N probabilistic goods, each one consisting of two adjacent component goods, with both of these component goods being equally likely to be assigned as the probabilistic good. Formally,* Π^{PS} *is maximized by introducing N probabilistic goods* ($k = 1$ *to* N), *where*

$$\begin{cases} \varphi_k^k = \varphi_{k+1}^k = 1/2, & \varphi_j^k = 0 \quad \forall\, j \notin \{k, k+1\} \quad \text{if } k < N \\ \varphi_1^N = \varphi_N^N = 1/2, & \varphi_j^k = 0 \quad \forall\, j \notin \{1, N\} \qquad \text{if } k = N. \end{cases}$$

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Proposition 3 shows that the benefits of probabilistic selling extend to a seller with numerous component products. As in the two-good Hotelling model, some consumers strongly prefer one of these goods while others have weaker preferences. By introducing a probabilistic good that randomly selects from the two component goods that are most near to a weak-preferenced consumer's ideal point, the seller can price discriminate on the basis of this differing strength of preferences. Furthermore, it is important to note that probabilistic selling is advantageous over a wider range of costs when there are more than two goods. As the number of component products grows, the maximum distance from one's ideal product to the two most suitable component products falls. Thus, larger N allows the seller to set higher prices for the probabilistic good, thereby achieving positive profit margins over a wider range of costs. Finally, it is clear that the benefits of probabilistic selling could be extended to many other spatial models, e.g., if consumer ideal points are uniformly distributed within a cube. In such a three-dimensional space, the number of probabilistic goods would grow exponentially, i.e., a probabilistic good could profitably be introduced for any two adjacent products (as long as costs are sufficiently low).

The results of these two model extensions show the robustness of the findings derived from the basic model, suggesting that these findings can hold in various more general settings. In the remainder of this paper, we examine several important market/product factors that may affect the profit advantage of probabilistic selling. In the process, we discover additional advantages of PS.

3. Impact of Demand Uncertainty and Capacity Constraints

In this section, we examine the impact of two important factors, (1) demand uncertainty and (2) capacity constraints, on the profit advantage of probabilistic selling. Such an examination is important because in many markets, e.g., travel services, entertainment tickets, educational classes, and holiday gift buying, demand uncertainty and capacity constraints play pivotal roles in a seller's pricing decision. Problems associated with excess demand or idle capacity can be particularly acute in service industries. For instance, consider a professional career development center that offers a particular computer training class at both of its locations within a city. Prior to the start of the session, the center may be uncertain how many people will show up to register at each location. Thus, there is a possibility that one session will be more popular than the other. Yet, not knowing which class will be more popular, it is difficult for the center to set prices appropriately and to ensure efficient use of its facilities.

In this section, we ask whether probabilistic selling exacerbates or lessens the problems that may be caused by demand uncertainty and/or limited capacity. To foreshadow our results, the answers to these questions may be somewhat surprising. Notice that in the previous section, we showed that market expansion is one potential benefit of the probabilistic selling strategy. At first glance, one would question whether a seller with limited capacity would ever benefit from introducing a probabilistic good since increasing the total quantity demanded does not seem to be particularly helpful. However, we will soon show that in the presence of demand uncertainty, the advantage of PS can be even larger when capacity is limited than when it is unlimited.

3.1. Model of Asymmetric Product Demand

In order to address the issues related to demand uncertainty and capacity constraints, we now focus on a model of asymmetry in product popularity. We adopt the same assumptions as in the basic model but allow asymmetric distribution of demand. In addition, to sharpen the focus and simplify the presentation, we assume the fit-cost-loss coefficient (t) is one, i.e., $t = 1$, and a zero marginal cost, $c = 0$.[4]

The demand given in Expression (1) can be rewritten as Equation (3):

$$\begin{cases} 1/2 \text{ of customers prefer product 1:} \\ \quad v_{1i} = 1 - x_i \quad v_{2i} = x_i \quad x_i \sim U[0, 1/2] \\ 1/2 \text{ of customers prefer product 2:} \\ \quad v_{1i} = x_i \quad v_{2i} = 1 - x_i \quad x_i \sim U[0, 1/2] \end{cases}. \tag{3}$$

To systematically incorporate asymmetric demand for the component products, we assume that α of customers ($1/2 \leq \alpha \leq 1$) prefer product 1 and $1 - \alpha$ of customers prefer product 2.[5] This implies the following distribution of consumer valuations:

$$\begin{cases} \alpha \text{ of customers prefer product 1:} \\ \quad v_{1i} = 1 - x_i \quad v_{2i} = x_i \quad x_i \sim U[0, 1/2] \\ 1 - \alpha \text{ of customers prefer product 2:} \\ \quad v_{1i} = x_i \quad v_{2i} = 1 - x_i \quad x_i \sim U[0, 1/2] \end{cases}. \tag{4}$$

Note that our basic model is a special case of Equation (4) because distribution (4) reduces to Equation (3) when $\alpha = 1 - \alpha = 1/2$. The boundary case of

[4] Allowing a positive cost does not qualitatively affect our results but does significantly complicate the analysis.

[5] Our analyses focus on asymmetry in product popularity. We have proved that the same results hold qualitatively in a model where the products differ in quality. Detailed analysis of this model can be obtained by contacting the authors.

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$\alpha = 1$ is a standard vertical differentiation model in which all consumers prefer product 1 to product 2 although they differ in the magnitude in which they prefer product 1 over product 2. Proposition 4 summarizes the impact of asymmetry in demand on the profit advantage of probabilistic selling.

Proposition 4 (Impact of Demand Asymmetry).
(a) *Differentiation in product popularity weakly reduces the profit advantage of probabilistic selling, i.e.,* $\partial \Delta\Pi / \partial\alpha \le 0$.
(b) *Introducing a probabilistic good reduces price differentiation between component goods, i.e.,* $p_1^{\mathrm{TS}} > p_2^{\mathrm{TS}}$, $p_1^{\mathrm{PS}} = p_2^{\mathrm{PS}}$.

Proposition 4(a) suggests that the seller gains less from introducing a probabilistic good when the component products differ in their popularity than in the absence of such asymmetry. This is because demand asymmetry allows the seller to price discriminate even under traditional selling (i.e., $p_1^{\mathrm{TS}} > p_2^{\mathrm{TS}}$) and hence reduces the value of introducing a probabilistic good. However, it is important to notice that, as exemplified by the example in §2.3.1, introducing a probabilistic good can improve profit even if all consumers prefer the same product (i.e., $\alpha = 1$).

Proposition 4(b) reveals an interesting result: The prices of the component products are less differentiated under PS than TS (i.e., $p_1^{\mathrm{TS}} > p_2^{\mathrm{TS}}, p_1^{\mathrm{PS}} = p_2^{\mathrm{PS}}$).[6] To understand this, please notice that under TS, the only way for the seller to segment the market is to adopt differentiated prices. However, under PS, the seller can segment the market by selling a discounted probabilistic good, i.e., sell the probabilistic good rather than the less popular component product (at a discounted price) to those consumers with weak preferences for the more popular item. This allows the seller to charge relatively high prices for both component products and, thus, maintain high margins on sales to consumers that strongly prefer the less popular product. This finding that the prices of the component products are less differentiated under PS than TS implies that knowing demand for individual products can be more critical under TS than under PS. The importance of this finding will soon become more apparent when we discuss the impact of demand uncertainty below.

3.2. Demand Uncertainty (DU)

In the preceding section, we assumed that product 1 is more popular than product 2 (i.e., $\alpha > 1/2$). In this section, we incorporate seller uncertainty by assuming that product 1 and product 2 are equally likely to be the more popular good and that the seller does not know which will be the more popular product when making its pricing decision. Specifically, the seller does not know if $\alpha > 1/2$ will apply to product 1 or product 2. Other assumptions remain the same as in §3.1.

Let L (where $L = \mathrm{DU}, \mathrm{NDU}$) denote the type of the market, i.e., with and without demand uncertainty. Let Π_L^s denote the profit under strategy s ($s = \mathrm{TS}, \mathrm{PS}$) in market L. Note that the difference $\Pi_{\mathrm{NDU}}^s - \Pi_{\mathrm{DU}}^s$ measures the profit loss caused by uncertainty under strategy s ($s = \mathrm{TS}, \mathrm{PS}$), and the difference $\Pi_L^{\mathrm{PS}} - \Pi_L^{\mathrm{TS}}$ measures the profit advantage of probabilistic selling in market L, respectively. Proposition 5 summarizes our key results.

Proposition 5 (Impact of Demand Uncertainty).
(a) *Sellers facing demand uncertainty benefit more from probabilistic selling than sellers without such uncertainty* (*i.e.*, $\Pi_{\mathrm{DU}}^{\mathrm{PS}} - \Pi_{\mathrm{DU}}^{\mathrm{TS}} \ge \Pi_{\mathrm{NDU}}^{\mathrm{PS}} - \Pi_{\mathrm{NDU}}^{\mathrm{TS}}$).
(b) *Demand uncertainty increases the profit advantage of probabilistic selling because introducing a probabilistic good weakens or eliminates the dependence of pricing decisions on the identity of the more popular product, which reduces the profit loss caused by demand uncertainty* (*i.e.*, $\Pi_{\mathrm{NDU}}^{\mathrm{TS}} - \Pi_{\mathrm{DU}}^{\mathrm{TS}} > \Pi_{\mathrm{NDU}}^{\mathrm{PS}} - \Pi_{\mathrm{DU}}^{\mathrm{PS}}$).

The finding that demand uncertainty increases the profit advantage of probabilistic selling is intriguing. Such a positive effect of demand uncertainty on the profit advantage of probabilistic selling occurs because probabilistic selling can buffer against demand uncertainty, thus offering an additional reason for introducing a probabilistic good. In the absence of demand uncertainty (see §3.1), under the traditional selling strategy, it is optimal to charge a price premium for the more popular product (i.e., $P_2^{\mathrm{TS}} < P_1^{\mathrm{TS}}$). However, in the presence of demand uncertainty, without knowing which good will be more popular, the seller is unable to set prices correctly based on the (unknown) demand for each product. This information disadvantage leads to a lower profit compared to the case without demand uncertainty. However, this information disadvantage can be removed by introducing a probabilistic good. First, the price of the probabilistic good is not affected by the identity of the "hot" product because the value of a probabilistic good is determined by the expected value of its component products (e.g., the price of a "probabilistic toy" is the same regardless if toy A or toy B will turn out to be the hot toy). Second, introducing a probabilistic good also makes the prices of the component goods less sensitive to the demand of individual products. As we pointed out in the discussion of Proposition 4(b), by introducing a probabilistic good, the seller has less need to use differentiated prices

[6] In the alternative model of demand based on asymmetric quality, the component good prices under PS are not identical. However, we still have the result that the prices of the component products are less differentiated under PS than TS (i.e., $p_1^{\mathrm{TS}} - p_2^{\mathrm{TS}} > p_1^{\mathrm{PS}} - p_2^{\mathrm{PS}}$).

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because she can segment the market using the probabilistic good. Furthermore, a probabilistic good can cushion the damage caused by suboptimal prices. For instance, under traditional selling, if a price is set too high due to the seller's failure to perfectly forecast the demand for its product, the firm loses an entire sale whereas, under probabilistic selling, a missed consumer will not be lost altogether but will instead purchase the probabilistic good (albeit at a lower margin than the traditional product).

3.3. Capacity Constraints and Demand Uncertainty (CC and DU)

We now consider a seller with both limited capacity and demand uncertainty. Let K denote the total number of units of each product that the seller has the capacity to produce. As in §3.2, the seller is uncertain about which product will be the more popular product. As the degree of the product asymmetry is higher (i.e., α is larger), the seller faces greater uncertainty.

Let $\Pi^s_{\text{DU\&CC}}$ denote profit of strategy s ($s = \text{TS}, \text{PS}$) with DU and CC. The seller faces the following maximization problem:

Traditional selling

$$\begin{cases} \Pi^{\text{TS}}_{\text{DU\&CC}} = \max\limits_{P_1^{\text{TS}},\, P_2^{\text{TS}}} P_1^{\text{TS}} D_1^{\text{TS}} + P_2^{\text{TS}} D_2^{\text{TS}} \\ \text{s.t.} \quad D_1^{\text{TS}} \le K, \quad D_2^{\text{TS}} \le K, \quad P_1^{\text{TS}} = P_2^{\text{TS}} \end{cases} \tag{5}$$

Probabilistic selling

$$\begin{cases} \Pi^{\text{PS}}_{\text{DU\&CC}} = \max\limits_{P_1^{\text{PS}},\, P_2^{\text{PS}},\, P_O^{\text{PS}},\, \varphi,\, X_o} P_1^{\text{PS}} D_1^{\text{PS}} + P_2^{\text{PS}} D_2^{\text{PS}} + P_O^{\text{PS}} X_O \\ \text{s.t.} \quad D_1^{\text{PS}} + \varphi X_O \le K, \quad D_2^{\text{PS}} + (1-\varphi) X_O \le K, \\ \qquad X_O \le D_O^{\text{PS}}, \quad P_1^{\text{PS}} = P_2^{\text{PS}}, \end{cases} \tag{6}$$

where X_O in Equation (6) is the number of probabilistic good sales. The first two constraints in Equation (6) ensure that total sales of each product (including each product's share of the probabilistic good sales) do not exceed capacity constraints. The third constraint ensures that the number of probabilistic good sales chosen by the seller does not exceed the demand for the probabilistic good. The last constraint restricts the seller from charging a price premium for the more popular product because it does not know which product will be more popular.

Recall that Π^s_{DU} denotes the profit under strategy s in market with DU but without CC. It is obvious that imposing an additional constraint on the seller can not improve profit under either strategy, i.e., $\Pi^s_{\text{DU\&CC}} - \Pi^s_{\text{DU}} \le 0$, $s = \text{TS}, \text{PS}$. Notice that $(\Pi^{\text{PS}}_{\text{DU\&CC}} - \Pi^{\text{TS}}_{\text{DU\&CC}})$ and $(\Pi^{\text{PS}}_{\text{DU}} - \Pi^{\text{TS}}_{\text{DU}})$ measure the profit advantage of probabilistic selling in markets with and without capacity constraints, respectively, when the market has demand uncertainty.

The impact of capacity constraints in the presence of DU is summarized in Proposition 6.

Proposition 6 (Effect of Capacity Constraints, with DU). *In the presence of demand uncertainty, capacity constraints*

(a) *Increase the profit advantage of probabilistic selling when capacity is in a midrange and demand uncertainty is sufficiently large.*

(b) *Decrease the profit advantage of probabilistic selling when capacity is very small.*

(c) *Have no effect on the profit advantage of probabilistic selling when capacity is sufficiently large.*

Formally,

$$(\Pi^{\text{PS}}_{\text{DU\&CC}} - \Pi^{\text{TS}}_{\text{DU\&CC}}) - (\Pi^{\text{PS}}_{\text{DU}} - \Pi^{\text{TS}}_{\text{DU}}) \cdot \begin{cases} > 0 & \text{if } \underline{K} < K < \bar{K} \text{ and } \alpha > \hat{\alpha} \\ = 0 & \text{if } K \ge \bar{K} \\ < 0 & \text{if } K \le \underline{K}. \end{cases}$$

Proposition 6 shows an important result: Capacity constraints can be a favorable factor for the profit advantage of probabilistic selling when a seller faces demand uncertainty. Demand uncertainty and capacity constraints combine to create two problems for the seller: (1) demand-capacity mismatch, and (2) inefficiency. Introducing a probabilistic good can help the seller to overcome both.

First, the coexistence of demand uncertainty and capacity constraints can create mismatch between demand and capacity (i.e., one product has unfulfilled demand but the other has excess capacity). Such a mismatch problem is hard to resolve under the traditional selling strategy because, without knowing which product will be perceived more favorably by consumers, the seller is unable to use prices to optimally shift demand towards the product with underutilized capacity. However, when the seller offers a probabilistic good, all consumers with weak preferences (although a majority of which may prefer the higher demand product) will buy the probabilistic good, but only some of these consumers will be assigned to the ex post high demand product. As a result, the seller, without knowing which product will have excess capacity, is able to spread aggregate demand across the two goods more evenly, which increases capacity utilization.

Second, the co-existence of demand uncertainty and capacity constraints can create inefficiency (i.e., consumers with weak preferences may take the capacity of the preferred product away from consumers with strong preferences). Such inefficiency is hard to resolve under the traditional selling strategy because the seller is unable to use prices to ration consumers. However, under probabilistic selling, the seller can use the probabilistic good to effectively move some consumers with weak preferences to consume the

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"inferior" product, which saves capacity of the "superior" product for consumers with strong preferences.

As shown in Proposition 6, the positive effect of capacity constraints requires a midrange capacity and a sufficiently high level of demand uncertainty. When demand uncertainty is very low, neither mismatch nor inefficiency is a major problem for the seller. When capacity is sufficiently large ($K \geq \bar{K}$), capacity constraints do not affect the profit advantage of PS because, in this case, the seller has sufficient capacity to implement the unconstrained solution for either PS or TS. When capacity is sufficiently small ($K \leq \underline{K}$), capacity constraints reduce the profit advantage of introducing a probabilistic good because the seller would allocate few (if any) units of its scarce capacity to the lower-margin probabilistic good, which weakens the benefit of PS.

4. Discussion and Conclusions

4.1. Managerial Implications

This paper proposes and analyzes a creative way of selling products and services: probabilistic selling. Our analyses illustrate that this is a general marketing tool that can benefit many sellers in broad industries because its advantage is fundamentally driven by buyer heterogeneity in the strength of their preferences, which occurs in almost all markets. The generality of probabilistic selling's advantage has important managerial implications, especially because probabilistic goods are not concrete but "virtual" goods. Offering probabilistic goods can be an innovative way to extend one's product line without incurring the cost of developing new products or services, which can be a significant benefit in situations where new product development or provision is difficult or impractical due to resource constraints. The "virtual" nature also allows flexibility in adjusting the length of product line (e.g., a cruise line can flexibly offer a four-day "Eastern or Western Bahamas" cruise only in the peak season but not in the off-peak season). Some examples of probabilistic selling already exist in practice. For example, Circus Circus in Las Vegas sells "run-of-the-house" rooms at a discounted rate through travel intermediaries (such as vacationstogo.com) but charges a premium to guarantee a room in a certain tower; many restaurants offer a house wine (from an undisclosed winery) at a discounted price but customers can pay a higher price to select a certain label. However, the number of additional potential applications for manufacturers, service providers, and retailers is almost limitless. For instance, a hotelier could charge a premium to guarantee a room on a specific floor; a university could set higher tuition for students who want to assure themselves a particular slate of courses rather than be subject to uncertainty about which sections or courses may be open this semester; or a dentist could charge an additional fee if the patient wants to receive service from a particular dental hygienist.[7]

The results in this paper also provide insight into which markets are likely to see the greatest benefit from introducing probabilistic goods. In particular, our findings suggest that factors which enhance the profitability of the probabilistic selling strategy include low marginal costs, buyer heterogeneity in their tastes, moderate horizontal differentiation between the component goods, similar aggregate demand across component products, and demand uncertainty (especially when combined with capacity constraints). A multiproduct seller should seek to implement the probabilistic selling strategy for products or services with the above characteristics. Consider a stage theatre that offers the same show over the course of a week. The probabilistic selling strategy could be applied by raising the current ticket prices and then offering a discounted probabilistic good(s), being careful to avoid grouping nights that are too asymmetric in popularity. For example, the theatre could offer a "weekend ticket" (i.e., the consumer is assigned to either a Friday or Saturday night performance) or a "midweek ticket" (i.e., the consumer is assigned to either a Tuesday or Wednesday night performance). The probabilistic selling strategy could be particularly appealing for service providers with high variability in capacity usage (e.g., an auto repair shop or an ice skating rink for which its consumers may face long waits on some days or times but capacity and the staff are underutilized on some other days or times), especially when these patterns are unpredictable or the service provider faces other barriers to adjusting prices in response to variations in demand.

It is important to point out several issues related to the implementation of probabilistic selling. Like several other marketing strategies (e.g., advance selling and quantity discounts), probabilistic selling can suffer from arbitrage. Sales of the probabilistic good need to be nonrefundable, nontransferable, and nonexchangeable so that a purchaser of the probabilistic good faces some risk, else the probabilistic good would cannibalize all component product sales. Recent developments in technology are increasing the seller's capability to meet these required conditions for implementing probabilistic selling. For example, new technologies such as smart cards, RFID chips, biometric palm readers, and electronic tickets enable the seller to successfully attach a particular product or service to a particular consumer, which drastically increases the seller's capability to limit arbitrage

[7] We thank Steven Shugan for suggesting the house wine and university examples.

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(Xie and Shugan 2001, Shugan and Xie 2005) and to enforce the special policies required for the sales of probabilistic goods (e.g., nonrefundable). Even without advanced technologies, arbitrage is easily avoided for service industries in which services can be delivered at the time of purchase. Furthermore, advanced technologies enable sellers to implement more complex pricing schemes (Rust and Chung 2006), such as those required under probabilistic selling.

4.2. Limitations and Future Research

In this paper, we assumed symmetric costs. With asymmetric costs, consumers may expect that once a payment is made, the seller may have an incentive to avoid assigning the high-cost good as the probabilistic good (or the good that is in short supply if the firm faces asymmetric capacity constraints). This can undermine the profitability of probabilistic selling since it is essential that consumers are uncertain about the assignments of the probabilistic good at the time of purchase and believe that it could turn out to be either product. Although this will not be a problem for sellers with endogenous credibility (i.e., due to a long-term reputation effect), it would be useful to develop possible mechanisms to endogenously establish a seller's "uncertainty credibility." Recent increased popularity of various independent product information created by professional product reviewers and consumers due to advances in technology (Chen and Xie 2005, 2008) may provide ways for sellers to obtain such "uncertainty credibility." For example, *Consumer Reports WebWatch* spent approximately $38,000 booking airline seats, hotel rooms, and rental cars in order to report differences between opaque travel web sites with nonopaque sites (McGee 2003). The existence of such independent information helps to establish sellers' credibility. Recent developments of various new forms of consumer social interactions also create increasing new opportunities for the seller to overcome the commitment problems. For example, biddingfortravel.com, which has experienced over 100 million visits, has a section devoted exclusively to posting "winning bids" in which *Priceline* users report what bids were accepted and what flight itinerary, hotel property, or car rental company was received. When random past assignments are reported in such an online forum, consumers may believe that future assignments are also random. Finally, an intermediary such as online travel agent might also serve as a commitment mechanism.

Another important dimension would be to incorporate risk aversion in the analysis. Attitudes toward the probabilistic good depend not only on the strength of one's preferences (as accounted for in this current paper), but also on one's disposition toward risk. Probabilistic selling may enable the seller to discriminate according to variation in risk aversion. A third important dimension is more elaborate market settings such as those with competition or an intermediate channel structure. As Liu and Zhang (2006) do in the context of personalized pricing, it would be interesting to consider how the probabilistic selling strategy would alter the market interactions between multiple sellers or channel members. A fourth potential extension would be to study settings where there are more complex interactions in the demand for the component products (e.g., variations in the value of each consumer's ideal product and/or nonlinear transportation costs).[8]

Finally, future research could generalize the probabilistic selling strategy as a hybrid with bundling. Venkatesh and Kamakura (2003) demonstrate that mixed bundling (i.e., selling a bundle in addition to the component products) is not strictly more profitable than pure component selling (i.e., selling only component products) if the component products are too close of substitutes. In particular, when consumers are interested in consuming only a single good (e.g., only want to see a single movie tonight—"Hannibal Rising" *or* "The Hitcher"), as is true for the Hotelling and Salop circle model, there is no advantage from mixed bundling. However, we demonstrate that the probabilistic selling strategy can be strictly advantageous to pure component selling in this setting. This suggests that it may be desirable to generalize the definition of a bundle. Traditionally, in the bundling literature, it is assumed that each component of the bundle is included with probability one. In contrast, a probabilistic good, as defined in the current paper, could be thought of as a bundle where the probabilities of getting each particular component sum to one. More generally, it might be possible to offer a "probabilistic bundle" where each component good has a probability less than one of being allocated to the buyer but the sum of the probabilities across all the component products is not necessarily equal to one, e.g., a book club that sends each out of 10 books with probability of 1/2 so that the member receives on average 5 books per month. It would be interesting to explore when such a probabilistic bundle might be optimal.[9]

[8] It is unclear whether it would still be optimal for the seller to make symmetric assignments of the probabilistic good in such settings. For example, with extreme risk aversion and/or nonlinear transportation costs, the probabilistic good can be a poor alternative even for consumers with only moderately strong preferences. Thus, the cannibalization effect from asymmetric assignments may not be as severe. We thank an anonymous reviewer for this insight.

[9] We thank the area editor for pointing out this interesting potential extension.

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Appendix

Summary of Notations

s	Type of strategy, $s = \text{TS}$ (Traditional selling), PS (Probabilistic selling)
J	Specified component product, $j = 1, 2$
x_i	A taste parameter that is i.i.d. across consumers
T	A fit-cost-loss coefficient that reflects the loss of utility from not consuming one's ideal product
v_{ji}	Value of product j to consumer i
c_j	Marginal cost of product j
P_j^s	Price of the specified product j under strategy s; $j = 1, 2$; $s = \text{TS}, \text{PS}$
P_o^{PS}	Price of the probabilistic ("opaque") good
φ	The probability that product 1 is allocated as the probabilistic good
$\hat{x}_2$	The maximum value of x_i for which a consumer will purchase product 2
$\hat{x}_1$	The minimum value of x_i for which a consumer will purchase product 1
D_j^{TS}	Demand for the specified product j under TS
D_j^{PS}	Demand for the specified product j under PS
D_o^{PS}	Demand for the probabilistic good under PS
$F(x)$	Cumulative distribution function in the general model
N	Number of component goods in the circle model
Π^s	Profit of the seller under strategy s; $s = \text{TS}, \text{PS}$
α	When there is differentiation in popularity, the fraction of consumers that prefer the high-demand product over the low-demand product
K	The total number of units of each product that the seller has the capacity to produce
X_O	The number of units of the probabilistic good sold
Π^s_{DU}	Profit of strategy s ($s = \text{TS}, \text{PS}$) when there is demand uncertainty and there are not capacity constraints
Π^s_{NDU}	Profit of strategy s ($s = \text{TS}, \text{PS}$) when there is not demand uncertainty and there are not capacity constraints
$\Pi^s_{\text{DU\&CC}}$	Profit of strategy s ($s = \text{TS}, \text{PS}$) when there is both demand uncertainty and constrained capacity

In what follows, we sketch the proofs for all of the propositions in the paper. Full details are available in the Technical Appendix online at http://mktsci.journal.informs.org.

Sketch of Proof of Lemma 1 and Proposition 1

Traditional Selling Strategy

For the value distribution in Equation (1), demand for each product is given by:

$$D_j^{\text{TS}} = \begin{cases} 0 & \text{if } P_j^{\text{TS}} \geq 1 \\ \dfrac{1 - P_j^{\text{TS}}}{t} & \text{if } 1 > P_j^{\text{TS}} \geq 2 - t - P_{-j}^{\text{TS}} \\ \dfrac{t - P_j^{\text{TS}} + P_{-j}^{\text{TS}}}{2t} & \text{if } P_j^{\text{TS}} < 2 - t - P_{-j}^{\text{TS}}. \end{cases} \tag{7}$$

Maximizing $\Pi^{\text{TS}} = \sum_{j=1}^{2} (P_j^{\text{TS}} - c) D_j^{\text{TS}}$ yields the profit and prices listed in Table A.1.

Probabilistic Selling Strategy

Without loss of generality, we restrict attention to $\varphi \geq 1/2$. We can divide the Hotelling line into three segments: consumers with $x_i \leq \hat{x}_1$ purchase product 1, consumers with $x_i \geq \hat{x}_2$ purchase product 2, and consumers with $\hat{x}_1 < x_i < x_0$ purchase the probabilistic good. For the probabilistic good to have positive sales, then x_0 must be strictly larger than $\hat{x}_1$. There are two possible outcomes: full coverage (i.e., $x_0 = \hat{x}_2$) and incomplete coverage (i.e., $x_0 < \hat{x}_2$).

Full coverage (FC): The seller's profit is given by

$$\Pi_{\text{FC}}^{\text{PS}} = (P_1^{\text{PS}} - c)(\hat{x}_1) + (P_2^{\text{PS}} - c)(1 - \hat{x}_2) + (P_O^{\text{PS}} - c)(\hat{x}_2 - \hat{x}_1). \tag{8}$$

For a given $\hat{x}_1$ and $\hat{x}_2$, the prices that extract the maximum possible consumer surplus are

$$P_O^{\text{PS}} = 1 - \varphi t \hat{x}_2 - (1 - \varphi) t (1 - \hat{x}_2) \tag{9}$$

$$P_2^{\text{PS}} = 1 - t(1 - \hat{x}_2) \tag{10}$$

$$P_1^{\text{PS}} = (1 - \varphi) t (1 - 2\hat{x}_1) + P_o^{\text{PS}}. \tag{11}$$

Maximizing Equation (8) with respect to $\hat{x}_1$ and $\hat{x}_2$ yields $\hat{x}_1 = 1/4$ and $\hat{x}_2 = (1 + \varphi)/(4\varphi)$, and a resulting profit of:

$$\Pi_{\text{FC}}^{\text{PS}} = 1 - c - \frac{t(5\varphi - 1)}{8\varphi}. \tag{12}$$

Notice that $\partial \Pi_{\text{FC}}^{\text{PS}} / \partial \varphi = -t/(8\varphi^2) < 0$. Comparing $\Pi_{\text{FC}}^{\text{PS}}$ to the profits under TS:

$$\Pi_{\text{FC}}^{\text{PS}} - \Pi^{\text{TS}} = \begin{cases} \dfrac{t(1-\varphi)}{8\varphi} > 0 \quad \text{if } c \leq 1 - t \\ 1 - c - \dfrac{(1-c)^2}{2t} - \dfrac{t(5\varphi - 1)}{8\varphi} \begin{cases} > 0 & \text{if } c < \hat{c} \\ < 0 & \text{if } c > \hat{c} \end{cases} \\ \qquad \text{otherwise} \end{cases},$$

$$\text{where } \hat{c} = 1 - t + \frac{t\sqrt{\varphi(1-\varphi)}}{2\varphi}, \quad \frac{\partial \hat{c}}{\partial \varphi} < 0. \tag{13}$$

Incomplete coverage (IC): The seller's profit is given by

$$\Pi_{\text{IC}}^{\text{PS}} = (P_1^{\text{PS}} - c)(\hat{x}_1) + (P_2^{\text{PS}} - c)(1 - \hat{x}_2) + (P_O^{\text{PS}} - c)(x_0 - \hat{x}_1). \tag{14}$$

For a given $\hat{x}_1$, $\hat{x}_2$, and x_0, the prices that extract the maximum possible consumer surplus are

$$P_2^{\text{PS}} = 1 - t(1 - \hat{x}_2) \tag{15}$$

$$P_O^{\text{PS}} = 1 - \varphi t x_o - (1 - \varphi) t (1 - x_o) \tag{16}$$

$$P_1^{\text{PS}} = (1 - \varphi) t (1 - 2\hat{x}_1) + P_o^{\text{PS}}. \tag{17}$$

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Table A.1 The Optimal Prices and Profits in the Basic Model

	Traditional selling	Probabilistic selling	Comparison
Specific goods			
Price	$P_j^{TS} = \begin{cases} \frac{2-t}{2} & \text{if } c \le 1-t \\ \frac{1+c}{2} & \text{if } c > 1-t \end{cases}$	$P_j^{TS} = \begin{cases} 1-\frac{t}{4} & \text{if } c < \bar{c} \\ \frac{1+c}{2} & \text{if } c \ge \bar{c} \end{cases}$	$\Delta p_j = \begin{cases} \frac{t}{4} > 0 & \text{if } c \le 1-t \\ \frac{2(1-c)-t}{4} > 0 & \text{if } 1-t < c < \bar{c} \\ 0 & \text{if } c \ge \bar{c} \end{cases}$
Sales	$D_j^{TS} = \begin{cases} \frac{1}{2} & \text{if } c \le 1-t \\ \frac{1-c}{2t} & \text{if } c > 1-t \end{cases}$	$D_j^{PS} = \begin{cases} \frac{1}{4} & \text{if } c < \bar{c} \\ \frac{1-c}{2t} & \text{if } c \ge \bar{c} \end{cases}$	$\Delta D_j = \begin{cases} -\frac{1}{4} < 0 & \text{if } c \le 1-t \\ -\frac{2(1-c)-t}{4t} < 0 & \text{if } 1-t < c < \bar{c} \\ 0 & \text{if } c \ge \bar{c} \end{cases}$
Probabilistic good			
Price		$P_o^{PS} = \begin{cases} 1-\frac{t}{2} & \text{if } c < \bar{c} \\ \text{N/A} & \text{if } c \ge \bar{c} \end{cases}$	
Sales		$D_o^{PS} = \begin{cases} \frac{1}{2} & \text{if } c < \bar{c} \\ 0 & \text{if } c \ge \bar{c} \end{cases}$	
φ^*		$\varphi = \begin{cases} \frac{1}{2} & \text{if } c < \bar{c} \\ \text{N/A} & \text{if } c \ge \bar{c} \end{cases}$	
Total demand	$D^{TS} = \begin{cases} 1 & \text{if } c \le 1-t \\ \frac{1-c}{t} & \text{if } c > 1-t \end{cases}$	$D^{PS} = \begin{cases} 1 & \text{if } c < \bar{c} \\ \frac{1-c}{t} & \text{if } c \ge \bar{c} \end{cases}$	$\Delta D = \begin{cases} 0 & \text{if } c \le 1-t \\ \frac{t-(1-c)}{t} > 0 & \text{if } 1-t < c < \bar{c} \\ 0 & \text{if } c \ge \bar{c} \end{cases}$
Total profit	$\Pi^{TS} = \begin{cases} 1-\frac{t}{2}-c & \text{if } c \le 1-t \\ \frac{(1-c)^2}{2t} & \text{if } c > 1-t \end{cases}$	$\Pi^{PS} = \begin{cases} 1-\frac{3t}{8}-c & \text{if } c < \bar{c} \\ \frac{(1-c)^2}{2t} & \text{if } c \ge \bar{c} \end{cases}$	$\Delta\Pi = \begin{cases} \frac{t}{8} > 0 & \text{if } c \le 1-t \\ \frac{AB}{8t} > 0 & \text{if } 1-t < c < \bar{c} \\ 0 & \text{if } c \ge \bar{c} \end{cases}$

Notes. $\bar{c} = 1 - t/2$, $A = 2(1-c) - t$, $B = 3t - 2(1-c)$, $\Delta p_j = P_j^{PS} - P_j^{TS}$, $\Delta D_j = D_j^{PS} - D_j^{TS}$, $\Delta D = D^{PS} - D^{TS}$, $\Delta\Pi = \Pi^{PS} - \Pi^{TS}$.

Maximizing Equation (14) w.r.t. $\hat{x}_1$, $\hat{x}_2$, and x_o yields $\hat{x}_1 = 1/4$, $\hat{x}_2 = 1-(1-c)/(2t)$, and $x_o = (1-c-t(1-\varphi))/(2t(2\varphi-1))$. This outcome represents positive sales of the probabilistic good, i.e., $x_0 > \hat{x}_1$, only if $c < \bar{c}$ where $\bar{c} = 1 - t/2$. Furthermore, the market is incompletely covered only if $x_0 < \hat{x}_2$ or:

$$c > 1 - \frac{t(3\varphi-1)}{2\varphi} \equiv \hat{\hat{c}}. \tag{18}$$

The resulting profit is:[10]

$$\Pi_{IC}^{PS} = \frac{4\varphi(1-c)^2 - 4t(1-c)(1-\varphi) + (1-\varphi)t^2}{8t(2\varphi-1)} \tag{19}$$

Notice that $\partial\Pi_{FC}^{PS}/\partial\varphi = -(2(1-c)-t)^2/(8t(2\varphi-1)^2) < 0$. Assuming $c < \bar{c}$ and condition (18) is met, i.e., the conditions under which there is incomplete coverage with positive sales of the probabilistic good, Π_{IC}^{PS} in comparison to Π^{TS} is

$$\Pi_{IC}^{PS} - \Pi^{TS} = \frac{(1-\varphi)(2(1-c)+t)^2}{8t(2\varphi-1)} > 0. \tag{20}$$

Under PS, the seller chooses either full coverage or incomplete coverage depending on which outcome yields the highest profit. Thus, PS is strictly preferred to TS if either of these two outcomes yields higher profit. Using the derivations above, this implies

$$\text{Max}[\Pi_{FC}^{PS}, \Pi_{IC}^{PS}] > \Pi^{TS} \quad \text{if} \begin{cases} \text{(a)} \ c \le 1-t; \\ \text{(b)} \ c > 1-t \ \& \ c < \hat{c}; \quad \text{or} \\ \text{(c)} \ 1-\frac{t}{2} > c > 1-t \ \& \ c < \hat{\hat{c}}. \end{cases} \tag{21}$$

Using the definitions in Equations (13) and (18), we have $\hat{\hat{c}} < \hat{c}$ and $\hat{c} < 1 - t/2 \ \forall \varphi \ge 1/2$, thus proving part (a) of Lemma 1. Part (c) is evident due to demand symmetry and because we have shown both $\partial\Pi_{FC}^{PS}/\partial\varphi < 0$ and $\partial\Pi_{IC}^{PS}/\partial\varphi < 0$

[10] The incomplete coverage case can only be optimal for a given φ if $\varphi \ne 1/2$. Thus, we do not need to be concerned that the denominator of Equation (18) is zero when $\varphi = 1/2$.

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for $\varphi > 1/2$. Table A.1 records the resulting prices, demand, and profit under PS, taking into account that the seller may wish not to offer the probabilistic good if costs are sufficiently large.

Part (b) of Lemma 1 involves straightforward comparisons of the demand and prices under PS and TS. Part (d) of Lemma 1 records additional comparative statics. For example, $\partial\Delta\Pi/\partial t$ is found using Π^{TS} from Table A.1 and Π^{PS}_{FC} and Π^{PS}_{IC} from Equations (12) and (19), respectively:

$$\frac{\partial(\Pi^{PS}_{FC}-\Pi^{TS})}{\partial t} = \begin{cases} \dfrac{1-\varphi}{8\varphi} > 0 & \text{if } t \le 1-c \\ \dfrac{1}{8}\left[\dfrac{4(1-c)^2}{t^2} - \dfrac{1}{\varphi} - 5\right] \begin{cases} > 0 & \text{if } t < \hat{t} \\ > 0 & \text{if } t > \hat{t} \end{cases} & \text{otherwise,} \end{cases}$$

$$\text{where } \hat{t} = 2(1-c)\sqrt{\frac{\varphi}{5\varphi-1}} \tag{22}$$

$$\frac{\partial(\Pi^{PS}_{IC}-\Pi^{TS})}{\partial t} = -\frac{(1-\varphi)(4(1-c)^2-t^2)}{8(2\varphi-1)t^2} < 0. \tag{23}$$

Equation (23) is only valid when PS leads to incomplete market coverage, i.e., $1-c < t < 2(1-c)$. Thus, the $\bar{t}$ referred to in Lemma 1(d) must lie within the interval $[1-c, \hat{t}]$. Proposition 1 summarizes the key results.

Sketch of Proof of Proposition 2

We start by proving part (b) of Proposition 2, i.e., that symmetric assignments of the probabilistic good are optimal. Suppose that $\varphi \ge 1/2$ so that under PS, the seller partitions the market so that any consumer with $x_i \le \hat{x}_1$ purchases product 1, any consumer with $\hat{x}_1 < x_i \le x_0$ purchases the probabilistic good, and any consumer with $x_i \ge \hat{x}_2$ purchases product 2, where $x_0 \le \hat{x}_2$. For any such set $(\hat{x}_1, \hat{x}_2, x_0)$, profit under PS is maximized at either $\varphi = 1/2$ or $\varphi = 1$.

The seller's profit is given by

$$\Pi^{PS}(\varphi) = (P_1^{PS}-c)F(\hat{x}_1) + (P_2^{PS}-c)(1-F(\hat{x}_2-\varepsilon)) + (P_o^{PS}-c)(F(x_0)-F(\hat{x}_1)), \tag{24}$$

where $F(x)$ is the cumulative distribution and ε is an arbitrarily small number.

To induce the desired consumer behavior, prices must be set accordingly:

$$P_o^{PS} = 1-\varphi t x_0 - (1-\varphi)t(1-x_0) \tag{25}$$

$$P_1^{PS} = (1-\varphi)t(1-2\hat{x}_1) + P_o^{PS} \tag{26}$$

$$P_2^{PS} = 1-t(1-\hat{x}_2). \tag{27}$$

If $x_0 > 1/2$, then $\partial P_o^{PS}/\partial\varphi < 0$, $\partial P_1^{PS}/\partial\varphi < 0$, and $\partial P_2^{PS}/\partial\varphi = 0$. Thus, for $x_0 > 1/2$, profit is higher at $\varphi = 1/2$ than at any $\varphi > 1/2$.

For $x_0 \le 1/2$, using the prices given in Equations (25), (26), and (27), we find

$$\frac{\partial\Pi^{PS}}{\partial\varphi} = F(x_o)(1-x_o(1+t)) - tF(\hat{x}_1)(1-2\hat{x}_1) \equiv \Omega. \tag{28}$$

Ω is not a function of φ. Thus, if $\Omega > 0$, profit under PS is maximized at $\varphi = 1$, i.e., TS and PS yield equivalent profit. If $\Omega < 0$, profit under PS is maximized at $\varphi = 1/2$.

Now turn to part (a) of Proposition 2. For part 1, at $\varphi = 1/2$, by setting a price for the probabilistic good at $P_o^{PS} = 1 - t/2$ and keeping the same prices for the component goods as under TS, the seller will sell the same number of component goods sales but will make additional sales to the customers who were unserved under TS. Such sales enhance profits as long as $c < \bar{c}$.

For part 2 of (a) in Proposition 2, the firm could sell the probabilistic good to any consumer located at $x_i = 1/2$ by setting a price $P_o^{PS} = 1 - t/2$, i.e., a price that generates as least as high a margin as was obtained under TS. Furthermore, the seller could raise prices of at least one component good and still make positive sales. Hence, profit under PS is strictly larger than profit under TS.

Sketch of Proof of Proposition 3

Traditional Selling Strategy

If each component product is priced at P_j^{TS}, profit is

$$\Pi^{TS}(P_j^{TS}) = \begin{cases} P_j^{TS} & \text{if } P_j^{TS} \le 1-\dfrac{t}{2N} \\ 2N\left(\dfrac{1-P_j^{TS}}{t}\right)(P_j^{TS}-c) & \text{if } 1 > P_j^{TS} \ge 1-\dfrac{t}{2N} \\ 0 & \text{if } P_j^{TS} \ge 1. \end{cases} \tag{29}$$

Maximizing $\Pi^{TS}(P_j^{TS})$ w.r.t. P_j^{TS}, we find the maximum profit that is available under TS:

$$\Pi^{TS} = \begin{cases} 1-\dfrac{t}{2N}-c & \text{if } t \le N(1-c) \\ \dfrac{N(1-c)^2}{2t} & \text{if } t > N(1-c). \end{cases} \tag{30}$$

Probabilistic Selling Strategy

We start by assuming the probabilistic goods are created according to part (c) of Proposition 3 in order to demonstrate parts (a) and (b) of Proposition 3. Then, we show that the seller can not improve its profit by creating additional or alternative probabilistic goods.

Since each consumer has an expected value of $1 - t/(2N)$ for the probabilistic good that combines the two component products that lie closest to his or her ideal point, the seller offers each probabilistic good at the price $P_o^{PS} = 1 - t/(2N)$. Profit under PS is given by

$$\Pi^{PS}(P_j^{PS}) = 2N\left(\frac{1-P_j^{PS}}{t}\right)(P_j^{PS}-c) + 2N\left(\frac{1}{2N}-\frac{1-P_j^{PS}}{t}\right)\left(1-\frac{t}{2N}-c\right). \tag{31}$$

This profit is maximized at $P_j^{PS} = 1 - t/(4N)$, yielding a profit of

$$\Pi^{PS} = 1-\frac{3t}{8N}-c. \tag{32}$$

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Comparing the profit under PS and TS:

$$\Pi^{PS}-\Pi^{TS}=\begin{cases}\dfrac{t}{8N}>0 & \text{if } c\le 1-\dfrac{t}{N}\\[2ex] 1-c-\dfrac{3t}{8N}-\dfrac{N(1-c)^2}{2t}\begin{cases}>0 & \text{if } c<\bar{c}\\ \le 0 & \text{if } c\ge\bar{c}\end{cases}, & \\ \qquad\text{if } c>1-\dfrac{t}{N} & \end{cases}$$

$$\text{where } \bar{c}=1-\frac{t}{2N}. \quad (33)$$

Offering these probabilistic goods strictly increases profit if $c<\bar{c}$, thus proving part (a) of Proposition 3. Furthermore, $\partial\bar{c}/\partial N=t/(2N^2)>0$, thus proving part (b) of Proposition 3.

Now we verify part (c) of Proposition 3. Under the proposed equilibrium, there is complete market coverage and the probabilistic goods are priced so that no consumer earns a strictly positive surplus from purchasing a probabilistic good. Thus, the only way an alternative probabilistic good could increase profit is if it has a higher expected value to some consumer. Without loss of generality (due to symmetry), consider the segment of consumers located between two component products j and $j+1$. The proof consists of two parts: First, we show that it is not optimal to sell these consumers a probabilistic good that consists of any component good other than j or $j+1$. Second, we show that symmetrically assignments of j and $j+1$ are optimal. See the Technical Appendix for further details online at http://mktsci.journal.informs.org.

Sketch of Proof of Proposition 4

The analysis assumes that $c=0$ and $t=1$. Profit under TS and PS are given in Table A.2. Proposition 4 summarizes these results.

Traditional Selling Strategy

There are two potential pricing strategies: (a) $P_1^{TS}\ge 1/2$, $P_2^{TS}\ge 1/2$ and (b) $P_1^{TS}\ge 1/2$, $P_2^{TS}<1/2$. In scenario (a), the maximum obtainable profit of $\Pi^{TS}=1/2$. In scenario (b) there are "crossover" sales, i.e., some consumers who prefer product 1 will buy product 2. Profit is maximized at a corner solution in which there is full market coverage. Here, profit is $\Pi^{TS}=(1+2\alpha)^2/(16\alpha)$ which is strictly larger than 1/2 if $\alpha>1/2$.

Probabilistic Selling Strategy

We derive the equilibrium profit under PS in two parts. First, we assume $\varphi^*=1/2$. Then, we verify that $\varphi=1/2$ maximizes profit. If $\varphi=1/2$, then $P_o^{PS}=1/2$. The profit function is:

$$\begin{aligned}\Pi^{PS}&=2\alpha P_1^{PS}(1-P_1^{PS})+2(1-\alpha)P_2^{PS}(1-P_2^{PS})\\&\quad+(1-2\alpha(1-P_1^{PS})-2(1-\alpha)(1-P_2^{PS}))\tfrac{1}{2}.\end{aligned}\quad (34)$$

The maximum obtainable profit is $\Pi^{PS}=5/8$. Comparing this to the profit earned under TS:

$$\Delta\Pi=\Pi^{PS}-\Pi^{TS}=\frac{-4\alpha^2+6\alpha-1}{16\alpha}>0. \quad (35)$$

The comparative statics in Table A.2 immediately follow.

To verify that $\varphi^*=1/2$, we show that for $\varphi\ge 1/2$, profit under PS is convex in φ and strictly decreasing in φ at $\varphi=1/2$. Thus, profit under PS is maximized either with symmetric assignments (i.e., $\varphi^*=1/2$) or not selling the probabilistic good at all (i.e., $\varphi^*=1$).

Table A.2 Profits with Product Differentiation in Popularity

Strategy	Price	Profit
Traditional selling (TS)	$P_2^{TS}<P_1^{TS}$	$\Pi^{TS}=\dfrac{(1+2\alpha)^2}{16\alpha}$
Probabilistic selling (PS)	$P_2^{PS}=P_1^{PS}$, $\varphi^*=\dfrac{1}{2}$, $P_o^{PS}=\dfrac{1}{2}$	$\Pi^{PS}=\dfrac{5}{8}$
Comparison	$\Delta\Pi=\Pi^{PS}-\Pi^{TS}=\dfrac{-4\alpha^2+6\alpha-1}{16\alpha}>0$, $\dfrac{\partial\Delta\Pi}{\partial\alpha}=\dfrac{1-4\alpha^2}{16a^2}\le 0$	

Sketch of Proof of Proposition 5

Table A.3 gives the profit under TS and PS with and without demand uncertainty. Proposition 5 summarizes these results.

Traditional Selling Strategy

Without demand uncertainty, Table A.2 shows that optimal profit is achieved with prices $P_1^{TS}>P_2^{TS}$. With demand uncertainty, the seller can not sell the popular good at a price premium because it does not know whether product 1 or product 2 is the popular one. Thus, the seller maximizes its profit such that $P_1^{TS}=P_2^{TS}$. This yields a profit of $\Pi^{TS}=1/2$.

Probabilistic Selling Strategy

Under PS, $\varphi^*=1/2$ and $P_O^{PS}=1/2$. With demand uncertainty, the seller maximizes (34) s.t. $P_1^{PS}=P_2^{PS}$. This leads to the same prices and profit ($\Pi^{PS}=5/8$) as in the no demand uncertainty case.

Sketch of Proof of Proposition 6

Here, the seller faces both capacity constraints and demand uncertainty. Starting with the case where the seller faces only demand uncertainty (studied above), we add the additional constraints that production of each good can not exceed K. If K is sufficiently large, these constraints are not binding under either TS or PS. Thus, the profit under each strategy is unaffected by capacity constraints, and we have $\Delta_{DU}^{CC-NCC}=0$. Furthermore, for moderately large K, capacity constraints force a deviation from the unconstrained outcome under TS but do not affect PS. Thus, in this range, it

Table A.3 Profit with and Without Demand Uncertainty

	Uncertainty about relative product popularity		
Strategy	No uncertainty (NDU)	With uncertainty (DU)	Impact of uncertainty $\Delta\Pi_{DU}^t=\Pi_{DU}^t-\Pi_{NDU}^t$
Traditional selling (TS)	$\Pi_{NDU}^{TS}=\dfrac{(1+2\alpha)^2}{16\alpha}$	$\Pi_{DU}^{TS}=\dfrac{1}{2}$	$\Delta\Pi_{DU}^{TS}=-\dfrac{(2\alpha-1)^2}{16\alpha}<0$
Probabilistic selling (PS)	$\Pi_{NDU}^{PS}=\dfrac{5}{8}$	$\Pi_{DU}^{PS}=\dfrac{5}{8}$	$\Delta\Pi_{DU}^{PS}=0$
Comparing	(a) $\Pi_{DU}^{PS}>\Pi_{DU}^{TS}$, (b) $(\Pi_{DU}^{PS}-\Pi_{DU}^{TS})>(\Pi_{NDU}^{PS}-\Pi_{NDU}^{TS})$, (c) $\Delta\Pi_{DU}^{TS}<\Delta\Pi_{DU}^{PS}$		

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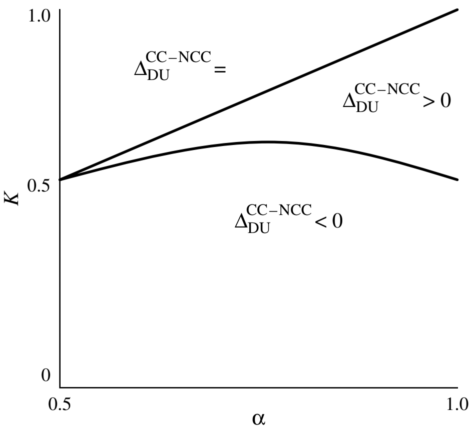
Figure A.1 Impact of Capacity Constraints When There Is Demand Uncertainty



must be the case that $\Delta_{DU}^{CC-NCC} > 0$. We then identify additional regions for which $\Delta_{DU}^{CC-NCC} > 0$.

Traditional Selling Strategy

The profit function under TS is given in the proof of Proposition 4. Facing capacity constraints and demand uncertainty, Equation (36) gives the maximum obtainable profit:

$$\Pi^{TS} = \begin{cases} \dfrac{K(2\alpha - K)}{2\alpha^2} & \text{if } K_f \le K < \dfrac{2\alpha+1}{4} \\ \dfrac{K(2(1-\alpha)-K)}{1-\alpha} & \text{if } K < K_f \end{cases},$$

$$\text{where } K_f = \frac{2\alpha(1-\alpha)}{1+\alpha}. \quad (36)$$

Probabilistic Selling Strategy

We focus only on identifying potential scenarios for which $\Delta_{DU}^{CC-NCC} > 0$. Using the Π^{TS} defined in Equation (36), we look at any probabilistic selling strategy that increases the magnitude of $(\Pi^{PS} - \Pi^{TS})$ (versus the value of $(\Pi_{DU}^{PS} - \Pi_{DU}^{TS})$ as reported in Table A.3).

If $K \ge 1/2$, the optimal strategy is to cover the entire market and set prices so that the capacity constraint for the popular good is just binding. Here, $\Delta_{DU}^{CC-NCC} > 0$ if $K > \alpha(3-2\alpha)/2 \equiv \underline{K}$.

If $K < 1/2$, full market coverage is not feasible. The seller has two feasible strategies in which positive sales of the probabilistic good would be sold: (a) sell all three products or (b) only sell the probabilistic product. Under each option, we find that $\Delta_{DU}^{CC-NCC} < 0$. Proposition 6 summarizes these results. Figure A.1 presents a graphical illustration.

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