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EFFECTS OF UNCERTAINTY REDUCTION ON WEIGHT OF COMPOSITE LAMINATES AT CRYOGENIC TEMPERATURES

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ABSTRACT

The effect of uncertainty reduction measures on the weight of laminates for cryogenic temperatures is investigated. The uncertainties in the problem are classified as error and variability. Probabilistic design is carried out to analyze the effect of reducing the uncertainty on the weight. For demonstration, variability reduction takes the form of quality control, while error is reduced by including the effect of chemical shrinkage in the analysis. It is found that the use of only error control leads to 12% weight reduction, the use of only quality control leads to 20% weight savings and the use of error and variability control measures together reduces the weight by 37%. In addition, the paper also investigates how to improve the accuracy and efficiency of probability of failure calculations (performed using Monte Carlo simulation technique). Approximating the cumulative distribution functions for strains is shown to lead to more accurate probability of failure estimations than the use of response surface approximations for strains.

NOMENCLATURE

E_1, E_2, G_{12} = elastic modulus along and transverse to fiber direction and shear modulus of a composite ply
v_{12} = major Poisson's ratio of a composite ply
T_{zero} = stress free temperature
T_{serv} = service temperature
α_1, α_2 = coefficient of thermal expansion along and transverse to fiber direction
θ, θ_1, θ_2 = ply orientation angles
t_1, t_2 = thickness of plies with angles θ_1 and θ_2, respectively
h = total laminate thickness
b_e = bound of error
ε_1, ε_2, γ_{12} = strains along and transverse to fiber direction and shear strain of a composite ply
N_x and N_y = mechanical loading in x and y directions
P_f and PSF = Probability of failure and probability sufficiency factor, respectively
Superscripts: U = upper limit; L = lower limit

1. INTRODUCTION

The design of composite laminates for liquid hydrogen tanks that operate at cryogenic temperatures involves several challenges. Large residual thermal strains develop because the thermal expansion coefficients are different in the fiber and the transverse directions. The residual strains lead to matrix cracking that may initiate delamination or cause hydrogen leakage. Park and McManus [1] proposed a micro-mechanical model based on fracture mechanics principles and verified their model by experiments in order to model the matrix cracking for composite laminates, and Kwon and Berner [2] analyzed the matrix damage of cross-ply laminates by combining a simplified micromechanics model with finite element analysis and show an increase in accuracy by taking the residual stress into account. Aoki et al. [3] used the micromechanics model of Park and McManus [1] to model matrix cracking of composite laminates under cryogenic temperatures. Aoki et al. [3] determined that the matrix cracking of the laminates was initiated by the transverse strains.

Using data from Aoki et al. [3], Qu et al. [4] performed deterministic and probabilistic design optimizations of composite laminates under cryogenic temperatures. They showed that small reductions in the variability of the transverse failure strains can lead to substantial savings in structural weight. They used response surface approximations for strains and probability of failure calculations. However, even small errors in strain values lead to large differences in probability of failure estimations. In this paper we propose the use of approximate probability density functions for strains and compare the probability of failure predictions with the ones obtained through the use of response surface approximations. The probabilities of failure are then obtained by Monte Carlo simulations (MCS). Response surface approximations are still used as functions of design variables for the probabilistic constraint, probabilistic sufficiency factor, PSF (Qu and Haftka [5]).

Researchers proposed different classifications for uncertainty over the years. Oberkampf et al. [6, 7] provided a good analysis of uncertainty in engineering modeling and simulations. In an attempt to explore the effects of reducing uncertainty, we use a simplified uncertainty classification that divides the uncertainty into two: a) error (mostly epistemic), and b) variability (aleatory part). Qu et al. [4] analyzed the effect of variability control on the weight saving from composite laminates under cryogenic conditions. Qu et al. [4] determined that quality control on the transverse failure strain is the most effective way of reducing the probability of failure, which in turn leads to reducing the weight of the laminates. In this paper, we consider also the effect of improved accuracy on the weight. Specifically, chemical shrinkage can be very important in the determination of transverse strains, but it is usually neglected due to paucity of data. Ifju et al. [10] developed the cure reference method (CRM) for measuring chemical shrinkage. In this paper, we explore the effect of this accuracy improvement on the weight along with the effect of reduced variability.

The next section of the paper presents the definition of the design problem. The methodology of probability of failure calculation is described in the Section 3. The solution method for the probabilistic design optimization problem is discussed in Section 4. Weight savings using error and variability control are presented in Section 5.

2. PROBLEM DEFINITION

We consider the design of a composite panel. The definition of the problem is taken directly from Qu et al. [4]. The laminate is subject to mechanical loading (N_x is 4,800 lb/inch and N_y is 2,400 lb/inch) and thermal loading due to operating temperature -423°F where the stress-free temperature is 300°F.

The objective is to optimize the weight of laminates with two ply angles $[\pm\theta_1/\pm\theta_2]_s$. The design variables are the ply angles θ_1, θ_2 and ply thicknesses t_1, t_2. The material used in the laminates is IM600/133 graphite-epoxy of ply thickness 0.005 inch. The minimum thickness necessary to prevent hydrogen leakage is assumed to be 0.04 inch. The geometry and loading condition are shown in Figure 1.

The deterministic design optimization of the problem was solved by Qu et al. [4]. Qu et al. used continuous design variables and rounded the thicknesses to integer multiples of the basic ply thickness 0.005 inches. In the deterministic optimization, Qu et al. multiplied the strains by the safety factor of S_F=1.4.

The deterministic optimization problem is formulated as

$$\begin{aligned} \min \quad & h = 4(t_1+t_2) \\ \text{s.t.} \quad & \varepsilon_1^L \le S_F\,\varepsilon_1 \le \varepsilon_1^U,\ \varepsilon_2^L \le S_F\,\varepsilon_2 \le \varepsilon_2^U,\ S_F\,|\gamma_{12}| \le \gamma_{12} \qquad (1) \\ & t_1,\ t_2 \ge 0.005 \end{aligned}$$

where the strain allowables are given in Table 1.

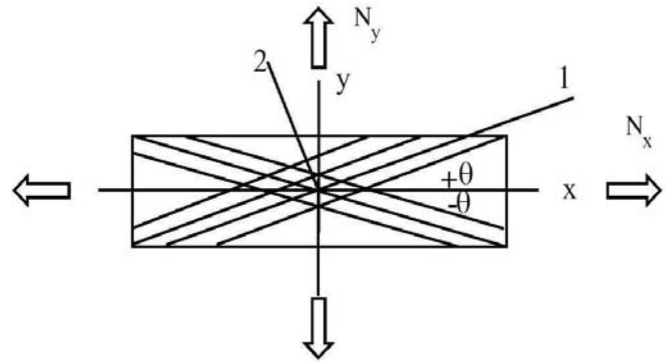


Figure 1. Geometry and loading.
(x-is the hoop direction and y is the axial direction)

Table 1. Strain allowables for IM600/133

ε_1^L	ε_1^U	ε_2^L	ε_2^U	γ_{12}^U
-0.0109	0.0103	-0.013	0.0154	0.0138

Since designs must be feasible for the entire range of temperatures, strain constraints were applied at twenty-one different temperatures, which were uniformly distributed from 77°F to –423°F. Qu et al. [4] found the three optima given in Table 2.

Table 2. Deterministic optimum designs by Qu et al. [4]. *The numbers inside the parentheses give the unrounded design thicknesses.*

θ_1 (deg)	θ_2 (deg)	t_1 (in)	t_2 (in)	h (in)
0.00	28.16	0.005	0.020	0.100 (0.103)
27.04	27.04	0.010	0.015	0.100 (0.095)
25.16	27.31	0.005	0.020	0.100 (0.094)

We analyze the problem that was addressed by Qu et al [4]; hence, the geometry, material parameters and the loading conditions are taken from that paper. The temperature dependent material properties as a function of temperature are shown in Figures 2-3.

3. CALCULATION OF PROBABILITY OF FAILURE

In this section, we calculate the probability of failure of the optimum designs listed in Table 1. The failure is defined as the first ply failure according to the maximum strain failure criterion. The strain allowables are from Qu et al. [4] and listed in Table 2. We assume that the strain allowables are the mean values of the failure strains. They are more likely to be A-basis or B-basis values. A-basis value is the value exceeded by 99% of the population with 95% confidence, while B-basis value is the value exceeded by 90% of the population with 95% confidence. Note that when there is redundancy, B-basis values are used.

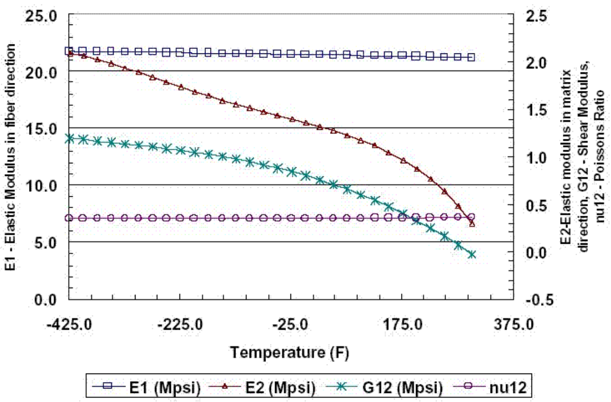


Figure 2. Material properties E_1, E_2, G_{12} and v_{12} as a function of temperature.

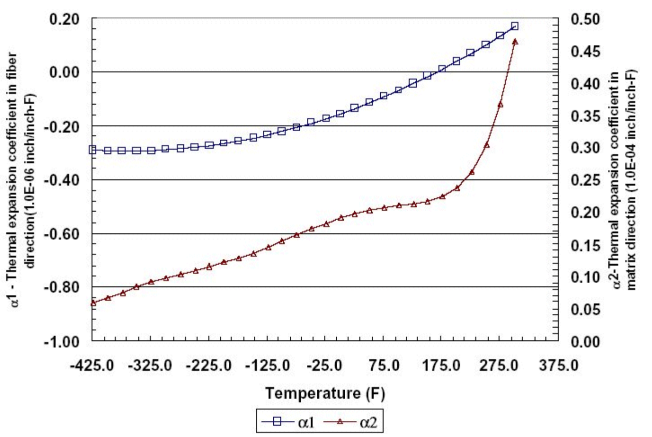


Figure 3. Material properties α_1 and α_2 as a function of temperature.

The first step in the calculation of probability of failure is to quantify uncertainties included in the problem. Here, we use a simple classification for uncertainty that we used in our pervious work (Kale et al. [8], Acar et al. [9]). Uncertainty is divided into two as error and variability to distinguish between the uncertainties that apply equally to the entire fleet of a structural model (error) and the uncertainties that vary for an individual structure (variability). Furthermore, this classification reduces the difficulty in analysis of the effects of uncertainty control.

The variability refers to the departure of a quantity in individual laminates that has the same design. Here, the elastic properties (E_1, E_2, G_{12} and v_{12}), coefficients of thermal expansion (α_1 and α_2), failure strains (ε_1^L, ε_1^U, ε_2^L, ε_2^U and γ_{12}^U) and stress-free temperature (T_{zero}) have variability. These random variables are all assumed to follow uncorrelated normal distributions. The coefficients of variation of the random variables are listed in Table 4.

Table 3. Uncertainty Classification

Type	Spread	Cause	Remedies
Error (mostly epistemic)	Departure of the average fleet of an aerospace structure model (e.g. Boeing 737-400 from an ideal)	Errors in predicting structural failure, construction errors, deliberate changes	Testing and simulation to improve math model and the solution.
Variability (aleatory)	Departure of an individual structure from fleet level average	Variability in tooling, manufacturing process, and flying environment	Improve tooling and construction. Quality control.

Table 4. Coefficients of variation of the random variables (normal distribution) having variability.

E_1, E_2, G_{12} and v_{12}	α_1 and α_2	T_{zero}	ε_1^L and ε_1^U	ε_2^L, ε_2^U and γ_{12}^U
0.035	0.035	0.030	0.06	0.09

In addition to variability, we also use a simple model of the errors in the problem. The calculated values of failure strains differ from the actual values due to experimental or measurement errors. The use of the standard classical lamination theory (CLT) for the ply strain calculation leads to errors, because the standard CLT does not take chemical shrinkage into account and has other approximations. Other sources of error may also be present. We use a simple error model to relate the actual values of the strains to the calculated values

$$\varepsilon_{calc} = (1+e)\varepsilon_{true} \tag{2}$$

where e is the representative error factor that includes the effect of all error sources on the values of strains and failure strains. For example, if the estimated failure strain is 10% too high, this is approximately equivalent to the strain being calculated as 10% too low. For the error factor e, we use a uniform distribution with bounds of $\pm b_e$. In the subsequent sections, we will investigate the effect of reducing b_e on the probability of failure and weight savings.

The error in failure strains can be reduced by using more accurate failure models. Here we analyze the effect of a more accurate calculation of strains based on the cure reference method [10] to account for the shrinkage due to chemical process. In Section 4, we analyze the effect of error reduction on probability of failure and the weight saving when errors are reduced for fixed probability of failure.

To calculate probability of failure, we use Monte Carlo simulations. For acceptable accuracy, a sufficient number of MCS needs to be performed, and for each simulation the strain values obtained through the standard CLT analysis must be used. However, this is computationally very expensive and need to be repeated many times during the optimization. In order to reduce the computational cost associated with the calculation of probability of failure, Qu et al. [4] used response surface approximations for strains (ε_1 in θ_1, ε_1 in θ_2, ε_2 in θ_1, ε_2 in θ_2, γ_{12} in θ_1, γ_{12} in θ_2) obtained via the standard CLT analysis.

Qu et al. fitted quadratic response surface approximations to strains in terms of four design variables (t_1, t_2, θ_1, θ_2), material parameters (E_1, E_2, G_{12}, v_{12}, α_1 and α_2) and service temperature T_{serv}. The range for ply thicknesses was taken between 0.0125 and 0.03 inches, and the range for ply angles were taken as between 20º and 30º while constructing the response surfaces. A quadratic response surface approximation in terms of 12 variables includes 91 coefficients; so, they used 182 samples obtained through Latin Hypercube Sampling (LHS) design. The evaluation of accuracy of response surface approximations of Qu et al. is given in Table 5. We see that the accuracy of response surfaces is quite good.

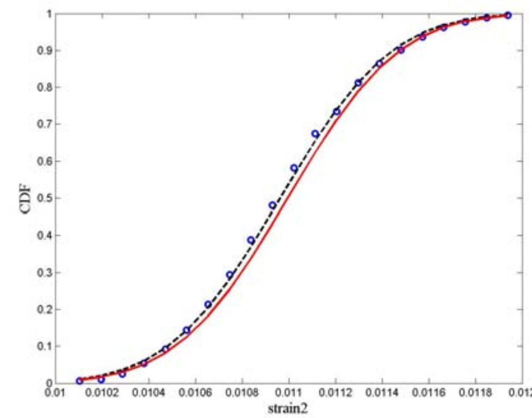
Table 5. Evaluation of accuracy of ARS used by Qu et al. [4]. *Note that the strains are millistrains.*

	ε_1 in θ_1	ε_1 in θ_2	ε_2 in θ_1	ε_2 in θ_2	γ_{12} in θ_1	γ_{12} in θ_2
R^2_{adj}	0.9977	0.9978	0.9956	0.9961	0.9991	0.9990
RMSE Predictor*	0.017	0.017	0.060	0.055	0.055	0.060
Mean of response	1.114	1.108	8.322	8.328	-3.13	-3.14

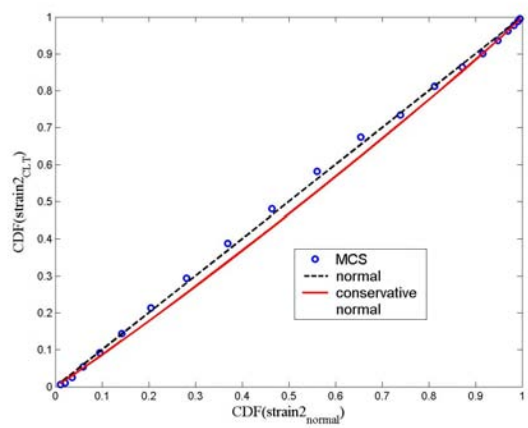
* standard error

The approach followed by Qu et al. [4] helps to reduce the computational cost, but even small errors in strain values may lead to large errors in probability of failure calculations. Instead, we propose the use of approximated cumulative distribution functions (CDF) of strains. We assume "conservative" normal distributions for strains and estimate the mean and the standard deviation of strains by MCS. The term conservative is used to denote that the mean and standard deviation of the assumed distribution is found in such a way that the CDF of the approximated distribution is smaller or equal to the CDF values calculated via MCS except for the strain values very near to the tail of the distribution. We use 1000 simulations, which are accurate to a few percent of the standard deviation for estimating the mean and standard deviation. CDF obtained through 1,000 MCS, the approximate normal distribution and the conservative approximate normal distributions for ε_2 corresponding to one of the deterministic optima (second design in Table 2) are compared in Figures 4(a) and 4(b).

Next, we compare the accuracy of the analysis response surface and approximate CDF approaches in Table 6. In Table 6, three approaches are listed for calculation of probability of failure of the second design in Table 2. The number of samples in MCS is 1,000,000 and the error bound b_e is taken as zero in Table 6. First, the strain values computed directly from the standard CLT analysis are compared with the failure strains, and the number of failures is recorded. Second, the analysis response surfaces (ARS) for the strains are generated, and the strains obtained from the ARS are used in the MCS*. Third, the strains obtained from fitted normal distribution of strains are used in MCS. We see in Table 6 that use of approximate CDF's for strains leads to more accurate probability of failure estimations compared to the use of response surface approximations for strains. However, we should note that the approximate CDF's were obtained by performing 1,000 MCS while the ARS were constrained by using only 277 MCS. In addition, while calculating P_f the approximate CDF's needs to be calculated for each design of experiments of the design response surface (DRS, the response surface approximation for probabilistic constraint, about which more detail is given in Section 4). It is possible that some combination of ARS with approximate CDF's may be more efficient and accurate than either, and this will be explored in future work.



(a) CDF versus strain



(b) Actual CDF versus fitted CDF

Figure 4. Comparison of the CDF obtained via 1,000 MCS, the approximate normal distribution and "conservative" approximate normal distributions for *ε_2* on *θ_1* corresponding to one of the deterministic optima (second design in Table 2).

* The term analysis response surface (ARS) is used following Qu et al. [4]. It designates the response surface approximations for the strains that replace the CLT analysis in the MCS.

Table 6. Comparison of probability of failure estimations for deterministic optimum of Qu et al. [4].

Approach followed	Probability of Failure ($x10^{-4}$)	Standard error in P_f due to limited sampling ($x10^{-4}$)
MCS with the standard CLT	10.21	0.320
MCS with RSA* to strains	16.83	0.410
MCS with approximation to CDF of strains	11.55	0.340

* *RSA: Response surface approximation*

4. PROBABILISTIC DESIGN OPTIMIZATION

The laminates are designed for a target failure probability of 10^{-4}. The optimization problem can be formulated as given in Eq. (3). The design variables are the ply thicknesses and angles.

$$\begin{aligned} \min \quad & h = 4(t_1+t_2) \\ s.t. \quad & P_f \le (P_f)_{target} \\ & t_1, t_2 \ge 0.005 \end{aligned} \tag{3}$$

For this optimization, we need to fit a design response surface (DRS)† to the probability of failure as a function of the design variables. The accuracy of the DRS may be improved by using an inverse safety measure. We use the probabilistic sufficiency factor (PSF) developed by Qu and Haftka [5]. They showed that a DRS fitted to PSF is more accurate than a DRS fitted to probability of failure. For a detailed review on inverse safety measures, the reader is referred to Ramu et al. [11].

4.1. Probabilistic sufficiency factor (PSF)

The safety factor S is defined as the ratio of the capacity G_C of the structure to the structural response G_R. The *PSF* is the probabilistic interpretation of the safety factor with the CDF defined as

$$F_S(s) = \text{Prob}\left(\frac{G_C}{G_R} \le s\right) \tag{4}$$

Given a target probability of failure, $\left(P_f\right)_{target}$, *PSF* can be found from

$$F_S(s) = \text{Prob}\left(\frac{G_C}{G_R} \le PSF\right) = \text{Prob}(S \le PSF) = \left(P_f\right)_{target} \tag{5}$$

That is, the *PSF* is found by equating the CDF of the safety factor to the target failure probability. The *PSF* takes values such that

$$PSF = \begin{cases} <1 \ \ if \ \ P_f > \left(P_f\right)_{target} \\ =1 \ \ if \ \ P_f = \left(P_f\right)_{target} \\ >1 \ \ if \ \ P_f < \left(P_f\right)_{target} \end{cases} \tag{6}$$

When MCS are used, the *PSF* can be estimated as the n^{th} smallest safety factor over all MCS, where $n = N\times(P_f)_{target}$. Using the *PSF*, the optimization problem can be formulated as

$$\begin{aligned} \min \quad & h = 4(t_1+t_2) \\ s.t. \quad & PSF \ge 1 \\ & t_1, t_2 \ge 0.005 \end{aligned} \tag{7}$$

The optimization problem given in Eq. (7) is solved by using Sequential Quadratic Programming (SQP) in MATLAB.

4.2. Design response surface (DRS)

We have three strain values of interest for each angle: ε_1, ε_2 and γ_{12}. The strain ε_2 and γ_{12} are more critical than ε_1. The mean and standard deviation of four strains (ε_2 in θ_1, ε_2 in θ_2, γ_{12} in θ_1 and γ_{12} in θ_2) are computed by using 1,000 MCS samples and fitted with normal distributions as shown in Figure 4. These distributions are used in MCS using 1,000,000 simulations at any design point to compute *PSF*. In order to perform the optimization, we need to approximate the *PSF* as a function of the design variables by a design response surface (DRS). Qu et al. [4] showed that using the combination of Face centered central composite design (FCCCD) and Latin hypercube sampling (LHS) designs gives accurate results; so, we follow the same procedure. We fit three DRS of the *PSF* as function of the four design variables (t_1, t_2, θ_1, θ_2) for three different error bound (b_e) values 0, 10% and 20%, respectively. The zero error bound case corresponds to a hypothetical condition that the strain calculation has no error. The error bound of 10% corresponds to the use of the cure reference method (CRM) that takes chemical shrinkage in the laminates into account while calculating the strains and the error bound of 20% corresponds to the use of the standard CLT that ignores chemical shrinkage while calculating the strains. A more detailed explanation regarding with the error bounds corresponding to the use of the standard CLT and the modified CLT is given in the next section of the paper. The ranges for design variables for DRS are decided as follows. The initial estimates of the ranges for design variables were taken from Qu et al. [4]. When we use the ranges from Qu et al. [4], we found that the prediction variances at the optimum points were larger than the root mean square error (RMSE) predictor of the response surfaces. The ranges for DRS were then reduced by zooming around the optimum values obtained from the wider ranges. After zooming, the prediction variances at the optimum designs were found to be smaller than the RMSE predictors. The ranges for response surfaces are given in Table 7.

† The term design response surface (DRS) follows Qu et al. [4] and indicates approximations to the probability of failure or other measures of safety as a function of design variables.

Table 7. The ranges of variables for the three DRS

	t_1 and t_2 (in)	θ_1 and θ_2 (deg)
b_e=0	0.012-0.017	24-27
b_e=10%	0.013-0.018	24-26
b_e=20%	0.015-0.022	22-25

Qu et al. [4] used a fifth-order DRS for probability of failure, and found that a fifth-order DRS is quite accurate. Thus, we also use a fifth-order DRS. A fifth-order response surface in terms of four variables has 126 coefficients. As noted earlier, Qu et al. [4] showed that using the combination of FCCCD and LHS designs gives accurate results; so, we follow the same procedure. We use 277 design points, 25 correspond to FCCCD and 252 are generated by LHS. The accuracy of response is evaluated with RMSE predictor and R^2_{adj}. In addition, three more DRS are also fitted to probability of failure for comparison purpose and the results are presented in Table 8. We see that the use of *PSF* response surface leads to more accurate fit compared to P_f response surface, and the ratio of mean square error to mean of response is smaller and also R^2_{adj} is larger. The errors in the probability of failure DRS are clearly unacceptable in view that the required probability of failure is 10^{-4}. The effect of the *PSF* error on error in probability of failure will be discussed in Section 5.

Table 8. Accuracies of DRS fitted to *PSF* and P_f in terms of four design variables (t_1,t_2,θ_1 and θ_2) for error bound b_e of 0, 10% and 20%

		R^2_{adj}	RMSE predictor*	Mean of response
b_e=0%	*PSF*	0.9978	4.655 x10^{-3}	1.077
	P_f	0.9839	9.447 x10^{-4}	8.081 x10^{-4}
b_e=10%	*PSF*	0.9973	4.645 x10^{-3}	0.9694
	P_f	0.9935	8.281 x10^{-4}	1.340 x10^{-3}
b_e=20%	*PSF*	0.9979	3.610 x10^{-3}	0.8621
	P_f	0.9984	7.664 x10^{-4}	4.103 x10^{-3}

* standard error

5. WEIGHT SAVING FROM THE LAMINATES BY REDUCING ERROR

As we noted earlier, the probabilistic design optimizations of the composite laminates are performed for three different values of error bound (b_e), namely 0, 10% and 20%. Schultz et al. [12] have shown that neglecting chemical shrinkage leads to substantial errors in strain calculations. We assume that the use of standard CLT without chemical shrinkage leads to 20% errors in strain calculations, while use of the modified CLT leads to the reduction of error bounds from 20% to 10%. The probability distribution function for the errors assumed to be uniformly distributed, which corresponds to maximum entropy (or randomness).

After assessing the error bounds for the strain calculations, we solve the optimization problem given in Eq. (7). The results of the optimization are presented in Table 9 and in Figure 6. Since there is a one-to-one correspondence between the thickness h and the weight of the laminate, the reduction in the laminates thickness gives the weight saving. We see that reducing the error bound from 20% to 10% leads to 12.4% weight saving. In addition, reducing error from 20% to 0 (clearly only a hypothetical case) leads to weight saving of 23.1%.

Table 9. Probabilistic optimum designs for different error bounds when only error control is applied. *The PSF and P_f given in the last two columns are calculated via Monte Carlo simulations of 10,000,000 samples where the strains are directly computed via the standard CLT analysis. The numbers in parentheses under PSF and P_f show the standard errors due to limited Monte Carlo sampling.*

Error bound	θ_1 θ_2 (deg)	t_1 t_2 (in)	h (in) [Δh* (%)]	*PSF*	P_f (x10^{-4})
0	25.47 26.06	0.0156 0.0137	0.1169 [23.1]	0.9986 (0.0030)	1.017 (0.032)
10%	25.59 25.53	0.0167 0.0167	0.1332 [12.4]	1.018 (0.0035)	0.598 (0.024)
20%	23.71 23.36	0.0189 0.0191	0.1520 [0.0]	0.9962 (0.0035)	1.111 (0.105)

**The optimum laminate thickness for 20% error bound is taken as the basis for Δh computations*

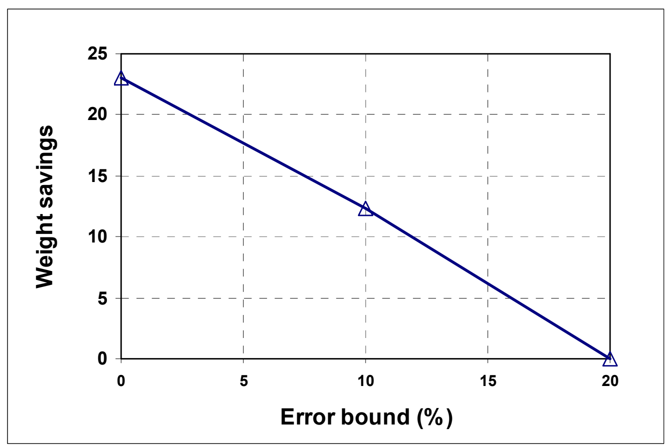


Figure 6. Reducing laminate thickness by error control (no variability control)

The standard errors in calculation of *PSF* and P_f due to limited MCS sample size are given in the last two columns of Table 9. The standard error for P_f is calculated from

$$\sigma = \sqrt{\frac{P(1-P)}{N}} \quad (8)$$

The standard error in *PSF* is calculated as follows. Assume that for calculating a probability of failure of 10^{-4}, we use 10^6 MCS samples. Then, the number of samples failed is 100 and the standard error for P_f calculation is 10^{-5}. Thus, 10 samples out of 100 represent the standard error. The standard error in *PSF* can be approximated as the difference between the 105th

smallest safety factor and 95th smallest safety factor. A better estimation for *PSF* can be done by utilizing the CDF of the safety factor S.

We have shown that it is possible to reduce the laminate thickness up to 12.4% by reducing the error from 20% to 10%. Now, we combine the error control with variability control and analyze the overall benefit of uncertainty control mechanisms. An example of the variability control is the testing of set of composite laminates and rejecting the laminates having lower failure strains as a form of quality control. We consider the case where the specimens that have transverse failure strain lower than two standard deviations below from the mean values are rejected (97%). We construct two new DRS of PSF corresponding to error bound of 0, 10% and 20%. All the properties such as the design of experiments, degree of polynomial are kept the same for the new response surfaces; the only change made is the ranges of design variables. The new ranges of design variables used while constructing the new response surfaces are given in Table 10.

Table 10. The new ranges of variables for the new three DRS

	t_1 *and* t_2 (in)	θ_1 and θ_2 (deg)
b_e=0	0.008-0.012	27-30
b_e=10%	0.009-0.014	26-29
b_e=20%	0.013-0.018	24-27

The probabilistic design optimizations of the composite laminates for three different values of error bound (b_e) are performed and the results are presented in Table 11 and in Figure 7. We note that when this form of variability control is applied, the laminate thickness can be reduced by 19.5%. If error control is also applied along with the variability control such that the error bound is reduced from 20% to 10%, the laminate thickness can be reduced by 36.2%. Note that the *PSF* and P_f given in the last two columns of Table 11 are calculated via MCS of 10,000,000 samples where the strains are directly computed via the standard CLT analysis. The numbers in parentheses under *PSF* and P_f show the standard errors due to limited sample size of MCS.

Table 11. Probabilistic optimum designs for different error bounds when both error and variability control is applied.

Error bound	θ_1 θ_1 (deg)	t_1 t_2 (in)	h (in) [Δh* (%)]	*PSF*	P_f (x10^{-4})
0	28.52 28.71	0.0089 0.0114	0.0813 [-46.6]	0.9965 (0.0014)	1.255 (0.035)
10%	27.34 27.37	0.0129 0.0114	0.0970 [-36.2]	1.0016 (0.0015)	0.906 (0.030)
20%	25.57 25.66	0.0168 0.0138	0.1224 [-19.5]	0.9968 (0.0015)	1.190 (0.109)

**The optimum laminate thickness for 20% error bound given in Table 9 (i.e. h=0.1520) is taken as the basis for Δh computations*

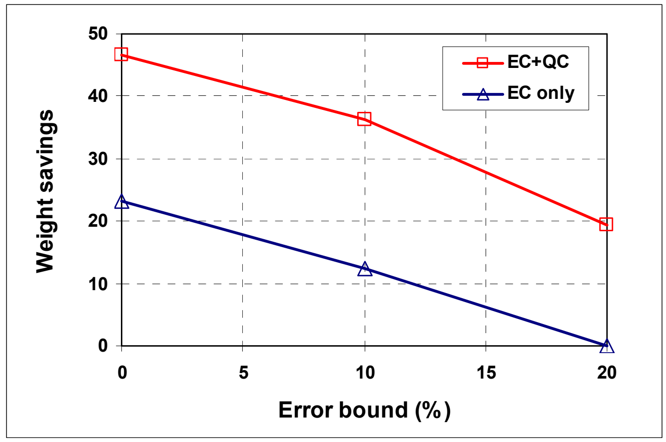


Figure 7. Reducing laminate thickness by error control (EC) and quality control (QC).

The numbers in the last two columns of Tables 9 and 11 show the *PSF* and P_f calculated by using the 10,000,000 MCS where strains are directly calculated through the standard CLT analysis. The actual values for *PSF* and P_f of the optimum designs are expected to be 1.0 and 10^{-4}. Discrepancies can be results of the following.

a) Error due to the use of normal distributions for strains which may not exactly follow normal distributions.
b) Error due to limited sample size of MCS while calculating the mean and standard distribution of strains.
c) Error due to limited sample size of MCS while computing the probabilistic sufficiency factor *PSF*.
d) Error associated with the use of response surface approximations for *PSF*.

We notice in Tables 9 and 10 that for the optimum designs, the two ply angles are similar in value. This is an indication that the problem may be simplified by setting the two ply angles equal and having a single thickness variable. Hence, the number of design variables can be reduced from four to two, which reduces the number of coefficients of the *PSF* response surface. Design of experiments required to construct *PSF* response surface also decreases.

Finally, the tradeoff plot for probability of failure (calculated via 10,000,000 MCS), weight and error control measures is shown in Figure 8. The optimum ply angles for the case with 20% error bound and no variability control are 25.59º and 25.53º degrees. Here we take both ply angles at 25º. We notice in Figure 8 that for our problem the specified error control is a more effective way of reducing weight compared to the specified variability control when the target probability of failure of the laminates is higher than 2x10^{-4}. However, since the laminates are designed for lower probabilities of failure variability control is more effective in reducing weight.

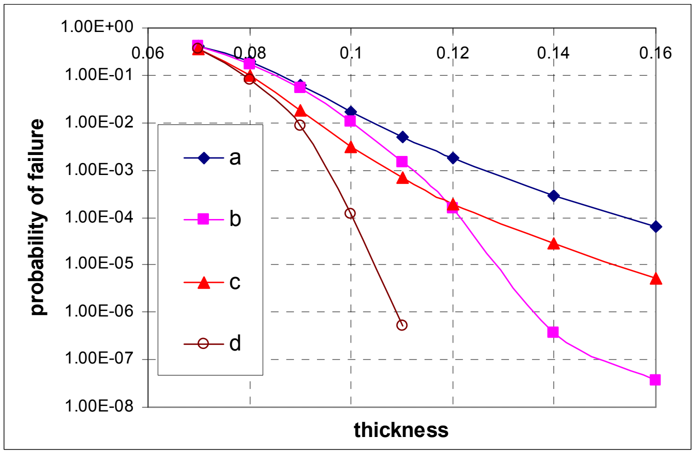


Figure 8. Trade-off plot for probability of failure, design thickness and uncertainty control measures.

6. CONCLUDING REMARKS

In this paper, the effects of uncertainty control measures on the weight saving from the laminates were investigated. The uncertainties in the problem are classified into error and variability, and the efficiencies of error and variability control measures for weight reduction are compared. For this problem, it is observed that a single measure of variability control (quality control below -2 sigma on failure strain) is superior to a form of error control (the use of modified CLT by taking chemical shrinkage into account instead of using standard CLT in calculating strains, which we assume reduces error bound from 20% to 10%) in reducing the weight. Error control leads to 12% weight reduction, the quality control leads to 20% weight savings and the use of error and variability control measures together reduces the weight by 37%.

In addition, the design optimization of composite laminates at cryogenic temperatures previously addressed by Qu et al. [4] is explored in more detail, and the accuracy and efficiency of probability of failure calculations are improved. To reduce the computational expense associated with the use of the standard CLT in each MCS, Qu et al. [4] used response surface approximations for strains. In this paper we approximated the cumulative distribution functions for strains in a conservative manner and found that this approach led to more accurate probability of failure estimates.

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