Why are Airplanes so Safe Structurally? Effect of Various Safety Measures on Structural Safety of Aircraft

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Passenger aircraft structural design is based on a safety factor of 1.5, and this safety factor alone is equivalent to a probability of failure of between 10^{-2} and 10^{-3}. Yet airliners are much safer, with crashes due to structural failure being extremely rare based on accident records. The probability of structural failure of transport aircraft is of the order of 10^{-8} per flight segment. This paper looks at two other contributions to safety--the use of conservative material properties and certification tests--using a simple model of structural failure. We find that the three safety measures together may be able to reduce the calculated probability of failure to about 10^{-7}, and that additional measures, such as conservative load specifications, may be responsible for the higher safety encountered in practice. In addition, the paper sheds light on the effectiveness of certification tests for improving safety. It is found that certification tests reduce the calculated failure probabilities by reducing the modeling error. We find that these tests are most effective when safety factors are low and when most of the uncertainty is due to systemic errors rather than variability.

I. Introduction

THIS paper explores the effects of various safety measures taken during aircraft structural design using the deterministic design approach based on FAA regulations. We use Monte Carlo simulations to calculate the effect of these safety measures on the probability of failure of a structural component. The safety measures that we include here are (1) the use of safety factors, (2) the use of conservative material properties (A-basis), and (3) the use of final certification tests. We do not include in this discussion the additional safety due to structural redundancy and due to conservative design load specification. The effect of the three individual safety measures and their combined effect on the probability of structural failure of the aircraft are demonstrated.

We start with a structural design employing all considered safety measures. The effect of variability in geometry, loads, and material properties is readily incorporated by the appropriate random variables. However, there is also uncertainty due to lack of knowledge (epistemic uncertainty), such as modeling errors in the analysis. To simulate these epistemic uncertainties, we transform the error into a random variable by considering the design of multiple aircraft models. As a consequence, for each model the structure is different. It is as if we pretend that there are hundreds of companies (Airbus, Boeing, etc.) each designing essentially the same airplane, but each having different errors in their structural analysis.

For each model we simulate certification testing. If the airplane passes the test, then an entire fleet of airplanes with the same design is assumed to be built with different members of the fleet having different geometry, loads, and

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material properties based on assumed models for variability in these properties. That is, the uncertainty due to variability is simulated by considering multiple realizations of the same design, and the uncertainty due to errors is simulated by designing different structures to carry the same loads.

We consider only stress failure due to extreme loads, which can be simulated by an unstiffened panel designed under uniaxial loads. No testing of components prior to certification is analyzed for this simple example.

II. Structural Uncertainties

A good analysis of different sources of uncertainty is provided by Oberkampf et al. (2000). Here we simplify the classification, with a view to the question of how to control uncertainty. We propose in Table 1 a classification that distinguishes between (1) uncertainties that apply equally to the entire fleet of an aircraft model and (2) uncertainties that vary for the individual aircraft. The distinction is important because safety measures usually target one or the other.

Similarly, the uncertainty in the failure of a structural member can also be divided into two types: systemic errors and variability. Systemic errors reflect inaccurate modeling of physical phenomena, errors in structural analysis, errors in load calculations, or use of materials and tooling in construction that are different from those specified by the designer. Systemic errors affect all the copies of the structural components made and are therefore fleet-level uncertainties. The other type of uncertainty reflects variability in material properties, geometry, or loading between different copies of the same structure and is called here individual uncertainty.

Table 1: Uncertainty Classification

Type of uncertainty	**Spread**	**Cause**	**Remedies**
Systemic error (modeling errors)	Entire fleet of components designed using the model	Errors due to imperfect math model and inaccuracies in solution of resulting equations.	Testing and simulation to improve math model and the solution.
Variability	Individual component level	Variability in tooling and manufacturing process.	Improve tooling and construction. Quality control.

III. Safety Measures

Aircraft structural design is still done by and large using code-based design rather than probabilistic approaches. Safety is improved through conservative design practices that include use of safety factors and conservative material properties. It is also improved by tests of components and certification tests that can reveal inadequacies in analysis or construction. In the following we detail some of these safety measures.

Safety Margin: Traditionally all aircraft structures are designed with a safety factor to withstand 1.5 times the limit load without failure.

A-Basis Properties: In order to account for uncertainty in material properties, the Federal Aviation Administration (FAA) recommends the use of conservative material properties. This is determined by testing a specified number of coupons selected at random from a batch of material. The A-basis property is determined by calculating the value of a material property exceeded by 99% of the population with 95% confidence.

Component and Certification tests: Component tests and certification tests of major structural components reduce stress and material uncertainties for given extreme loads due to inadequate structural models. These tests are conducted in a building block procedure. First, individual coupons are tested, and then a sub assembly is tested followed by a full-scale test of the entire structure. Since these tests cannot apply every load condition to the structure, they leave uncertainties with respect to some loading conditions. It is possible to reduce the probability of failure by performing more tests to reduce uncertainty or by extra structural weight to reduce stresses. If certification tests were designed together with the structure, it is possible that additional tests would become cost effective because they would allow reduced structural weight.

We simulate the effect of these safety measures by assuming the statistical distribution of the uncertainties and incorporating them in approximate probability calculations and Monte Carlo simulation. For variability the simulation is straightforward. However, while systemic errors are uncertain at the time of the design, they will not vary for a single structural component on a particular aircraft. Therefore, to simulate the uncertainty, we assume that we have a large number of nominally identical aircraft being designed (e.g. by Airbus, Boeing, Bombardier, etc.), with the errors being fixed for each aircraft. This creates a two-level Monte Carlo simulation, with different aircraft models being considered at the upper level, and different instances of the same aircraft at the lower level.

To illustrate the procedure we consider a simple example of an unstiffened panel designed for strength under uniaxial tensile loads. This will still simulate reasonably well more complex configuration, such as stiffened panels subject to stress constraints. Aircraft structures have more complex failure modes, such as due to fatigue and fracture, which require substantially different treatment and the consideration of the effects of inspections (See Kale et al. 2003). However, this simple example serves to further our understanding of the interaction between various safety measures. The procedure is summarized in Figure 1, which is described in detail in the next section.

IV. Panel Example Definition

Design and certification testing: We assume that we have *N* different aircraft models, corresponding to the outer loop in Figure 1. We consider a generic panel to represent the entire aircraft structure. The true stress (σ_{true}) is found from the equation

$$\sigma_{true} = \frac{P}{w \ t} \tag{1}$$

where P is the applied load on the panel of width w and thickness t. In a more general situation, Eq. (1) may apply to a small element in a more complex component.

When errors are included in the analysis, the true stress in the panel is different from the calculated stress. We include the errors by introducing an error factor e while computing the stress as

$$\sigma_{calc} = (1+e) \quad \sigma_{true} \tag{2}$$

Positive values of e yield conservative estimates of the true stress and negative values yield unconservative stress estimation. The other random variables account for variability. Combining Eqs. (1) and (2), the stress in the panel is calculated as

$$\sigma_{calc} = (1+e)\frac{P}{w \ t} \tag{3}$$

The design thickness is determined so that the calculated stress in the panel is equal to material allowable stress for a design load P_d multiplied by a safety factor S_F, hence the design thickness of the panel is calculated from Eq. (3) as

$$t_{design} = (1+e)\frac{S_F P_d}{w\sigma_a} \tag{4}$$

where the panel width, w, is taken here to be 1.0 meter, and σ_a is the material stress allowable obtained from testing a batch of coupons according to procedures that depend on design practices. Here, we assume that A-basis properties are used (Appendix I). During the design process, the only random quantities are σ_a and e. The thickness obtained from Eq. (4) (step A in Fig. 1) is the nominal thickness for a given aircraft model. The actual thickness will vary due to individual–level manufacturing uncertainties.

After the panel has been designed (that is, thickness determined) from Eq. (4), we simulate certification testing for the aircraft. Here we assume that the panel will not be built with complete fidelity to the design due to variability in geometry (width and thickness). The panel is then loaded with the design axial force of (S_F times P_d), and the stress in the panel is recorded. If this stress exceeds the failure stress (itself a random variable, see Table 2.) then the design is rejected, otherwise it is certified for use. That is, the airplane is certified (step B in Fig. 1) if the following inequality is satisfied

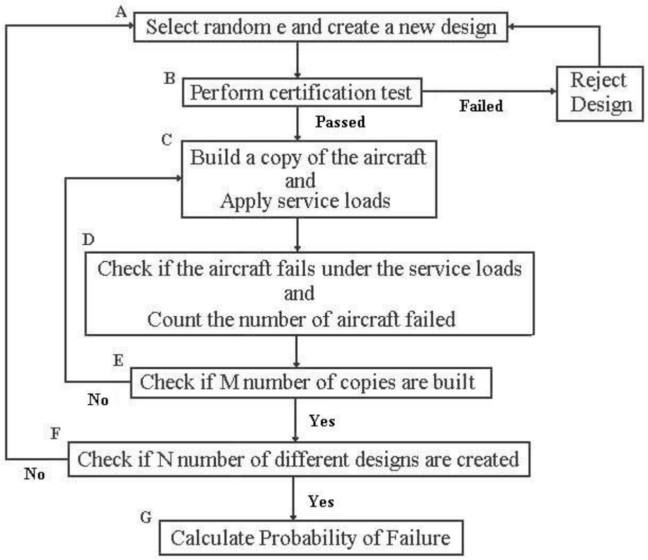

Figure 1: Flowchart for Monte Carlo simulation of panel design and failure

$$\sigma - \sigma_f = \frac{S_F P_d}{wt} - \sigma_f \leq 0 \tag{5}$$

and we can build multiple copies of the airplane. We subject the panel in each airplane to actual random maximum (over a lifetime) service loads (step D) and decide whether it fails using Eq. (6).

$$P \geq R = t\, w \sigma_f \tag{6}$$

Here, P is the applied load, and R is the resistance or load capacity of the structure in terms of the random width w, thickness t and failure stress σ_f. A summary of the distributions for the random variables used in design and certification is listed in Table 2.

This procedure of design and testing is repeated (steps A-B) for N different aircraft models. For each new model, a different random error factor e is picked for the design, and different allowable properties are generated from coupon testing (Appendix I). Then in the testing, different thicknesses and widths, and different failure stresses are generated at random from their distributions.

Table 2: Distribution of random variables used for panel design and certification

Variables	Distribution	Mean	Scatter
Plate width (w)	Uniform	1.0	(1%) bounds
Plate thickness (t)	Uniform	t_{design}	(3%) bounds
Failure stress (σ_f)	Lognormal	150.0	10 % coefficient of variation
Service Load (P)	Lognormal	100.0	10 % coefficient of variation
Error factor (e)	Uniform	0.0	----

Effect of Certification Tests on distribution of Error Factor *e*

One can argue that the way certification tests reduce the probability of failure is by changing the distribution of the error factor e. Without certification testing, we assume symmetric distribution of this error factor. However, designs based on unconservative models are more likely to fail certification, and so the distribution of e becomes conservative for structures that pass certification. In order to quantify this effect, we calculated the updated distribution of the error factor e. The updated distribution is calculated analytically by Bayesian updating by making some approximations, and Monte Carlo simulations are conducted to check the validity of those approximations.

Bayesian updating is a commonly used technique to obtain updated (or posterior) distribution of a random variable upon obtaining new information about the random variable. The new information here is that the panel has passed the certification test.

Using Bayes' Theorem, the updated (posterior) distribution $f^U(\theta)$ of a random variable θ is obtained from the initial (prior) distribution $f^I(\theta)$ based on new information as

$$f^U(\theta) = \frac{P(\in|\,\theta)\, f^I(\theta)}{\int_{-\infty}^{\infty} P(\in|\,\theta)\, f^I(\theta)\, d\theta} \tag{7}$$

where $P(\in|\theta)$ is the conditional probability of observing the experimental data $\in$ given that the value of the random variable is θ.

For our case, the posterior distribution $f^U(e)$ of the error factor e is given as

$$f^U(e) = \frac{P(C|e)\, f^I(e)}{\int_{-b}^{b} P(C|e)\, f^I(e)de} \tag{8}$$

where C is the event of passing certification, and $P(C|e)$ is the probability of passing certification for a given e. Initially, e is assumed to be uniformly distributed. The procedure of calculation of $P(C|e)$ is described in Appendix II, where we approximate the distribution of the geometrical variables, t and w as lognormal, taking advantage of the fact that their coefficient of variation is small compared to that of the failure stress (see Table 2).

We illustrate the effect of certification tests for the panels designed with A-Basis material properties. An initial and updated distribution plot of error factor e with 50 % bound is shown in Fig. 2. Monte Carlo simulation with 50,000 aircraft models is also shown. Figure 2 shows that the certification tests greatly reduce the probability of negative error, hence eliminating most unconservative designs. As seen from the figure, the approximate distribution calculated by the analytical approach matches well with the distribution obtained from Monte Carlo simulations.

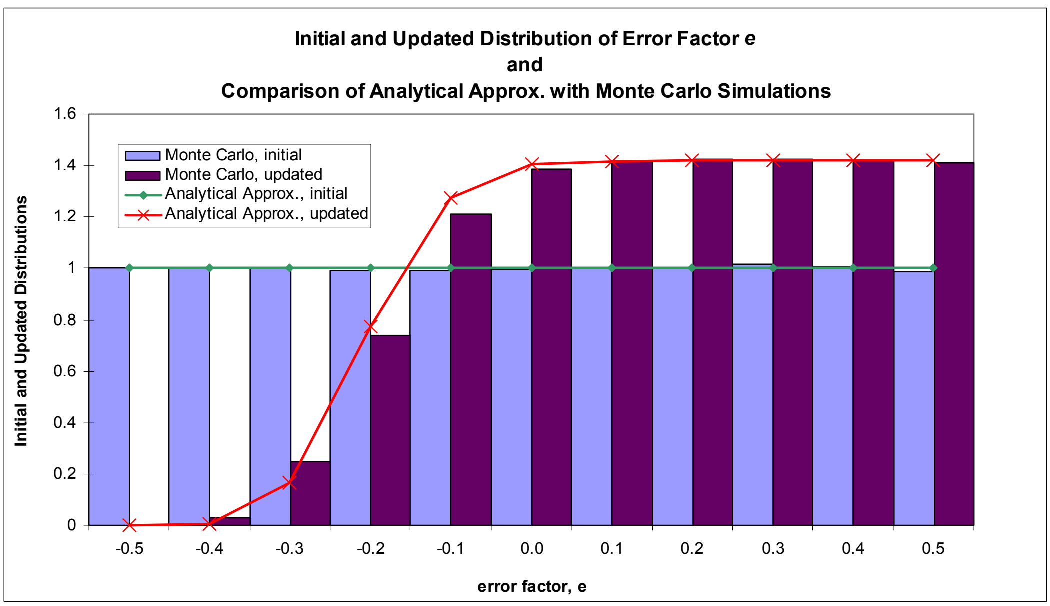

Figure 2: **Initial and updated probability distribution functions of error factor *e***
Error bound is 50% and Monte Carlo simulation done with a sample of 50,000.

Probability of Failure Calculation by Analytical Approximation

The stress analysis represented by Eq. (1) is trivial, so that the computational cost of Monte Carlo simulation of the probability of failure is not high. However, it is desirable to obtain also analytical probabilities that may be used for more complex stress analysis and to check the Monte Carlo simulations.

In order to take advantage of simplifying approximations of the distribution of the geometry parameters, it is convenient to perform the probability calculation in two stages, corresponding to the inner and outer loops of Fig. 1. That is, we first obtain expressions for the probability of failure of a single aircraft model (that is, given e and allowable stress). We then calculate the probability of failure over all aircraft models.

The mean value of the probability of failure over all aircraft models is calculated as

$$\hat{P}_f = \int P_f(t_{design}) f(t_{design}) dt_{design} \tag{9}$$

where t_{design} is the non-deterministic distribution parameter, and $f(t_{design})$ is the probability density function of parameter t_{design}.

It is important to have a measure of variability in this probability from one aircraft model to another. The standard deviation of failure probability gives a measure of this variability. In addition, it provides information on how accurate is the probability of failure obtained from Monte Carlo simulations. The standard deviation can be calculated from

$$\sigma_{P_f} = \left[\int \left(P_f(t_{design}) - \hat{P}_f \right)^2 f(t_{design}) dt_{design} \right]^{1/2} \tag{10}$$

Probability of Failure Calculation by Monte Carlo Simulations

The inner loop in Fig. 1 (steps C-E) represents the simulation of a population of *M* airplanes (hence panels) that all have the same design. However, each panel is different due to variability in geometry, failure stress, and loading (step D). We subject the panel in each airplane to actual random maximum (over a lifetime) service loads (step E) and calculate whether it fails using Eq. (6).

We count the number of panels failed for each airplane, and add up all the failures. The failure probability is calculated by dividing the number of failures by the number of airplane models that passed certification, times the number of copies of each model.

The analytical approximation for the probability of failure suffers due to the approximations used, while the Monte Carlo simulation is subject to sampling errors, especially for low probabilities of failure. Using large samples, though, can reduce the latter. Therefore, we compared the two methods for a relatively large sample of 1,000 aircraft models with 100,000 instances of each model. In addition, the comparison is performed for the case where mean material properties (rather than A-basis properties) are used for the design, so that the probability of failure is high enough for the Monte Carlo simulation to capture it accurately. Table 3 shows the results for this case..

Table 3: Comparison of probability of failures (P_f's) for panels designed using safety factor of 1.5, mean value for allowable stress and error bound of 50%

Value	Analytical Approximation	Monte Carlo Simulation*	% error
Average Value of P_f without certification (P_{nt})	1.741×10^{-1}	1.787×10^{-1}	2.6
Standard Deviation of P_{nt}	3.006×10^{-1}	3.035×10^{-1}	1.0
Average Value of P_f with certification (P_t)	1.010×10^{-3}	1.094×10^{-3}	7.6
Standard Deviation of P_t	6.16×10^{-3}	5.622×10^{-3}	9.6
Average Value of Initial error factor (e^i)	0.0000	-0.0058	---
Standard Deviation of e^i	0.2887	0.2876	0.4
Average Value of Updated error factor (e^{up})	0.2444	0.2416	1.2
Standard Deviation of e^{up}	0.1578	0.1546	2.1

*N = 1000 and M = 100,000 is used in the Monte Carlo Simulations

The last column of Table 3 shows the percent error of the analytical approximation compared to Monte Carlo simulations. It is seen that the analytical approximation is in good agreement with the values obtained through Monte Carlo simulations. It is remarkable that the standard deviation of the probability of failure is almost twice the average value of the probability (the ratio, the coefficient of variation, is about 170%) before certification, and about six times larger after certification. This indicates huge variability in the probability of failure for different aircraft models, and this is due to the large error bound, *e=0.5*. With 1000 different aircraft models (N), the standard deviation in the Monte Carlo estimates is about 3%, and the differences between the Monte Carlo simulation and the analytical approximation are of that order.

V. Effect of three safety measures on probability of failure

We next investigate the effect of other safety measures on failure probability of the panels using Monte Carlo simulations. We performed the simulation for a range of variability in error factor *e* for 500 airplane models (*N* samples in outer loop) and 20,000 copies of each airplane model (*M* samples in inner loop). Here, we compare the probability of failure of a structure designed with three safety measures (safety factor, conservative material property and certification testing) to that of a structure designed without safety measures.

Table 4: Probability of failure for different bounds on error *e* for panels designed using safety factor of 1.5 and A-basis property for allowable stress. Numbers in parenthesis denote the coefficient of variation of the quantity, as obtained from the analytical approximation

Error bound*e*	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t) x10^{-4}	Probability of failure without certification x10^{-4}(P_{nt})	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
50%	1.279 (0.29)	30.3	6.04 (8.30)	447 (2.63)	1.35×10^{-2}	4.41×10^{-2}
40%	1.274 (0.23)	21.4	5.74 (7.01)	96.3 (2.78)	5.96×10^{-2}	9.06×10^{-3}
30%	1.273 (0.17)	16.2	3.76 (4.21)	12.7 (2.65)	2.96×10^{-1}	8.94×10^{-4}
20%	1.265 (0.12)	8.4	0.972 (2.55)	1.14 (2.24)	8.53×10^{-1}	1.68×10^{-5}
10%	1.271 (0.06)	1.5	0.101 (1.53)	0.117 (1.50)	8.66×10^{-1}	1.57×10^{-6}

*Average over N=500 models

Table 4 presents the results when all safety measures are used for different bounds on the error. The second column shows the mean and standard deviation of design thicknesses generated for *N* generic panels calculated using Eq. (4). These panels correspond to the outer loop of Fig. 1. The variability in design thickness is due to the randomness in the error *e*, and in the stress allowable. The third column shows the percentage of panels that failed certification testing, and as expected, for large errors more aircraft fail certification.

The effectiveness of the certification tests can be expressed by two measures of probability improvement. The first measure is the ratio of the probability of failure with the test, P_t, to the probability of failure without tests, P_{nt}. The second measure is the difference of these probabilities. The ratio is a more useful indicator for low probabilities of failure, while the difference is more meaningful for high probabilities of failure. However, when P_t is high, the ratio can mislead. That is, an improvement from a probability of failure of 0.5 to 0.1 is more substantial than an improvement in probability of failure of 0.1 to 0.01, because it "saves" more airplanes. However, the ratio is more useful when the probabilities are small, and the difference is not very informative.

Table 4 shows that certification testing is more important for large error bounds *e*. For these higher values, the number of panels that fail certification is higher, thereby reducing the failure probability. While the effect of component tests (building block tests) is not simulated, their main effect is to reduce the error magnitude *e*. This is primarily due to the usefulness of component tests in improving analytical models and revealing unmodeled failure modes. With that in mind, we note that the failure probability for the 50% error range is 6.0×10^{-4}, and it reduces to 1.0×10^{-5} for the 10% error range-that is, by a factor of 60.

The actual failure probability of aircraft panels is expected to be of the order of 10^{-8} per flight, much lower than the best number of in the fourth column of Table 4. However, that number is for a lifetime for a single structural component. Assuming about 10,000 flights in the life of a panel and 100 independent structural components, this will translate to a per flight probability of failure of 10^{-7} per airplane. This factor of 10 discrepancy is exacerbated by other failure modes like fatigue that have not been considered. However, other safety measures, such as conservative load specifications may reduce this discrepancy.

Table 5: Probability of failure for different bounds on error *e* for panels designed using safety factor of 1.5 and mean value for allowable stress. Numbers in parenthesis denote the coefficient of variation of the quantity, as obtained from the analytical approximation.

Error bound *e*	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t) x 10^{-4}	Probability of failure without certification (P_{nt}) x10^{-4}	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
50%	(1.004, 0.29)	50.1	9.44 (6.10)	1780 (1.73)	5.32×10^{-3}	1.77×10^{-1}
40%	(0.996, 0.23)	51.5	10.9 (5.43)	1060 (1.86)	1.02×10^{-2}	1.05×10^{-1}
30%	(1.003, 0.17)	52.1	15.1 (4.59)	451 (1.86)	3.36×10^{-2}	4.36×10^{-2}
20%	(0.998, 0.11)	52.9	22.9 (3.05)	142 (1.64)	1.61×10^{-1}	1.19×10^{-2}
10%	(0.999, 0.05)	47.0	27.1 (1.34)	41.8 (1.08)	6.48×10^{-1}	1.47×10^{-3}

*Average over N models

Table 5 shows results when average rather than conservative material properties are used. It can be seen from Table 5 that the average thickness determined using the mean value of allowable stress is lower than that determined using the A-basis value of allowable stress (Table 4). This is equivalent to adding an additional safety factor over an already existing safety factor of 1.5. For the distribution considered in this paper, a typical value of the safety factor due to A-Basis property is around 1.27.

Comparing Table 4 and Table 5, we see that for large errors (40% or 50%), testing is more effective than the additional 27% increase in thickness for reducing the probability of failure.

Strangely, the probability of failure after certification increases with decrease in the error *e*. However, large errors produce some super-strong and some super-weak panels (see Fig. 3b). The super-weak panels are mostly caught by the certification tests, leaving the super-strong panels to reduce the probability of failure. Another way of looking at this effect is to note that when there are no errors, there is no point to the tests. Indeed, it can be seen that the probability of failure without certification tests improves with reduced error bound *e*, but that the reduced effect of the certification tests reverses the trend.

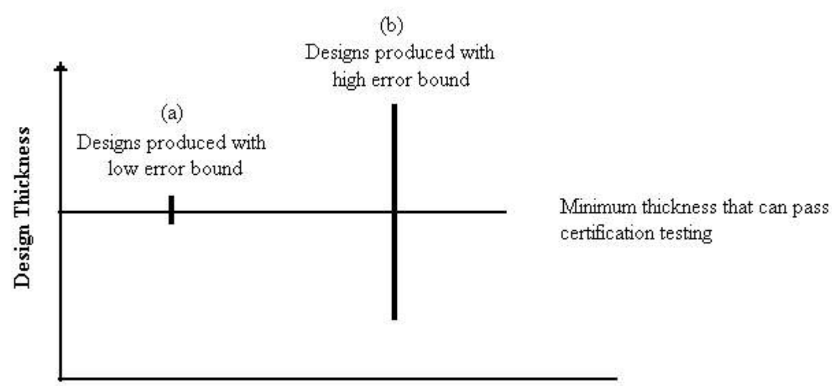

Figure 3: Design thickness variation with low and high error bounds. Note that after certification testing only the designs above the minimum thickness are built and flown. Those on the right have a much higher average design thickness than those on the left.

It is also observed that when the error bound *e* increases, (P_{nt}-P_t) increases and (P_t/P_{nt}) decreases revealing that the efficiency of testing increases.

Table 6: Probability of failure for different bounds on error *e* for safety factor of 1.0 and A-basis property for allowable stress. Numbers in parenthesis denote the coefficient of variation of the quantity, as obtained from the analytical approximation

Error bound *e*	Design thickness*	Certification failure rate %	Failure probability after certification (P_t) x10^{-2}	Failure probability with no certification (P_{nt}) x10^{-2}	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
50%	(0.842, 0.24)	52.3	6.76 (2.00)	25.8 (1.29)	2.62×10^{-1}	1.91×10^{-1}
40%	(0.836, 0.19)	24.0	10.0 (1.81)	23.7 (1.33)	4.22×10^{-1}	1.37×10^{-1}
30%	(0.851, 0.15)	16.0	10.8 (1.58)	18.6 (1.33)	5.82×10^{-1}	7.50×10^{-2}
20%	(0.840, 0.10)	7.5	10.1 (1.25)	11.7 (1.18)	8.60×10^{-1}	7.00×10^{-3}
10%	(0.846, 0.05)	2.0	6.006 (0.76)	6.096 (0.76)	9.85×10^{-1}	9.01×10^{-4}

*Average over N models

Table 6 shows the effect of not using a safety factor. Although certification tests improve the reliability, again in a general trend of high improvement with high error, the lack of safety factor of 1.5 limits the improvement. Comparing Tables 4 and 6 it can be seen that the safety factor reduces the probability of failure by two to three orders of magnitudes.

Table 7: Probability of failure for different error bounds for panels designed using safety factor of 1.0 and mean value for allowable stress

Error bound *e*	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t)	Probability of failure without certification (P_{nt})	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
50%	(0.676, 0.19)	50.7	0.151 (1.32)	0.598 (1.19)	2.53×10^{-1}	4.47×10^{-1}
40%	(0.665, 0.15)	51.6	0.185 (1.10)	0.519 (0.76)	3.56×10^{-1}	3.35×10^{-1}
30%	(0.662, 0.11)	53.2	0.237 (0.86)	0.514 (0.67)	4.61×10^{-1}	2.77×10^{-1}
20%	(0.663, 0.07)	49.8	0.333 (0.60)	0.510 (0.53)	6.53×10^{-1}	1.77×10^{-1}
10%	(0.667, 0.04)	50.4	0.429 (0.32)	0.510 (0.31)	8.41×10^{-1}	8.08×10^{-2}

*Average over N models

Table 7, shows results when the only safety measure is certification testing. Certification tests can reduce the probability of failure of panels by 45%, the highest improvement corresponds to the highest error. As can be expected, without certification tests and safety measures, the probability of failure is near 50%.

Tables 4 – 7 illustrate the probability of failure for a fixed 10 % coefficient of variation in failure stress. The general conclusion that can be drawn from these results is that the error bound *e* is one of the main parameters affecting the efficiency of certification tests to improve reliability of panels. Next, we will explore how another parameter, variability, influences the efficacy of tests. This is accomplished by changing the coefficient of variation on failure stress between 0 – 20% and keeping the error bound constant.

Table 8: Probability of failure for uncertainty in failure stress for panels designed using safety factor of 1.5, 50% error bounds *e* and A-basis property for allowable stress. Numbers in parenthesis denote the coefficient of variation of the quantity, as obtained from the analytical approximation

Coefficient of variation of σ_f	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t) x10^{-4}	Probability of failure without certification (P_{nt}) x10^{-4}	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
0 %	(0.993, 0.29)	52.6	0.011 (5.37)	1690 (3.19)	6.26×10^{-6}	1.69×10^{-1}
5%	(1.131, 0.33)	37.5	1.52 (10.69)	1060 (3.01)	1.44×10^{-3}	1.06×10^{-1}
10 %	(1.274, 0.37)	35.5	6.05 (8.30)	540 (2.63)	1.12×10^{-2}	5.34×10^{-2}
15 %	(1.376, 0.40)	24.0	21.9 (5.19)	222 (2.26)	9.88×10^{-2}	2.00×10^{-2}
20%	(1.608, 0.48)	16.5	47.5 (3.54)	133 (1.94)	3.58×10^{-1}	8.53×10^{-3}

*Average over N models

Table 9: Probability of failure for uncertainty in failure stress for panels designed using safety factor of 1.5, 30% error bound *e* and A-basis properties

Coefficient of variation of σ_f	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t) x10^{-4}	Probability of failure without certification (P_{nt}) x10^{-4}	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
0 %	(0.997, 0.17)	49.9	0.022 (4.85)	245 (3.67)	8.98×10^{-5}	2.45×10^{-2}
5 %	(1.139, 0.20)	31.0	0.145 (6.89)	45.5 (3.30)	3.19×10^{-3}	4.54×10^{-3}
10 %	(1.271, 0.22)	16.0	3.93 (4.21)	12.1 (2.65)	3.26×10^{-1}	8.12×10^{-4}
15 %	(1.439, 0.24)	7.4	6.79 (2.88)	10.1 (2.08)	6.70×10^{-1}	3.34×10^{-4}
20%	(1.624, 0.27)	4.5	7.37 (2.11)	8.96 (1.66)	8.23×10^{-1}	1.58×10^{-4}

*Average over N models

Table 10: Probability of failure for uncertainty in failure stress for panels designed using safety factor of 1.5, 20% error bounds *e* and A-basis properties

Coefficient of variation of σ_f	Design thickness*	Certification failure rate %	Probability of failure after certification (P_t) x10^{-4}	Probability of failure without certification (P_{nt}) x10^{-4}	Probability ratio (P_t/P_{nt})	Probability difference (P_{nt}-P_t)
0 %	(1.007, 0.11)	47.9	0.04 (3.25)	25.2 (3.21)	1.60×10^{-3}	2.51×10^{-3}
5 %	(1.127, 0.12)	21.0	0.253 (3.14)	2.33 (2.86)	1.09×10^{-1}	2.08×10^{-4}
10%	(1.279, 0.15)	7.0	0.538 (2.55)	1.55 (2.24)	3.47×10^{-1}	1.01×10^{-4}
15 %	(1.436, 0.16)	3.5	1.17 (1.93)	1.70 (1.71)	6.91×10^{-1}	5.26×10^{-5}
20%	(1.629, 0.18)	3.1	1.91 (1.45)	2.40 (1.31)	7.93×10^{-1}	4.96×10^{-5}

*Average over N models

The increase in the variability in failure stress has a large effect on the allowable stress because A-basis properties specify an allowable that is below 99% of the sample. Increased variability reduces the allowable stress and therefore increases the design thickness. It is seen from Tables 8-10 that when the variability increases from 0% to 20%, the design thickness increases by more than 60%. In spite of this, the probability of failure still deteriorates. That is, the use of A-basis properties fails to fully compensate for the variability in material properties.

The variability in failure stress greatly changes the effect of certification tests. When the variability is large, the value of the tests is reduced because the tested aircraft can be greatly different from the airplanes in actual service. We indeed see from the Tables 8-10 that the effect of certification tests is reduced as the variability in the failure stress increases. Recall that the effect of certification tests is also reduced when the error *e* decreases. Indeed, Table 8 shows a much smaller effect of the tests than Table 10.

P_{nt} and P_t results of Tables 8-10 corresponding to a 10% coefficient of variation of σ_f are slightly different from the results presented in Table 3 with 50%, 30% and 20% error bounds. This is an indication of the accuracy of Monte Carlo simulations. More accurate results may be obtained by increasing the sample size.

Up to now, both the probability difference (P_{nt}-P_t) and the probability ratio (P_t/P_{nt}) seem to be good indicators of efficiency of tests. To allow easy visualization, we combined the errors and the variability in a single ratio (Bounds on e) / $V_R(\sigma/\sigma_f)$ ratio (ratio of error bound e to the coefficient of variation of the stress ratio). The denominator accounts for the major contributors to the variability. The value in the denominator is a function of four variables; service load P, width w, thickness t, and failure stress σ_f. Here, P and σ_f have lognormal distributions but w and t are uniformly distributed. Since the coefficient of variations of w and t is very small, they can also be treated as lognormally distributed to make calculation of the denominator easy while plotting the graphs. Since the standard deviations of the variables are small, the denominator is now the square root of the sum of the squares of coefficient of variations of the four variables mentioned above, that is

$$V_R(\sigma/\sigma_f) \cong \sqrt{V_R^2(P)+V_R^2(w)+V_R^2(t)+V_R^2(\sigma_f)} \tag{11}$$

The effective safety factor is the ratio of the design thickness of the component when safety measures (such as usage of A-basis values for material properties and safety factor) are applied to the thickness of the component when no safety measures are taken.

Figures 4 and 5, present the ratio P_t/P_{nt} ratio in visual formats. It can be seen that as expected, the ratio decreases as the (Bounds on e)/$V_R(\sigma/\sigma_f)$ ratio increases. However, these two figures do not give a clear indication of how certification tests are influenced by the effective safety factor.

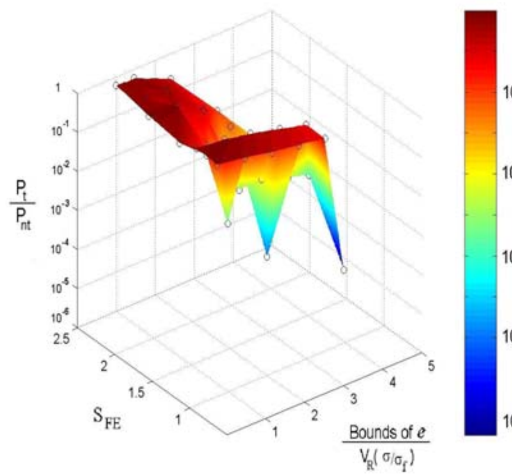

Figure 4: Influence of Effective Safety Factor, Error, and Variability on the Probability Ratio (3-D View)

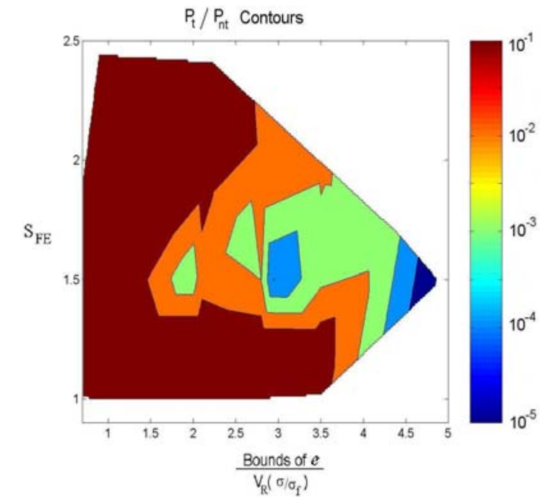

Figure 5: Influence of Effective Safety Factor, Error and Variability on the Probability Ratio (2-D Contour Plot)

Figures 6 and 7 show the probability difference, P_{nt}-P_t. This time, the dependence on the effective safety factor is monotonic. As expected, it is seen that as the effective safety factor increases, the improvement in the safety of component decreases; meaning that the tests become less useful. The probability difference is more decisive as it is proportional to the number of aircraft failures prevented by certification testing. The probability ratio lacks such clear physical interpretation, even though it is a more attractive measure when the probability of failure is very small.

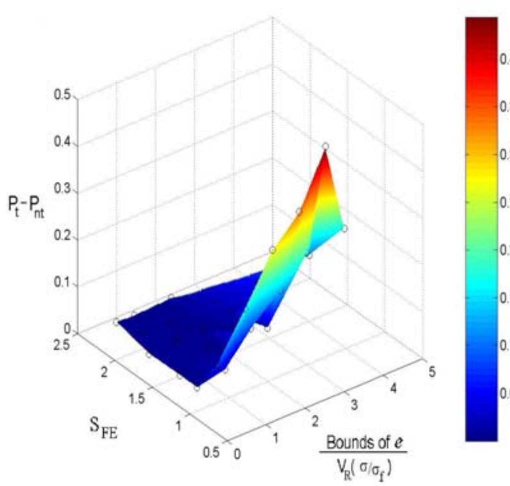

Figure 6: Influence of Effective Safety Factor, Error and Variability on the Probability Difference (3-D View)

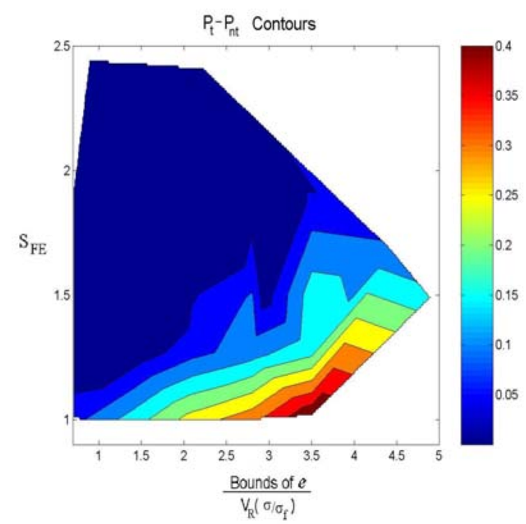

Figure 7: Influence of Effective Safety Factor, Error and Variability on the Probability Difference (2-D Contour Plot)

Considering the results presented by Figures 4-7, the probability difference (P_{nt}-P_t) is the more appropriate choice for expressing the effectiveness of tests.

VI. Concluding Remarks

We have used a simple example of an unstiffened panel design for yield to illustrate the effects of several safety measures taken in aircraft design: safety factors, conservative material properties, and certification tests. Analytical calculations and Monte Carlo simulation were performed to account for both fleet-level uncertainties (such as errors in analytical models) and individual uncertainties (such as variability in material properties).

It was seen that increase of the systemic errors in the analysis manifests itself by increasing the probability of failure. We found that the systemic errors can be reduced by the use of certification tests, thereby reducing the probability of failure.

We found that the effect of tests is most important when errors in analytical models are high and when the variability between airplanes is low. This leads to the surprising result that in some situations larger error variability in analytical models reduces the probability of failure if certification tests are conducted. For the simple example analyzed here, the use of conservative (A-basis) properties was equivalent to a safety factor of up to 1.6, depending on the scatter in failure stresses.

The effectiveness of the certification tests is expressed by two measures of probability improvement . The ratio of the probability of failure with the test, P_t, to the probability of failure without tests, P_{nt}, is useful when P_t is small. The difference is more meaningful when the probability is high. Using these measures we have shown that the effectiveness of certification tests increases when the ratio of error to variability is large and when the effective safety factor is small.

The effect of building-block type tests that are conducted before certification was not assessed here. However, these tests reduce the errors in the analytical models, and on that basis we determined that they can reduce the probability of failure by one or two orders of magnitude.

The calculated probabilities of failure with all the considered safety margins were still high—about 10^{-7}—compared to the probability of failure of actual aircraft structural components—about 10^{-8}. This may be due to additional safety measures, such as conservative design loads or to the effect of design against additional failure modes.

Appendix I

A-Basis property – A-basis value is the value exceeded by 99% of the population with 95% confidence. This is given by

$$\text{A-basis} = \mu - \sigma \times k_1 \tag{A1}$$

where μ is the mean, σ is the standard deviation and k_1 is the tolerance coefficient for normal distribution given by Eq.(A2)

$$k_1 = \frac{z_{1-p} + \sqrt{z_{1-p}^2 - ab}}{a}$$

$$a = 1 - \frac{z_{1-\gamma}^2}{2(N-1)} \; ; \; b = z_{1-p}^2 - \frac{z_{1-\gamma}^2}{N} \tag{A2}$$

where, N is the sample size and z_{1-p} is the critical value of normal distribution that is exceeded with a probability of $1-p$. The tolerance coefficient k_1 for a lognormal distribution is obtained by first transforming the lognormally distributed variable to a normally distributed variable. Equation A1 and A2 can be used to obtain an intermediate value. This value is then converted back to the lognormally distributed variable using inverse transformation.

In order to obtain the A-basis values, 15 panels are randomly selected from a batch. Here the uncertainty in material property is due to allowable stress. The mean and standard deviation of 15 random values of allowable stress is calculated and used in determining the A-basis value of allowable stress.

Appendix II

Calculations of P(C|e), the probability of passing certification test

$$P(C \mid e) = P(\sigma_f > \sigma) = P\left(\sigma_f > \frac{S_F P_d}{wt}\right) = P\left(\sigma_f wt > S_F P_d\right) = P(R > S) \tag{A3}$$

where $R = \sigma_f\, t\, w$ and $S = S_F\ P_d$. (A4)

S is a deterministic value, and since the coefficient of variations of t and w is small compared to the coefficient of variation of σ_f, R can be treated as lognormal. In order to utilize the properties of lognormal distribution for calculating the distribution parameters, we can take S as a lognormally distributed random variable with zero coefficient of variation. Hence, both R and S are lognormally distributed random variables with distribution parameters λ_R, ζ_R, λ_S and ζ_S. Then,

$$\lambda_S = \ln(S_F P_d) \text{ and } \zeta_S = 0 \tag{A5}$$

$$\lambda_R(e) = \lambda_{\sigma_f} + \lambda_t(e) + \lambda_w \text{ and } \zeta_R^2 = \zeta_{\sigma_f}^2 + \zeta_t^2 + \zeta_w^2 \tag{A6}$$

where $\lambda_t(e) = \ln(t_{design}(e)) - 0.5\zeta_t^2 = \ln\left((1+e)\frac{S_F P_d}{w\sigma_a}\right) - 0.5\zeta_t^2$ (A7)

Then, P(C|e) can be calculated as

$$P(C \mid e) = P(R > S) = \Phi\left(\frac{\lambda_R(e) - \lambda_S}{\zeta_R^2 + \zeta_S^2}\right) = \Phi\big(\beta(e)\big) = \int_{-\infty}^{\beta(e)} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx \tag{A8}$$

Calculations of mean value and standard deviation of Probability of Failure

Failure is predicted to occur when the resistance of the structure (R) of the problem is less than the load (P), see Eq. (6). Then, the probability of failure is given as:

$$P_f = \Pr(R < P) \tag{A9}$$

The load P is lognormally distributed, and as explained in Appendix II, the distribution of R can also be approximated by a lognormal distribution, which allows us to immediately obtain the probability of failure of a single aircraft model.

To calculate the probability of failure over all aircraft models, we take into account the fact that that t_{design} is a random variable. Then, the expected value of probability of failure is given as:

$$\hat{P}_f = \int P_f(t_{design})\, f(t_{design})\, dt_{design} \tag{A10}$$

where t_{design} is the non-deterministic distribution parameter, and $f(t_{design})$ is the probability density function of parameter t_{design}.

The standard deviation of failure probability can be calculated from

$$\sigma_{P_f} = \left[\int \left(P_f - \hat{P}_f \right)^2 f(P_f)\, dP_f \right]^{1/2} \tag{A11}$$

where

$$\begin{aligned} P_f &= P_f(t_{design}) \\ f(P_f) &= f(t_{design}) \left| \frac{dt_{design}}{dP_f} \right| \\ dP_f &= \frac{1}{dt_{design}/dP_f} dt_{design} \end{aligned} \tag{A12}$$

Hence, Eq. (A11) can be re-written as

$$\sigma_{P_f} = \left[\int \left(P_f(t_{design}) - \hat{P}_f \right)^2 f(t_{design})\, dt_{design} \right]^{1/2} \tag{A13}$$

As seen from Eqs. (A10) and (A13), the mean and standard deviation of the probability of failure can be expressed in terms of the probability density function (pdf) f of the design thickness, t_{design}. Therefore, we can perform the failure probability estimations to after calculating the pdf of t_{design}. The random variables contributing to t_{design} are (see Eq. (4)) e, w and σ_a. Since the variations of w and σ_a are small compared to e, we neglect their contribution and calculate the pdf of t_{design} from the pdf of error factor e from

$$f(t_{design}) = f_e(e) \frac{de}{dt_{design}} \tag{A14}$$

where $f_e(e)$ is the updated pdf of e.

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