EGM 6812 - Fluid Mechanics 1
Spring 2008
Qualifying Exam

1. The drag force on a solid sphere of diameter d moving at a velocity of u in a still incompressible fluid with density ρ_∞ and dynamic viscosity μ is given by

$$F_D = 3\pi\mu d u \tag{1}$$

provided that $\mathrm{Re} = \rho_\infty u d/\mu \ll 1$.

(a) Use Eq. (1) to determine the terminal velocity of a sphere falling in a still medium, making no assumptions about the latter's density relative to that of the sphere.

(b) Estimate the terminal velocity of a raindrop of $d = 1\,\mathrm{mm}$ falling in air at standard atmospheric conditions at sea level. Comment on whether your result is realistic and, if it is not, why not.

(c) Given a transparent tube filled with an unknown fluid, a ruler, and a stopwatch, how could you estimate the viscosity of the fluid? What would the minimum length of the tube have to be for you to obtain a meaningful estimate?

(d) Assuming that a sphere of diameter d is traveling at its terminal velocity through a tube of diameter D, determine the average velocity of the fluid at the midsection of the sphere.

2. (a) Starting from the momentum equation

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p \tag{2}$$

where ρ is the density, $\mathbf{u}$ is the velocity vector, and p is the pressure, use the vector-analysis identity

$$\mathbf{u} \cdot \nabla \mathbf{u} = \nabla \left(\frac{1}{2} \mathbf{u} \cdot \mathbf{u} \right) - \mathbf{u} \times \boldsymbol{\omega} \tag{3}$$

where $\boldsymbol{\omega}$ is the vorticity, show that for steady, incompressible, and irrotational flow Eq. (2) reduces to Bernoulli's equation

$$p + \frac{1}{2} \rho \mathbf{u} \cdot \mathbf{u} = \text{constant} \tag{4}$$

(b) The flow in a two-dimensional solid-propellant rocket can be modeled by the velocity field

$$\frac{u}{v_i} = \frac{\pi}{2} \frac{x}{h} \cos\left(\frac{\pi}{2} \frac{y}{h} \right) \tag{5}$$

$$\frac{v}{v_i} = -\sin\left(\frac{\pi}{2} \frac{y}{h} \right) \tag{6}$$

defined in the domain $x \geq 0$ and $-h \leq y \leq h$, where v_i is a constant reference velocity.

i. Is Bernoulli's equation applicable to this flow field? If yes, why? If not, why not?

ii. Determine the pressure field $p = p(x, y)$.