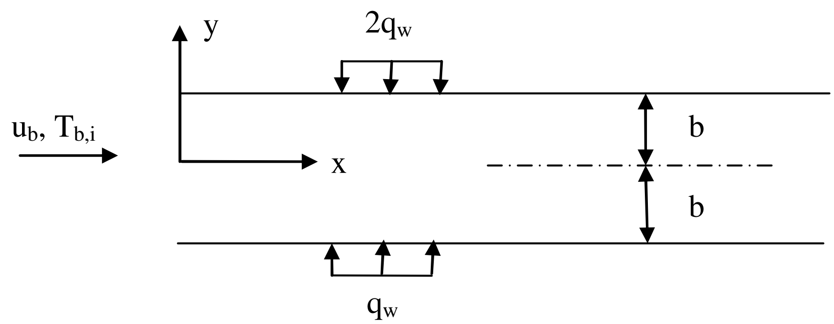
Convection Heat Transfer Qualifier – Spring 2008

Problem 1) Consider a laminar slug flow flowing between two semi-infinite parallel plates separated by a distance 2b. The lower plate is subjected to a uniform heat flux, q_w, and the upper plate is subjected to a uniform heat flux, $2q_w$. The inlet bulk temperature of the fluid is $T_{b,i}$. The velocity, u_b, in the FULLY DEVELOPED thermal boundary layer is approximately uniform and slug flow may be assumed.

(a) Find an expression for the FULLY DEVELOPED temperature profile, T(y).

(b) Evaluate the Nusselt Number at the upper surface, where Nu=hb/k.



Problem 2) Consider steady laminar flow over a flat plate that is maintained at a constant wall temperature. You are given the similarity parameter, $\eta = y\sqrt{\frac{u_\infty}{\nu x}}$ and the stream function, $\psi(x, y) = \sqrt{\nu u_\infty x}F(\eta)$. Recall that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

(a) Show that the energy equation may be reduced to the following ordinary differential equation

$$\tau'' + \frac{1}{2}\Pr F\tau' = 0,$$

where (‘) denotes differentiation with respect to η and $\tau = \frac{T_w - T}{T_w - T_\infty}$.

(b) For **high** Prandtl number fluids (Pr>>1), the velocity profile inside the thermal boundary layer may be assumed linear, $u(y) = \frac{\tau_w}{\mu} y$. Use this information to show that, $F' = F''(0)\eta$.

(c) Given that F”(0)=0.332, derive an expression for the Nusselt number. Leave your answer in terms of the gamma function defined below.

Note: $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x^2} dx$; also Γ(n+1)=nΓ(n).

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Incompressible Fluid

Conservation of Mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Conservation of Momentum

x-momentum

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+\mathrm{X}$$

y-momentum

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial v}+\nu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+Y$$

Conservation of Energy

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\mu\Phi+q'''$$

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left[\left(\frac{\partial u}{\partial y}\right)+\left(\frac{\partial v}{\partial x}\right)\right]^2$$

$$\mathrm{Re}=\frac{ux}{\nu}$$

$$\mathrm{Pr}=\frac{\nu}{\alpha}$$

$$Nu=\frac{hx}{k}$$

$$St=\frac{h}{\rho c_p u}$$