

Problem I (25 points)

- (a) Nondimensionalize Navier-Stokes equation ($\rho \frac{D\vec{V}}{Dt} = -\nabla p + \mu \nabla^2 \vec{V}$) for an oscillatory flow with a high Reynolds number. Clearly list all typical scales you use and show all solution steps. **(8 points)**
- (b) Explain the physical meaning of the dimensionless groups you obtain. **(5 points)**
- (c) Nondimensionalize Navier-Stokes equation for a non-oscillatory, unsteady flow. Clearly list all typical scales you use and show all solution steps. **(7 points)**
- (d) Use the non-dimensionalized equation to explain mathematically why the viscous term can't be dropped for solving the velocity field of a laminar high-Re# flow over a plate. **(5 points)**

Problem II (25 points)

Given:

A thin film of a Newtonian liquid flows over an inclined flat plate as shown.

Find:

- (a) List your assumptions (3 points)
- (b) Simplify the continuity equation as much as possible. (4 points)
- (c) Simplify the x momentum equation. (5 points)
- (d) What are the boundary conditions? (4 points)
- (e) Find the velocity distribution in the liquid. (3 points)
- (f) What is the maximum velocity? (3 points)
- (g) What is the wall shear stress? (3 points)

Schematic:

