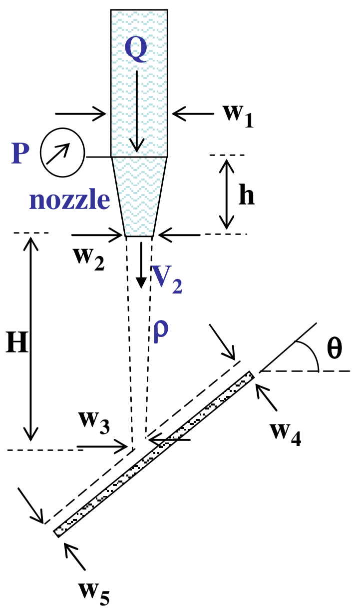
Problem I (25 points)

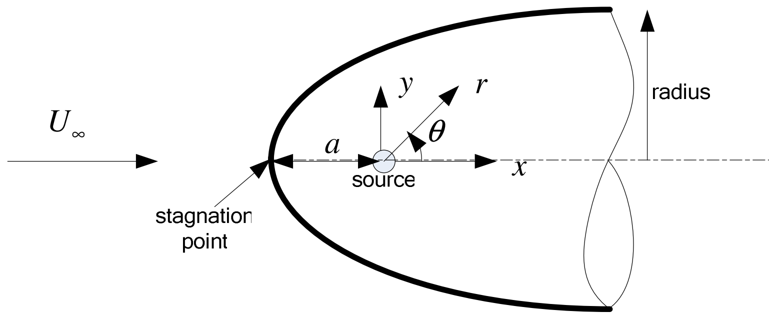
A plane nozzle discharge vertically downward to atmosphere. The nozzle is supplied with a steady flow of water. A stationary, inclined, flat plate, located beneath the nozzle, is struck by the water stream. The water stream divides and flows along the inclined plate; the two streams leaving the plate are of unequal thickness. Frictional effects are negligible in the nozzle and in the flow along the plate surface. As indicated in the figure, the width of the nozzle is W_1 at the top, W_2 at the bottom, and the height of the nozzle is h. The water stream exits the nozzle at a velocity of V_2 and the water density is ρ. The volume flow rate at the nozzle is Q. The distance from the nozzle to the plate is H and the plate is inclined at θ with respect to the horizontal level.



a) List all assumptions. **(2 points)**
b) Draw your control volume. **(3 points)**
c) Evaluate the minimum gage pressure (P) required at the nozzle inlet. **(4 points)**
d) Determine the velocity, V_3, of the flow hitting on the plate. **(3 points)**
e) Determine the magnitude and direction of the force exerted by the water stream on the inclined plate (in the axes parallel with and perpendicular to the plate). **(5 points)**
f) If the width of the water stream is W_3 when it hits the plate, determine the flow width W_4 (as a function of θ and W_3). The plate is very short; thus you can assume that the velocities at W_4 and W_5 are the same as V_3. **(8 points)**

Problem II (25 points)

A uniform stream $\left(\psi = -\frac{U_\infty}{2} r^2 \sin^2\theta\right)$ is superimposed with a source of strength m ($\psi = m\cos\theta$) at the origin to form what is called a Rankine half-body of revolution.



(a) Using $u_r = \frac{-1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}$ and $u_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}$, find expressions for u_r and u_θ over the body. (5 points).

(b) Find the source strength to place the stagnation point at $(r,\theta) = (a,\pi)$. (3 points)

(c) Use the result of (b) to find an expression for u_r (2 points).

(d) Find an expression for the normalized stream function becomes $\frac{\psi}{U_\infty a^2}$ and determine its value on the surface (2 points). Hint: The stagnation point is on the surface.

(e) Find an expression for $\frac{r}{a}$ as a function of θ along the surface. (2 points)

(f) Far downstream, the body reaches a constant radius. What is the radius? (3 points)

(g) Explain how you would find an expression for the surface pressure coefficient $C_p = \frac{P_S - P_\infty}{\frac{1}{2}\rho U_\infty^2}$.

You don't have to come up with an analytical expression. Just list the steps and explain. (3 points)

(h) It turns out that the flow reaches its maximum velocity at $(r,\theta) = \left(a\sqrt{3}, 70.5^0\right)$ where $U_s = 1.155U_\infty$. What is C_p there? (2 points)

(i) In reality, a boundary layer develops over the body. Is there a favorable or adverse pressure gradient or both along the body? Explain. (3 points)