Convection Exam – Spring 2007

Problem 1) Consider that low Prandtl number fluid is injected between two parallel plates as shown below and separated by a distance H and having width w. The lower plate is maintained at temperature T_w and the upper plate is insulated. Fluid is injected with mass flow rate m_o and temperature T_o that laminar flow is maintained at all times. At the stagnation point, a thermal boundary layer forms in the positive and negative x-directions as shown. Assume that the axial component of velocity does not vary in the direction normal to the wall (slug flow assumption).

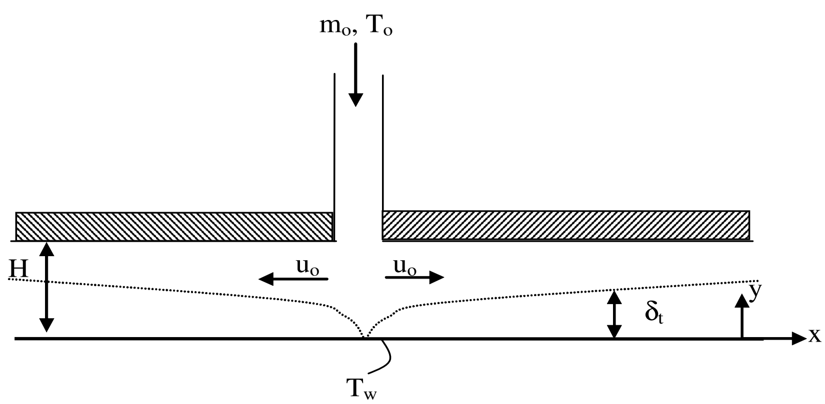
a) Use conservation of mass to evaluate u_o and Reynolds number where $\mathrm{Re} = \dfrac{\rho u_o H}{\mu}$

b) Assume the temperature profile in the thermal boundary layer varies as $\dfrac{T - T_o}{T_w - T_o} = a + b\eta + c\eta^2$,

where $\eta = \dfrac{y}{\delta_t}$. Use the integral energy equation to evaluate the variation of the thermal boundary layer thickness, δ_t, with x. Express relationship in terms of Re, Pr, x, and H.

Integral Energy Equation: $-\alpha \left.\dfrac{\partial T}{\partial y}\right|_{y=0} = \dfrac{d}{dx}\left[\int_0^{\delta_t} u(T - T_o)dy\right]$

c) Evaluate the heat transfer coefficient. Define an appropriate Nusselt number and evaluate.



Problem 2) A fluid is placed between two vertical walls of different temperature ($T=T_1$ at $y=b$ and $T=T_2$ at $y=-b$: $T_1<T_2$). Due to temperature gradients the fluid near the hot wall rises and the fluid near the cold wall descends.

Derive expressions for:

a) the temperature profile
b) the velocity profile
c) the heat transfer coefficient, where the driving potential is (T_2-T_1)

Note: the following may be assumed: the volumetric flow rate of the upward moving stream is the same as that in the downward moving stream; u=u(y); and T=T(y), i.e. u and T only vary in the y direction.

Governing Eqs.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_c)+\nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

where T_c is the centerline temperature.