Fluids1: Problem A (estimated time 40 minutes, 25 total points)

Given:

- A semi-infinite reservoir possessing a constant fluid height has a gravity driven fluid jet issuing from a circular orifice of diameter *D* (known) into an ambient fluid (fluid jet and ambient fluid are different materials) as shown below. Assume atmospheric pressure acts everywhere.
- As this liquid jet falls under the force of gravity, it accelerates and decreases in diameter. At some distance, H_2 , the jet diameter is d (unknown). At some point beyond H_2 , the jet undergoes an instability process and breaks up into spherical droplets that possess a velocity V_{drop} and are separated by a distance L. This problem focuses on the fluid dynamic prior to the break up process.

Schematic:



Find:

- a) Assuming that the <u>ambient fluid is air and the fluid reservoir contains</u> <u>water</u>, determine the velocity at the center of the orifice exit V_D . List your assumptions in making this calculation. (5 points)
- b) What is the boundary condition at the ambient fluid / fluid jet interface? Can any approximations be made for an air / water interface? (5 points)
- c) For the fluid jet in a), determine the velocity at the bottom of the jet V_d . List your assumptions in making this calculation. (4 points)
- d) For the fluid jet in a), determine the diameter at the bottom of the jet d. List your assumptions in making this calculation. (5 points)
- e) If the ambient fluid was changed to oil and the fluid reservoir remained water, would the calculation methods and assumptions in a) d) change? If so, how?
 (6 points)

Fluids1: Problem B (estimated time 45 minutes, 25 total points)

Given:

• Fully-developed flow between two infinite, parallel plates separated by a distance h and driven by an oscillating pressure gradient $\frac{1}{\rho} \frac{\partial p}{\partial x} = \operatorname{Re}\left\{Ke^{j\omega t}\right\}$.

Find:

- a) Reduce the Navier-Stokes equations to the appropriate PDE governing this flow and state the boundary and initial conditions. (3 points)
- b) Solve this PDE in terms of the axial velocity field for the sinusoidal steady state solution. (6 points)
- c) Consider the Stokes number for this problem $S = \sqrt{h^2 \omega / 4\nu}$. Explain the physical meaning. (3 points)
- d) Physically (NOT MATHEMATICALLY) discuss the limit of the solution for very large frequencies and very small frequencies. Discuss the two classic Navier-Stokes solutions at very high and very low oscillation frequencies (4 points)
- e) Sketch the magnitude of non-dimensional velocity field for several values of the Stokes number (0,2,10, and 30). (3 points)
- f) At S = 30, the maximum velocity magnitude does not occur at the centerline. Rather, there is a velocity overshoot near the wall. Physically, describe the underlying cause of this phenomenon.

(3 points)

g) Discuss the generation and decay (destruction) of voriticity generation for both cases in d). (3 points)

Schematic:

