Convection Heat Transfer – Fall 2007

Problem 1

Consider incompressible forced flow over a flat plate with a free stream velocity of U_∞ and free stream temperature T_∞. The boundary condition for the surface of the plate is:

$$T_w - T_\infty = Ax^m$$

Here A and m are constants. The x-coordinate denotes the position along the plate and the y-coordinate denotes the position normal to the plate.

a) In order to analyze the boundary layer thermal field associated with this problem, it is assumed at $u(x,y)=U_\infty$ everywhere in the boundary layer. For what range of Prandtl number is such an assumption valid?

b) Using the assumption that $u(x,y)=U_\infty$ everywhere, show that a similarity solution is available and the governing energy equation may be reduced to an ordinary differential equation with dependent variable F and independent variable η. The similarity variables are given as,

$$Let\ \theta = \frac{T - T_\infty}{T_w - T_\infty} \quad where \quad T_w - T_\infty = Ax^m$$

$$\theta = x^m F(\eta)$$

$$\eta = \frac{y}{\sqrt{x}}$$

(Caution: when substituting variables in energy equation, note that T_w-T_∞ varies with x)

c) State the boundary conditions and use the solution to the energy equation to evaluate the Nusselt Number, Nu_x as a function of Reynolds number and Prandtl number.

Note: the error function $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$; erf(0)=0 and erf(∞)=1

Convection Heat Transfer – Fall 2007

Problem 2

Consider a turbulent flow boundary layer near a wall. Implementing the law of the wall, the momentum and energy equations may be expressed as,

Momentum

$$\frac{\tau_w}{\rho} = (\nu + \varepsilon_M)\frac{du}{dy}$$

Energy

$$q_w = -\rho C_p(\alpha + \varepsilon_H)\frac{dT}{dy}$$

where ε_M is the eddy viscosity and ε_H is the eddy thermal diffusivity. Assume that in the laminar sublayer ($y^+<5$) the turbulent stresses and heat flux are negligibly small and in the outer layer ($y^+>5$), the turbulent stresses and heat flux are dominant. Find an expression for the Stanton number as a function of the friction coefficient and Prandtl number. Assume $Pr_t=0.8$. Ignore the buffer layer for this problem.

(recall that $y^+ = \frac{yu^*}{\nu}$ and $u^* = \sqrt{\frac{\tau_w}{\rho}}$)