Thermal Sciences Qualifying Exam – Spring 2005

Closed book and closed notes. Formula sheet provided. Select and solve one of the (**A**) problems, one of the (**B**) problems, and one of the (**C**) problems for a total of three problems. Select and solve any one of the remaining three problems. Each problem is worth 25 points, for a total of 100 points. Do not solve more than four problems.

Each solution must contain a clear statement of assumptions.

Number all pages consecutively in the upper right corner.

Problem (A1) Using a differential control volume approach, derive the heat diffusion equation for a cylindrical coordinate system, namely r, ϕ, z, t. Include a schematic clearly labeling your coordinate system and the differential volume element. Include all necessary steps and all assumptions.

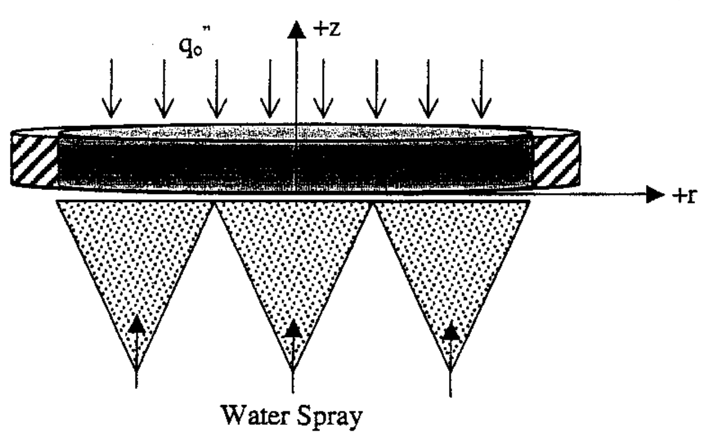
Problem (A2) Consider a solid cylinder as shown in the figure below. The cylinder is of radius R and of height H, and may be considered perfectly insulated along the outer edge at r=R. The upper surface (z=H) of the cylinder will be used for chemical vapor deposition (CVD), which will be accomplished by applying a uniform heat flux q_o'' (W/m^2) to the upper surface using a high-temperature, reactive stream. Cooling will be accomplished at the lower surface (z=0) using a high-volume spray of fine droplets that may undergo phase-change.

The CVD process is initiated while the spray cooling is turned OFF. This results in a non-uniform, high temperature distribution characterized by T=F(r,z), which passivates the upper surface for CVD. At this time, the spray cooling is initiated, which instantaneously brings the entire sprayed surface to the saturation temperature of the coolant, T_{sat}, at which it remains.

(i) Considering the moment that the spray is initiated as t=0, and the corresponding temperature distribution F(r,z) as the initial condition, solve for the transient temperature distribution in the cylinder. You may leave integrals in you final expressions if necessary.

(ii) What is the steady-state temperature distribution? <u>Sketch</u> the steady-state temperature distribution. Discuss the steady-state solution in terms of your solution for part (i).

(iii) Given the high-volume spray of water, which implies excess cooling capacity, discuss the heat transfer at the sprayed surface, and the effect of varying the spray-cooling rate on the CVD surface (z=H) temperature?



Problem (B1) Consider steady laminar flow through two parallel plates separated by a distance H. A constant heat flux, q''_w (W/m^2), is maintained on both the upper and lower plate. A slug flow assumption may be made, i.e. in the energy equation $u=u_b$, where u_b is a constant bulk axial velocity.

(i) Evaluate the fully-devloped temperature profile between the plates.
(ii) Evaluate the Nusselt Number.

Hint: Remember that $T_b = \frac{1}{u_b A}\int_A TudA$, where T_b is the bulk fluid temperature.

Problem (B2) Consider steady laminar flow over a flat plate that is maintained at a constant wall temperature. You are given the similarity parameter, $\eta = y\sqrt{\frac{u_\infty}{\nu x}}$ and the stream function, $\psi(x,y) = \sqrt{\nu u_\infty x}F(\eta)$. Recall that $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$.

(i) Show that the energy equation may be reduced to the following ordinary differential equation

$$\tau'' + \frac{1}{2}\Pr F\tau' = 0,$$

where (') denotes differentiation with respect to η and $\tau = \frac{T_w - T}{T_w - T_\infty}$.

(ii) For low Prandtl number fluids, slug flow may be assumed ($u=u_\infty$) inside the thermal boundary layer. For such a condition evaluate the temperature profile.

Note: $erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-z^2}dz$, where the $erf(\infty) = 1$ and $erf(0) = 0$.

Problem (C1) A rigid box contains N particles of a gas at low density and elevated temperature, with molecular weight M and number of moles n.

(i) If the gas is monatomic, show that the internal energy of the system is given by

$$U = \frac{3}{2} NkT$$

where k is the Boltzmann constant (R/N_o).

(ii) Show that the specific heat ratio for the monatomic gas is 7/5.

(iii) For polyatomic species, discuss the reasons for their lower values of specific heat ratio.

Problem (C2) A geothermal plant is supplied with 10 kg/s of steam (assumed pure) at 500°C and 8 MPa. The ambient conditions are 0°C and 0.1 MPa.

(i) Find the maximum power which the plant could produce.

(ii) Show the process on a T-s diagram that the steam would have to undergo in order to achieve the maximum power production from part (i).

(iii) Typically, a real geothermal plant simply expands the steam in a turbine from the inlet state to ambient pressure. Discuss the sources of irreversibility, which limit the power output to below the maximum.

Given from steam tables:

	v (m^3/kg)	u (kJ/kg)	h (kJ/kg)	s (kJ/kg K)
At 500°C, 8 MPa	.04175	3064.3	3398.3	6.7248
At 0.1 MPa saturated liquid	0.001043	417.3	417.4	1.3029
At 0°C saturated liquid	0.00100	0.0	0.0	0.0