Thermal Sciences Qualifying Exam - Spring 2004

Closed book and closed notes. Formula sheet provided. Select and solve one of the (**A**) problems, one of the (**B**) problems, and one of the (**C**) problems for a total of three problems. Select and solve any one of the remaining three problems. Each problem is worth 25 points, for a total of 100 points. Do <u>not</u> solve more than four problems.

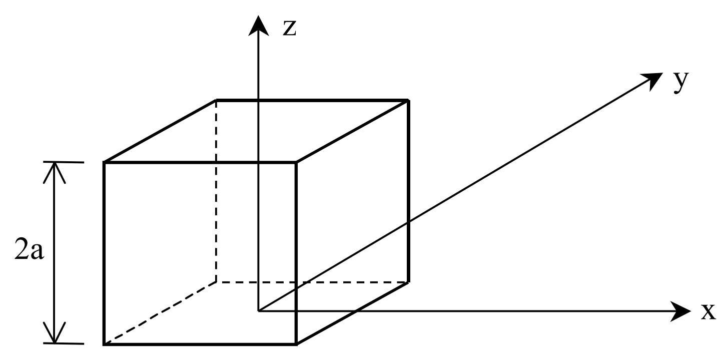
<u>Each solution must contain a clear statement of assumptions</u>

(A1) A large solid cube (dimension 2*a*) of stainless steel (k=15 W/m K) is dropped into a constant temperature oil bath (i.e. large reservoir bath with circulation). The cube rapidly settles flat on the bottom. The temperature of the oil bath T_{bath} is much less than the initial uniform temperature of the cube T_o.

i) Using the coordinate axis shown in the figure, solve the problem for the temperature distribution T = T(x, y, z, t). Fourier series constants may be left in integral form but must be clearly defined.

ii) Using the temperature distribution from part (i), calculate an expression for the total rate of heat removed (W) through the top surface (z=2a) as a function of time.

iii) Take a line parallel to the x-axis through the center of the cube (y=0 and z=a) and sketch the temperature as a function of x along this line (-a<x<a) for a time just after zero, an intermediate time, and a long time.

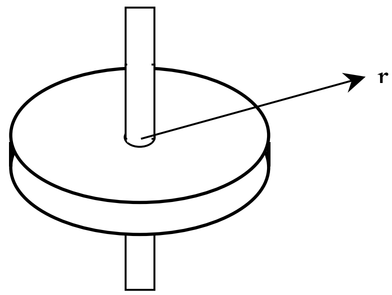


(A2) A radial fin is fixed to a heated pipe such that the temperature at the base of the fin ($r=r_1$) remains at a constant temperature T_b. The fin has a thickness δ and an outer radius of r_2, such that $\delta << r_2$. The fin is placed in a well-mixed air stream of temperature T_{oo} with average convection coefficient of h (W/m^2 K).

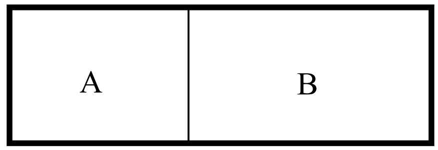
(i) Derive a suitable energy equation for the steady-state, one-dimensional heat transfer problem through the fin, including boundary conditions.

(ii) Solve the energy equation derived in part (i) to yield an expression for the temperature as a function of radius, T=T(r).

(iii) Using the temperature distribution derived in part (ii), calculate the total heat rate (W) removed from the pipe by the fin.



(B1) A container is separated by a removable partition into two regions A and B. Both A and B contain ideal gases at the same pressure. Let n_A and n_B be the initial number of molecules in A and B, respectively.



Let A and B initially contain identical chemical species at different temperatures:

(i) Derive an expression for the total change in entropy, using classical models, after the partition is removed and the system is allowed to reach equilibrium.

(ii) If the partition was instead left in place, but heat was allowed to be transferred through it until equilibrium was achieved, would the same change in entropy occur? Why or why not?

(iii) From a statistical (microscopic) point of view, argue why entropy increases for cases (i) and (ii).

Now consider the case where A and B initially contain different species at the same temperature and pressure:

(iv) If the partition is removed, argue qualitatively why entropy is higher at equilibrium.

(v) During the transition, use statistical arguments to show why the system evolves toward equilibrium. Hint: consider probability theory to show which way the individual species migrate across the A/B boundary.

(B2) A heat engine is in thermal communication with two thermal energy reservoirs at temperatures T_H and T_L, and produces net work W_{net}. For the first three parts of this problem, the thermal energy reservoirs are infinite, so that T_H and T_L are constant.

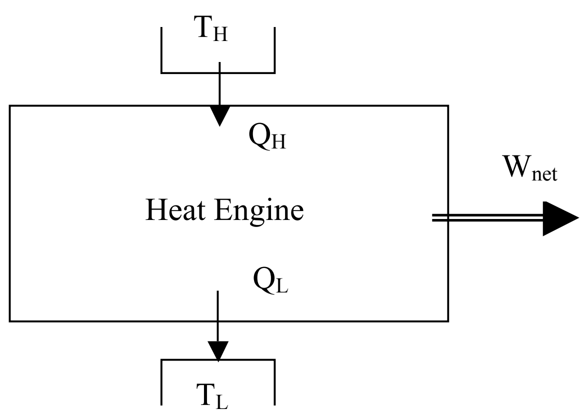
(i) Identify and discuss sources of irreversibility associated with the heat transfer processes. Distinguish between those internal to the working fluid and those external to the working fluid.

(ii) For many model power cycles (Brayton, Otto, etc.) higher compression ratio results in higher thermal efficiency. Use the Second Law of Thermodynamics to explain this effect.

(iii) Discuss the effect of variable working fluid temperature during heating (as in e.g. a Brayton cycle), and relate to part i).

(iv) Consider a case with two high temperature thermal energy reservoirs at different temperatures. Describe how the efficiency of a reversible cycle would be determined. Hint: you may wish to define two Carnot cycles that interact.

(v) Now consider T_H to be variable, as in the case of heat supplied by a limited supply of hot fluid. How does improved heat exchanger effectiveness raise the thermal efficiency? Discuss in terms of heat exchanger irreversibilities and the effect on the Carnot efficiency (based on fluid states).

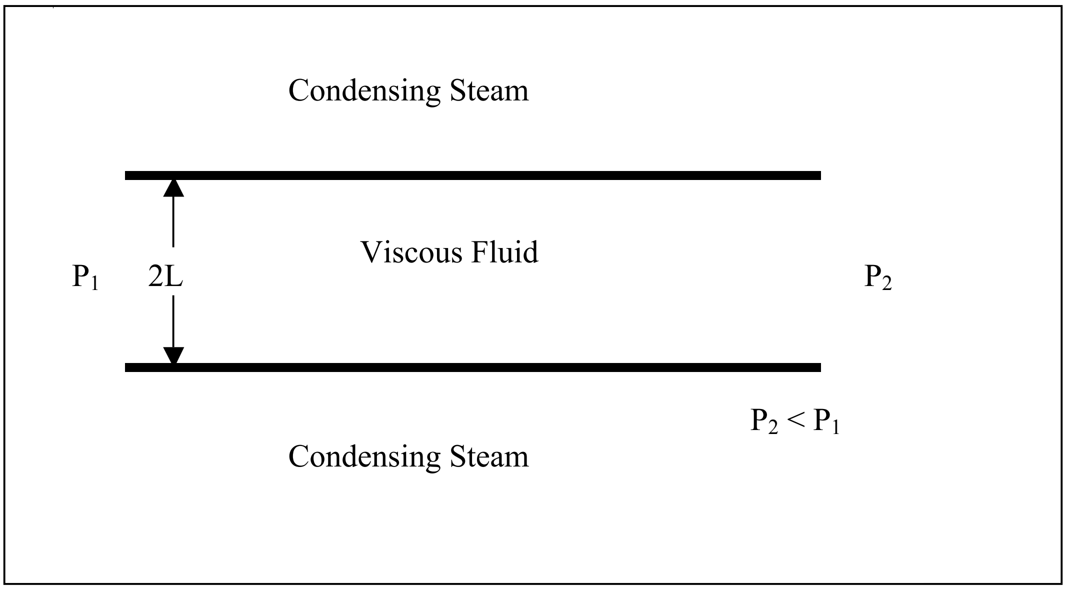


(C1) Parallel plates separated contain a highly viscous fluid, as shown below in the figure. The plates are exposed on the top and bottom to condensing steam. The velocity profile between the plates is fully developed and is given as:

[illegible]

(i) Simplify the energy equation to a form appropriate for this problem. Explicitly state the reason for neglecting any term.

(ii) Find an expression for the Nusselt number.



(C2) Derive parameters that indicate when the various terms in the energy conservation equation are important. Clearly state the physical significance of each.