Value of Game Theory

Value - makes you specify the game (timing, assumptions, rules, players, etc.); sometimes theorists who don't use the structure of game theory miss implicit assumptions (or change those assumptions) in their analysis

Law of one Price - doesn't hold in practice, but implied by perfectly competitive model; theorists tried to develop model to mirror reality (not one price) for a long time

- **Stigler** - early paper using search to justify different prices; criticized as a "partial partial equilibrium" because he didn't address firms (only looked at half the market)
- **Diamond** - first "full partial equilibrium" model (meaning he addressed the full market, but ignored the rest of the economy; this is what we typically refer to as a partial equilibrium model); used multiperiod model with firms possibly choosing new prices each period so search from previous period is no good; result converged to single price where all firms charged monopoly price so concluded that search with cost doesn't necessarily buy anything
- **Salop & Stiglitz** (1977) - finally found model resulting in different prices, but didn't specify assumptions so had a logical flaw
  - **Model** - not clearly specified, but here are the basics:
    - Identical firms with U-shaped average total cost (ATC)
    - **Reservation Price Demand** - each consumer buys 1 unit if the price is below his reservation price
    - Consumers have identical demand, but some have low search cost and others have high search cost; only 2 types of consumers (not continuous like previous models)

      These two can be generalized & result doesn't change
    - **Complete Information** - consumers know ATC and firms know distribution of types of consumers
      - **Imperfect Information** - consumers don't know prices (firm's decision) and firms' don't know type of a specific consumer (that would be perfect info)
    - **Single Search** - consumers can search and learn complete distribution of prices
      - **Sample Scenario** - consumers drive into a town with several gas stations; they don't know which gas stations charge what price; search could be buying a newspaper that lists all gas stations and their prices
      - **Sequential Search** - consumers go from store to store searching prices; complicates the model because we have to worry about what consumer's remember and what they learn
        - **One Arm Bandit** - statistics problem; could end up on inferior machine (lower probability of winning) using finite sample (there is a positive probability that the inferior machine could pay out more than a better machine in a finite sample); in this case, a consumer could show up to one gas station with a high price; then goes to a second station that also has high price; consumer decides this is the low price and doesn't search anymore
      - **When to Search** - consumers who search get the minimum price, $p_{\min}$; consumers who don't search expect to pay average price
        - **Benefit to Search** - BS $= \frac{1}{n}\sum_{i=1}^{n} p_i - p_{\min}$
        - **Net Benefit to Search** - NB = BS – cost of search
        - If NB > 0, consumers will search; if NB < 0, consumers won't search

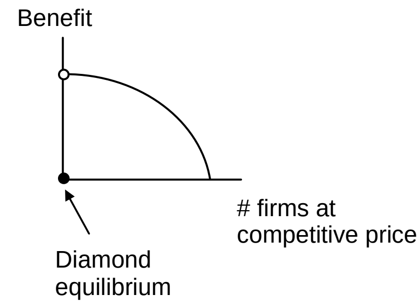
**** Assumption **** - this formula for BS assumes no rationing (i.e., firms sell to everyone who shows up to buy from them)

- **Firms** - take other firms prices as given (similar to simultaneous Nash equilibrium), but have Stackelberg (leader-follower) approach to consumers

  High Price - if high price firm lowers price, it lowers NB & induces some consumers to stop searching (increases sales at high price firm)

  Low Price - if low price firm lowers price, it increases NB & induces people to search

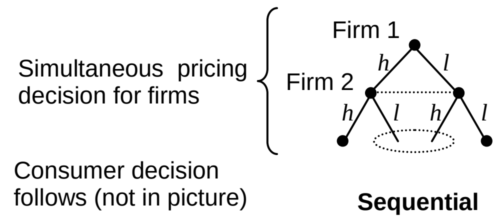
  ∴ both types of firms have incentive to lower price, unless all firms are at the high price in which case NB = 0 (and lowering price would increase it)



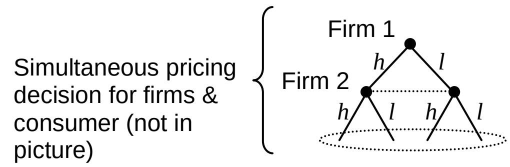
Game Theory - Salop & Stiglitz used some of the theoretic language, but didn't set it up as a game; we will:

- Firms move simultaneously in setting price
- Consumers observe some aspect but not all (weird timing issues)

Example - using 2 firms & 2 prices shown on right; top game is what Salop & Stiglitz used; consumer has 3 information sets (knows distribution of prices, but not which firm is which); lower game is what Stiglitz later used when he rederived the results for another paper



Sequential
3 info sets for consumer



Simultaneous
Single info set for consumer

Problem 1 - not sensible; why would consumers know distribution of prices, but not know who charges what price?

  Only Part - consumers really only need to know average and min price, not the full distribution of prices

  Bill Brainerd - came by Stiglitz's department on Mondays: "I saw a great price, but I don't remember where" ⇒ know distribution (part of it), but don't know who charges what price

  Simultaneous Story - actually preferred by Stiglitz, but he used above two cases to defend sequential to Slutsky; simultaneous (i.e., consumers choose whether to search &/or which firm to buy from at same time as firms choose prices) is more computationally demanding of consumers; consumers don't know distribution, but deduce it knowing firms price simultaneously knowing consumers may search knowing firms price (etc.)

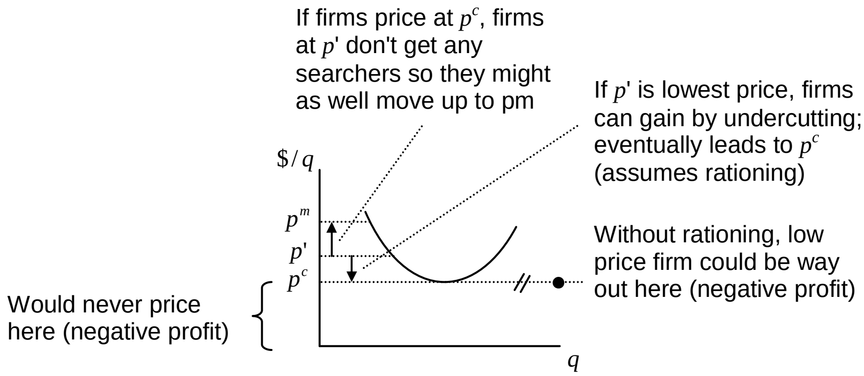
    Unrealistic - it's still a complete information game so it's just as unrealistic as the sequential version; imperfect information is more realistic but even harder to solve

Problem 2 - Salop & Stiglitz implicitly assumed no rationing in order to derive the formula for NB; this assumption is OK for constant marginal costs, but the model specifically says firms have U-shaped ATC so they will ration

  Rain Checks - in some areas, the law requires firms to guarantee sale prices to all consumers by issuing rain checks; in that case no rationing would apply

  Analysis - Salop & Stiglitz used intuitive micro argument and got two prices: min ATC and monopoly price (or reservation price)

**** Assumption **** - in their analysis, Salop & Stiglitz argued firms would undercut until they reached min ATC... this argument assumed rationing



Game Theory Analysis - with n firms and m consumers, it's difficult to find equilibrium (intersection of $n+m$ best replies); instead we'll aggregate the firms and the consumers to get reduced from best replies so we only deal with two (this is a "standard trick")

More Detailed - look at Golding & Slutsky "Price Dispersion in a Simultaneous Move Salop-Stiglitz Search Model Without Rationing" (1999)

Assumptions - clearly identify some of the things Salop & Stiglitz left out

(1) Market for homogenous durable commodity q

(2) Free entry with identical technology for each firm; cost function $C(q) = T - v(q)$ where $v'(q) > 0$ and $v''(q) > 0$ (i.e., decreasing rate of return; MC increases at increasing rate)

(3) Firm announces price p when it enters the market and must sell to every consumer who wishes to purchase from it (i.e., no rationing)

(4) Large number (L) of consumers who have identical indirect utility functions $U(p)$ and demands $D(p)$

General Demand - this are general downward sloping demand curves, not the more restrictive reservation price demand used by Salop & Stiglitz

No Income Effects - so $D(p) = -\partial U / \partial p$

Expected Utility - if price is uncertain, consumers maximize expected utility

(5) Consumers have search cost c^i; if paid allows consumer to determine the firm(s) with the lowest price; if multiple firms have the lowest price, consumers who search select form them at random with equal probability

Cost Distribution - $F(c) = \Pr[c^i \le c]$

(6) Consumers who don't search choose a firm at random for all firms in market with equal probability

(7) Consumers search with probability 1 if perceived benefits from search are strictly greater than the cost, with probability 0 if perceived benefits are strictly less, and with arbitrary probability if perceived benefits equal costs (this of this as a mixed strategy for an individual consumer or as a fraction of the consumers search and a fraction doesn't; denote search decision by $d(c)$ ... function of search cost:

$$d(c) = \begin{cases} 1 & \text{if NB} > 0 \\ \in [0,1] & \text{if NB} = 0 \\ 0 & \text{if NB} < 0 \end{cases}$$

(8) All decisions (firms & consumers) are simultaneous "in game time" (that means firms can actually make the decision first, but consumers make their decision before observing firm decisions)

Complete Information - all firms and consumers know cost curves, utility function, number of consumers, distribution of search costs, and fact that everyone else knows

Nash Equilibrium - look at conditions for each player where they don't want to change their decision; define:

$n^* =$ # of firms that choose to enter

$\mathbf{p}^* = (p_1^*, p_1^*, \ldots p_{n^*}^*) =$ prices firms charge

$\alpha^* =$ fraction of consumers who decide to search

Conditions i-iii say no firm can gain by changing what it's doing; iv is for consumers

(i) $\pi(p_i^* \mid \mathbf{p}_{\sim i}^*, \alpha^*) \geq \pi(p_i \mid \mathbf{p}_{\sim i}^*, \alpha^*) \;\forall\; p_i$

(i.e., firm that enters doesn't want to change price)

(ii) $\pi(p_i^* \mid \mathbf{p}_{\sim i}^*, \alpha^*) \geq 0 \;\forall\; i = 1, 2, \ldots n^*$

(iii) $\pi(p_{n^*+1}^* \mid \mathbf{p}^*, \alpha^*) \leq 0 \;\forall\; p_{n^*+1}$

(ii & iii are discrete free entry condition)

(iv) same as $d(c)$ formula on previous page $\forall\; c$

Reduced Form - trick we'll use (point of studying this paper)... can get derive best replies in tractable form (i.e., in two dimensions so we can graph them) if we have identical players in some respect

Consumers - since consumers are identical (except for search cost), firms only care about the number of consumers that search (i.e., α^*)

Benefit of Search - same for all consumers; we defined it in terms of dollars on bottom of p.1, but here we'll use utility (no assumption about risk tolerance):

$$\beta^* = U(p_{\min}) - \frac{1}{n}\sum_{i=1}^{n} U(p_i^*)$$

$\therefore$ Consumers who search have cost $\leq \beta^*$; since we're using α^* (proportion), this is the cumulative distribution of cost (see (5) on previous page)

 # Consumers Who Search - $\boxed{\lim_{\beta \to \beta^{*-}} F(\beta) \leq \alpha^* \leq F(\beta^*)}$

(This looks complicated because it's accounting for the possibility of having an atom [i.e., positive probability; vertical jump in CDF] at β^*)

This is the <u>best reply</u> of α wrt β^*

Producers -

Lemma 1 - if we fix α (i.e., fraction who search is given) at most two prices can be charged by firms; if there are 2 prices, the high price must be the monopoly price ($p^m(n, \alpha)$); the low price can be anything below the monopoly price and down to and including the competitive price:

$p^c \leq \overline{p} < p^m(n, \alpha)$

<u>Proof</u>: assume there are three prices $p_2 > p_1 > p_{\min}$

Firm charging p_1 will want to raise price to p_2... doesn't change number of consumers he serves so he increases profit $\therefore$ at most 2 prices

Firm charging any price > $p_{\min}$ will gain by raising price to $p^m(n,\alpha)$...
same argument: doesn't change number of consumers so this increases profit

Note: $p^m(n,\alpha)$ is not constant; depends on number of firms (and fraction who search, but that's fixed right now)

Lemma 2 - The monopoly price when the number of firms adjusts to ensure zero profits is independent of α

Proof: number of firms is adjusting so we can use $n(\alpha)$

$\therefore$ $p^m(n(\alpha),\alpha)$... if α changes, $n(\alpha)$ also changes, negating the effect on monopoly price so $\partial p^m / \partial \alpha = 0$ (this isn't really a proof)

Mathy version is in the paper, but it's nearly incomprehensible (looks like it's written in secret code!)

Two Price Equilibrium - can have some firms charging monopoly price and other firms charging a lower price $\overline{p}$ $\therefore$ benefit of search becomes:

$$\beta(\overline{p},\alpha) = U(\overline{p}) - \left[\gamma U(p^m) + (1-\gamma)U(\overline{p})\right] = \gamma\left[U(p^m) - U(\overline{p})\right]$$

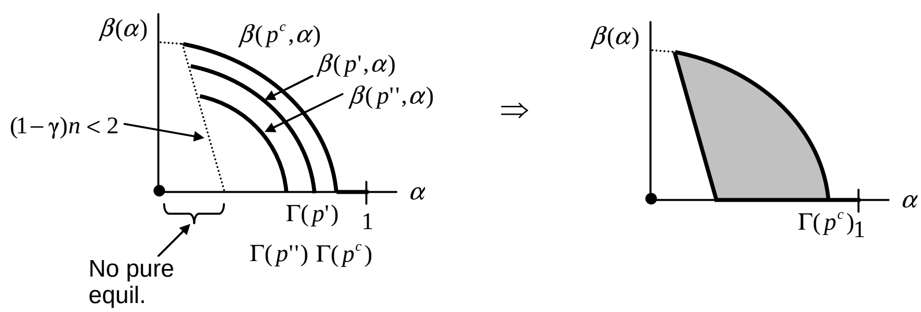
where $\gamma =$ fraction of firms charging p^m

Note: $U(p^m) - U(\overline{p}) > 0$ $\therefore$ $\beta \uparrow$ as $\gamma \uparrow$ (until $\gamma = 1$; then $\overline{p} = p^m$ so $\beta = 0$)
(i.e., if more firms charge monopoly price, benefit of search increases)

This is the best reply of β wrt α (in the paper they work out a more detailed equation)

Graphically - graphing $\beta(\overline{p},\alpha)$... use various values for the low price;

- Highest benefit occurs when $\overline{p} = p^c$ (competitive price, the lowest it can go)
- Only way a firm will charge the price (vs. monopoly price) is when at least two firms are charging the low price $\therefore$ discontinuity when $(1-\gamma)n < 2$
- As $\alpha \downarrow$ (less searchers), $\gamma \uparrow$ (more firms charge p^m) and $\beta \uparrow$

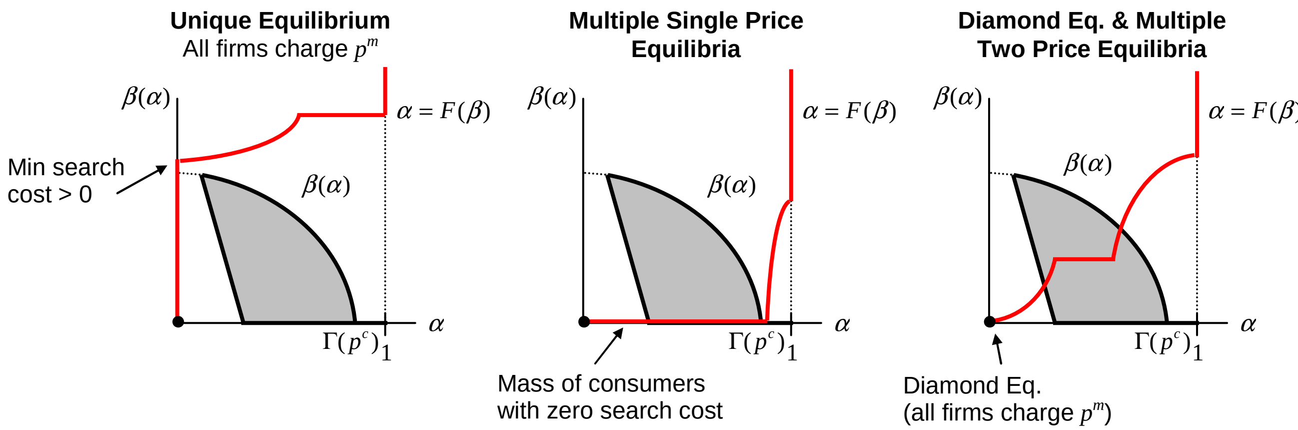


Combine Best Replies - since we reduced the best replies to two equations in two variables, we can plot them and see where they intersect; three cases:

(1) Unique equilibrium with all firms charging monopoly price; this is the original Diamond equilibrium; this occurs when all customers have sufficiently high search costs

(2) Multiple single price equilibria; all firms either charge the monopoly price or the low price

(3) Diamond equilibrium and multiple two price equilibria



Refinements - using "trembling hand" we can rule out (p^m, p^c) as a two price equilibrium because any mistake by another firm causes the low price firm to have negative profits (so entering at low price is weakly dominated by not entering)

Why Did We Do This?

(1) Game theory helps prevent mistakes; forces you to write out all the details
(2) Reduce game to form that's tractable to solve (usually requires identical players in some respect)

Intentional? - can't always tell if logical inconsistencies in papers are mistakes (like Salop & Stiglitz) or intentional; Slutsky talked about a paper that assumes a, b, & c which is "spurious generality"; taken individually, they look general, but in order for a, b, & c to hold at the same time there has to be a specific utility function... get a couple pitchers for Slutsky and he'll tell you who did this