Externalities and Public Goods

Welfare Theorems - basic assumptions we had to make:
1. **Price Takers** - core argument shows this assumption is valid for large economies; people can't lie ("misrepresent their preferences") to get out of the core.
2. **Convexity** - given single individual with non-convex preferences, he could use a mixed strategy, but it doesn't make sense to talk about his average consumption; given a million of this same type of individual with half choosing one point and the other half choosing the other, and we can talk about the average consumption.; for large economies, the convexity assumption is valid (i.e., a large economy "convexifies" individual non-convexities).
3. **No Externalities** - decisions that affect others' welfare directly (not pecuniary [through prices]).

Externalities
1st FTWE - fails if person 1 doesn't take other people's benefits/costs into account; this leads to inefficiency
**Inducing Efficiency** - try to get people to internalize extra benefits/costs
- **Pigovian Approach** - government steps in with corrective subsidies/taxes; efficiency is restored if the subsidy/tax is the correct amount.
  - **Note**: assumes lump sum taxes so we can ignore distortions from taxes.
- **Coasian Approach** - Chicago approach; government doesn't have to intervene; people can negotiate on their own.
  - **Note**: assumes zero transaction/bargaining costs.

Definitions -
- **Non-Excludable** - can't keep somebody from getting the good once it's supplied.
- **Non-Rival** - one person's consumption doesn't interfere with another persons; marginal cost is zero.
- **Jointness of Supply** - if the good is supplied, it's available to everyone even if excludable (e.g., non-congested bridge).
- **Inefficiency** - good that is non-rival, but excludable can be supplied privately, but may be inefficient because MC = 0 (i.e., if firm charges price > 0, people will consume less than the socially optimal level).

**Pure Public Good** - is both non-rival and non-excludable.

Notation -
- \( x^i \) - vector of private good consumption for person \( i \)
- \( \sum_{i=1}^{n} x^i \) - aggregate consumption of private goods
- \( y \) - supply of private goods
- \( z^i \) - vector of public good consumption for person \( i \)
- \( z \) - supply of public goods
Pareto Optimality -  
\[
\max_{x', x} W\left(u^1(x^1, z^1), \ldots, u^n(x^n, z^n)\right)
\]
\[
\text{s.t. } \sum_{i=1}^n x_i' = y \quad \text{(market clearing for private goods; demand = supply)}
\]
\[
z_i' = z_i \quad \forall i \quad \text{(market clearing for public goods; everyone consumes same amount)}
\]
\[
F(y, z) = 0 \quad \text{(aggregate technology; production of private and public goods is feasible)}
\]

Note: think of objective as weighted sum of \(u^i\)'s to generate the utility possibilities frontier (i.e., we're not limiting it to a specific definition of social efficiency like utilitarian or Rawlsian)

Assumptions -
1) only consider interior solution; there are some interesting results with non-negativity constraints, but we don't have time
2) at least 1 private good and 1 public good (makes the math easier)

Simplify - the formulation above is the general case which allows selective excludability so we showed how much public good each consume gets... technically could use \(\leq\) if some are excluded; if we assume there is no one excluded, we can simplify the formulation:
\[
\max_{x', x} W\left(u^1(x^1, z^1), \ldots, u^n(x^n, z^n)\right)
\]
\[
\text{s.t. } F\left(\sum_{i=1}^n x_i', z\right) = 0
\]

Lagrangian - \(\ell = W\left(u^1(x^1, z), \ldots, u^n(x^n, z)\right) - \lambda F\left(\sum_{i=1}^n x_i', z\right)\)

FOCs -
\(k^{th}\) private good consumed by \(j^{th}\) person:
\[
\frac{\partial \ell}{\partial x_k^j} = \frac{\partial W}{\partial u^i} \frac{\partial u^i}{\partial x_k^j} - \lambda \frac{\partial F}{\partial x_k^j} = 0, \quad \forall j = 1, \ldots, n \quad \text{and} \quad k = 1, \ldots, K
\]
\(h^{th}\) public good consumed by any person (they all consume the same amount)
\[
\frac{\partial \ell}{\partial z_h} = \sum_{j=1}^n \frac{\partial W}{\partial u^i} \frac{\partial u^i}{\partial z_h^j} - \lambda \frac{\partial F}{\partial z_h} = 0, \quad \forall h = 1, \ldots, m
\]

Two Private Goods - \(k \& l\) for same individual \(i\)
\[
\frac{\partial W}{\partial u^i} \frac{\partial u^i}{\partial x_k^i} = \frac{\partial F}{\partial x_k^i} \quad \Rightarrow \quad \frac{\partial u^i}{\partial x_k^i} = \frac{\partial F}{\partial u^i} / \frac{\partial F}{\partial x_k^i} \Rightarrow \text{MRS}_{k, l} \neq \text{MRT}_{k, l}
\]

Marginal rate of substitution between goods \(k \& l\) = marginal rate of transformation between those two goods

Note: since MRS for any consumer = MRT (which is always the same), MRS is the same for all consumers; this is the same result as before (see p.4 of "Simple Models" notes)

Public and Private - public good \(h\) and private good \(1\)

Take FOC of public good \(h\) (shown above):
\[
\sum_{j=1}^n \frac{\partial W}{\partial u^i} \frac{\partial u^i}{\partial z_h^j} = \lambda \frac{\partial F}{\partial z_h}
\]
Divide both sides by \( \lambda \frac{\partial F}{\partial x_i} \):
\[
\sum_{j=1}^{n} \frac{\partial W}{\partial u^j} \frac{\partial u^j}{\partial z_h} \frac{\partial F}{\partial x_i} = \lambda \frac{\partial F}{\partial z_h} \frac{\partial F}{\partial x_i}.
\]

\( \partial F / \partial x_i \) is not dependent on the consumer, so it can be put inside the summation:
\[
\sum_{j=1}^{n} \frac{\partial W}{\partial u^j} \frac{\partial u^j}{\partial z_h} = \frac{\partial F}{\partial z_h} = \text{MRT}_{z_h, x_i}
\]

Now use FOC for private good 1 and consumer j:
\[
\lambda \frac{\partial F}{\partial x_i} = \frac{\partial W}{\partial u^j} \frac{\partial u^j}{\partial x_i}
\]
\[
\sum_{j=1}^{n} \frac{\partial W}{\partial u^j} \frac{\partial u^j}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial u^j}{\partial x_i} = \sum_{j=1}^{n} \text{MRS}_{z_h, x_i} = \text{MRT}_{z_h, x_i}
\]

**Samuelson Condition for Public Goods** - \( \sum_{j=1}^{n} \text{MRS}_{z_h, x_i} = \text{MRT}_{z_h, x_i} \) ... sum of MRS between public and private goods for all consumers is equal to the MRT between those goods

**Caution** - sometimes theorists are casual about how they specify MRS

\[
\text{MRS}_{ij} = \left. \frac{-dx_j}{dx_i} \right|_{y=\text{constant}} \quad \text{or} \quad \left. \frac{-dx_i}{dx_j} \right|_{y=\text{constant}}
\]

But with public and private good, it matters:
\[
\text{MRS}_{z_h, x_i} = \left. \frac{-dx_i}{dz_h} \right|_{y=\text{constant}}
\]

which is equal to marginal willingness to pay for the public good (i.e., holding person j’s utility constant, how much \( x_i \) does person j have to give up for more public good \( z_h \)).

\[
\sum_{j=1}^{n} \text{MRS}_{z_h, x_i} = \text{total social willingness to pay} \text{ for public good}
\]

**Over Supply** - \( \sum_{j=1}^{n} \text{MRS}_{z_h, x_i} < \text{MRT}_{z_h, x_i} \) (i.e., marginal value is less than marginal cost)

**Under Supply** - \( \sum_{j=1}^{n} \text{MRS}_{z_h, x_i} > \text{MRT}_{z_h, x_i} \) (i.e., marginal value exceeds marginal cost)

**Duality** - between public and private goods:

**Quantity Conditions** - \( \sum_{j=1}^{n} x' = y \) (private) and \( z' = z \) (public)

**PO Conditions** - \( \text{MRS}_{k,j} = \text{MRT}_{k,j} \) (private) and \( \sum_{j=1}^{n} \text{MRS}_{z_h, x_i} = \text{MRT}_{z_h, x_i} \) (public)
View MRS as price ratio so $\text{MRS}_{k,j} = \text{MRT}_{k,j}$ for private goods basically says "all individuals pay the same price"; for public goods: $\sum_{j=1}^{n} \text{MRS}_{x_j, x_j'} = \text{MRT}_{x_j, x_j'}$ says that price is equal to the sum of what everybody pays

**Result** - "private good prices are like public good quantities" and "public good prices are like private good quantities" (mathematically)

**Free Riders** - since public goods are non-excludable, consumers have incentive to free-ride (not pay their "fair share" and still consume the good); for private goods, the free rider problems results in efficiency because collusion breaks down, but is the source of inefficiency for public goods

**Inefficiency** - because of free rider problem "voluntary approach" (asking consumers to pay for public goods by paying for the value they receive) doesn't work; usual solution is compulsory benefit taxation, but even that doesn't work because government can't determine MRS for each individual (because of free rider problem); that means government turns to income taxation assuming income is perfectly correlated with preferences for public goods

**Can't Solve Inefficiency** - because of duality, if someone manages to find a mechanism to solve the free rider problem in public goods (hence solving the inefficiency), that same mechanism could be used to enforce cartels for private goods (resulting in inefficiency)

**Samuelson's View** - argued if a good is not a pure private good (i.e., excludable and rival with no externalities), then it should be treated as a public good in that sum of MRS should equal MRT (i.e., FOC accounts for all costs/benefits)

**Not So Extreme** - need to distinguish solution to inefficiencies; flowers in your yard is not the same as national defense

**Private Goods**

- **For Person $j$** - $\text{MRS}_{x_j', x_j} = \text{MRT}_{x_j', x_j}$

- **For Person $k$** - $\text{MRS}_{x_k', x_k} = 0$ ... i.e., how much person $k$ is willing to give up (willingness to pay) of private good 2 in order for person $j$ to consume more of private good 1 (nothing!)

**Result** - we can add these two equations together: $\text{MRS}_{x_j', x_j} + \text{MRS}_{x_k', x_k} = \text{MRT}_{x_j', x_j}$

**Public Good** - $\text{MRS}_{x_j', x_j'} = \text{MRS}_{x_k', x_k'}$ ... i.e., doesn't matter who consumes the public good (because everyone consumes the same amount)

**Extremes** - pure private good: $\text{MRS}_{x_j', x_j'} = 0$; pure public good: $\text{MRS}_{x_k', x_k'} = \text{MRS}_{x_k', x_k'}$ ... other goods are between these extremes

**Slutsky's Papers** - focus on 1st and 2nd FTWE with Coasian bargaining

**Paper 1** - "Production Externalities and Long-Run Equilibria: Bargaining and Pigovian Taxation"

**Short Version** - perfect information with free entry; 1st FTWE fails; Coasian bargaining doesn't work because 2nd order conditions fail

**Scenario** - 1 lake; 2 types of firms: fishing & chemical plant; chemicals dumped in lake affect fishing (but fishing doesn’t impact chemical plant)

**Simplifying Assumptions & Notation** - don't really affect results, but simplify the math and make the result more obvious

1. Labor is the only input in both industries
2. Labor is numeraire (i.e., wage rate = 1)
3. \( X = \) total production from chemical plants (polluting industry)
4. \( Y = \) total production from fishing firms (polluted industry)
5. \( F(x) = \) amount of labor needed for individual chemical firm to generate output \( x \)
6. Identical chemical firms \( \therefore X = n_x X \)
7. \( G(y, X) = \) amount of labor needed for individual fishing firm to generate output \( y \); assume for given level of output \( y \), cost increases with \( X \)
   Atmospheric Externality - only total amount matters, not who or where it’s produced
8. Second order conditions... \( F(x) \) strictly convex with \( \lim_{x \to 0} F(x) > 0 \) (i.e., fixed cost of entry)... u-shaped average cost curve so we’ll have limited number of chemical firms
9. SOC for fishing firms... \( G(y, X) \) strictly convex with \( \lim_{y \to 0} G(y, X) > 0 \)
10. Identical consumers with population \( n \)
11. Quasilinear utility - \( u \left( \frac{X}{n_x}, \frac{Y}{n_y} \right) = \frac{L}{n} \) ... linear in labor so we can think of economy consisting of single individual: \( \hat{u}(X, Y) = L \) (standard trick when focusing on production; eliminates income and substitution effects); assume \( \hat{u}(X, Y) \) is strictly concave

Pareto Optimality - get unconstrained optimization by substituting supply of labor =

demand for labor: \( L = n_x F \left( \frac{X}{n_x} \right) + n_y G \left( \frac{Y}{n_y}, X \right) \)

\[
\max_{X,Y,n_x,n_y} W = \hat{u}(X, Y) - n_x F \left( \frac{X}{n_x} \right) - n_y G \left( \frac{Y}{n_y}, X \right)
\]

Assumption - interior solution
Second Order Conditions - \( \hat{u}(X, Y) \) is strictly concave; \( F \) and \( G \) are convex so the objective function is strictly concave.... wrong! no guarantee it’s concave with respect to all four variables; check Hessian is negative definite:

\[
H = \begin{bmatrix}
\frac{\partial^2 W}{\partial X^2} & \frac{\partial^2 W}{\partial X \partial Y} & \frac{\partial^2 W}{\partial X \partial n_x} & \frac{\partial^2 W}{\partial X \partial n_y} \\
\frac{\partial^2 W}{\partial X \partial Y} & \frac{\partial^2 W}{\partial Y^2} & \frac{\partial^2 W}{\partial Y \partial n_x} & \frac{\partial^2 W}{\partial Y \partial n_y} \\
\frac{\partial^2 W}{\partial X \partial n_x} & \frac{\partial^2 W}{\partial Y \partial n_x} & \frac{\partial^2 W}{\partial n_x^2} & \frac{\partial^2 W}{\partial n_x \partial n_y} \\
\frac{\partial^2 W}{\partial X \partial n_y} & \frac{\partial^2 W}{\partial Y \partial n_y} & \frac{\partial^2 W}{\partial n_y^2} & \frac{\partial^2 W}{\partial n_y \partial n_x}
\end{bmatrix}
\]

Need - each diagonal element < 0; determinant of 2x2 minors > 0; determinant of 3x3 minors > 0; determinant of matrix > 0

Assumption - some of these determinants are indeterminate (can be > 0 or < 0), but \( \exists \) some \( \hat{u}(X, Y), F \) and \( G \) such that \( W \) is concave
First Order Conditions - 

\( \frac{\partial W}{\partial X} = \dot{u}_x (X, Y) - F\left( \frac{X}{n_x} \right) - n_x G_X \left( \frac{Y}{n_y}, X \right) = 0 \)

\( \frac{\partial W}{\partial Y} = \dot{u}_y (X, Y) - G_Y \left( \frac{Y}{n_y}, X \right) = 0 \)

\( \frac{\partial W}{\partial n_x} = \frac{X}{n_x} F\left( \frac{X}{n_x} \right) - F\left( \frac{X}{n_x} \right) = xF'(x) - F(x) = 0 \)

\( \frac{\partial W}{\partial n_y} = \frac{Y}{n_y} G_Y \left( \frac{Y}{n_y}, X \right) - G \left( \frac{Y}{n_y}, X \right) = y G_Y (y, X) - G(y, X) = 0 \)

**Note:** for (3) and (4), we assumed \( n_x \) and \( n_y \) are continuous variables (so we could take derivatives), but they're actually integers

From (3): \( xF'(x) = F(x) \Rightarrow F'(x) = \frac{F(x)}{x} \) ... marginal cost of chem = average cost

From (4): \( y G_Y (y, X) = G(y, X) \Rightarrow G_Y (y, X) = \frac{G(y, X)}{y} \) ... MC of fish = AC

From (2): \( \dot{u}_y (X, Y) = G_Y \left( \frac{Y}{n_y}, X \right) \) ... marginal benefit of fish = marginal cost

From (1): \( \dot{u}_x (X, Y) = F\left( \frac{X}{n_x} \right) + n_x G_X \left( \frac{Y}{n_y}, X \right) \) ... MB of chemicals = social MC (the sum of all the costs; similar to Samuelson condition [middle of p.3])

**Competitive Equilibrium** - "without getting into details"

**Chemical Firm** - \( \max_x p_x x - F(x) \) ... FOC: (a) \( p_x = F'(x) \)

**Fishing Firm** - \( \max_y p_y y - G(y, X) \) ... FOC: (b) \( p_y = G_Y (y, X) \)

**Consumers** - \( \max_{X,Y,L} \hat{u}(X,Y) - L \) s.t. \( p_x X + p_y Y = L \) ... embed constraint:

\( \max_{X,Y} \hat{u}(X,Y) - p_x X - p_y Y \) ... FOCs: (c) \( \hat{u}_x = p_x \); (d) \( \hat{u}_y = p_y \)

Combine (b) and (d) to get (2)... \( \hat{u}_y = p_y = G_Y (y, X) \)

Combine (a) and (c)... \( \hat{u}_x = p_x = F'(x) \neq F\left( \frac{X}{n_x} \right) + n_x G_X \left( \frac{Y}{n_y}, X \right) \) (1)... ... the CE is not PO (1st FTWE fails)

**Pigovian Solution** - tax chemical firms: \( \max_x p_x x - F(x) - t_s x \) ... FOC: (a) \( p_x = F'(x) + t_s \);

set \( t_s = n_x G_X \left( \frac{Y}{n_y}, X \right) \) and CE will be PO

**Coasian Solution** - unclear who's involved because of free entry (Coase's paper only dealt with fixed number of participants); instead of bargaining, assume lake is owned by single individual who completely controls the lake (i.e. has property rights); assume no monitoring or enforcement costs for owner
Owners Revenue -

Optimal Fees - takes all profit from both firms: \( I_x = p_x x - F(x) \) and \( I_y = p_y y - G(y, X) \) (want to focus on output per firm)

\[
\max_{n_x, n_y, x, y} R = n_x I_x + n_y I_y = n_x \left( p_x x - F(x) \right) + n_y \left( p_y y - G(y, n_x, x) \right)
\]

FOC - will compare these to PO FOCs on previous page

(i) \[
\frac{\partial R}{\partial n_x} = p_x x - F(x) - n_y x G_x (y, n_x, x) = 0
\]

(ii) \[
\frac{\partial R}{\partial n_y} = p_y y - G(y, n_x, x) = 0
\]

(iii) \[
\frac{\partial R}{\partial x} = n_y \left( p_x - F'(x) \right) - n_y n_x G_x (y, n_x, x) = 0
\]

(iv) \[
\frac{\partial R}{\partial y} = n_y \left( p_y - G_y (y, n_x, x) \right) = 0
\]

Combine (i) and (iii) to get (3)...

\[
p_x - \frac{F(x)}{x} - n_y G_x (y, n_x, x) = 0
\]

\[
\Rightarrow \frac{F(x)}{x} = F'(x) \quad \text{... MC of chemicals = AC of chemicals}
\]

Combine (ii) and (iv) to get (4)...

\[
p_y - \frac{G(y, n_x, x)}{y} = 0
\]

\[
\Rightarrow \frac{G(y, X)}{y} = G_y (y, X)
\]

Combine (iii) with consumer FOC (c) to get (1) \( \hat{u}_x = p_x = F'(x) + n_y G_x (y, X) \)

Combine (iv) with consumer FOC (d) to get (2) \( \hat{u}_y = p_y = G_y (y, X) \)

∴ FOCS for Coasian bargaining (private owner of lake) are same as FOCS for PO solution, but...

SOC - full results in paper, but here’ the basics

\[
\frac{\partial^2 R}{\partial n_x} = 0 \quad \text{... supposed to be < 0 (sufficient), but \leq 0 is necessary... may be a problem}
\]

Check 2x2 minors... determinant must be \( \geq 0 \):

\[
\begin{vmatrix}
\frac{\partial^2 R}{\partial n_x^2} & \frac{\partial^2 R}{\partial n_x \partial n_y} \\
\frac{\partial^2 R}{\partial n_y \partial n_x} & \frac{\partial^2 R}{\partial n_y^2}
\end{vmatrix} = \begin{vmatrix}
\frac{\partial^2 R}{\partial n_x^2} & \frac{\partial^2 R}{\partial n_x \partial n_y} \\
\frac{\partial^2 R}{\partial n_y \partial n_x} & \frac{\partial^2 R}{\partial n_y^2}
\end{vmatrix} \leq 0
\]

(as long as \( \frac{\partial^2 R}{\partial n_x \partial n_y} = -x G_x \neq 0 \))
SOC fail and there's no assumption we can impose on $F$ or $G$ to make SOC hold

**Problem** - free entry results in zero profit for both firms so lake owner's revenue is zero; he can do better by removing a chemical firm which gives positive profits to fishing firms

**Note:** with fixed set of participants, both Coasian and Pigovian approaches work; problem with Coasian approach arises when there is not a fixed number of participants (i.e., doesn't work in long run)

**Paper 2** - "Private Information, Coasian Bargaining, and the Second Welfare Theorem"

**Short Version** - private information with no entry; 2nd FTWE fails; Coasian bargaining restricts government's ability to redistribute

Max social welfare subject to technology and self selection constraints

Private owner limits redistribution government can do. ∴ 2nd FTWE fails (can't get to all PO points with CE)

Even if private owner increases efficiency, will have people hurt who can no longer get government redistribution