Consumer Theory - Random Topics

"Random topics... in no particular sensible order" -Slutsky

Consumer Surplus

Diamond-Water Paradox - by many forms of measurement, diamonds are more valuable than water; paradox because water is more important for life to exist; so how do we measure the value of a commodity since it's not equal to market price?

Price = Marginal Value - prices signify marginal value, the value of a little more: (from first order conditions) $P_W = U_W/\lambda$ and $P_D = U_D/\lambda$ ∴ prices say diamonds are worth more on the margin; that is, a little more water is not as valuable as a little more diamond; prices say nothing about total worth of water or diamonds

Measuring Total Value - want to measure how much people would pay to keep a commodity from being taken away from them; can't really measure in terms of utility (because utility is ordinal and this is asking a cardinal question); one way is to set price really high relative to current price then two options (both are $ amounts):

1. **Compensating Variation (CV)** - how much compensation is needed if price was high to maintain same utility of low price

   Use expenditure function; fix utility level and change prices; says how much income needs to change

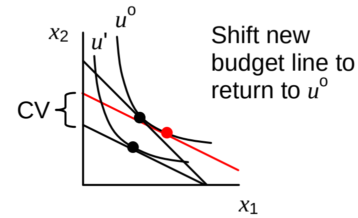
   $V(\mathbf{P}^o, I) \equiv \max U(\mathbf{x})$ st $\mathbf{P}^o \cdot \mathbf{x} \le I \;\Rightarrow\; u^o = V(\mathbf{P}^o, I)$

   Let $\mathbf{P}' = (P_1^o, P_1^o, \ldots, P_{n-1}^o, P_n')$, where $P_n' > P_n^o$ (only 1 price is higher)

   $E(\mathbf{P}', u^o)$ = how much money consumer needs to get to old level of utility with the new (higher) price

   $CV \equiv E(\mathbf{P}', u^o) - I = E(\mathbf{P}', u^o) - E(\mathbf{P}^o, u^o)$

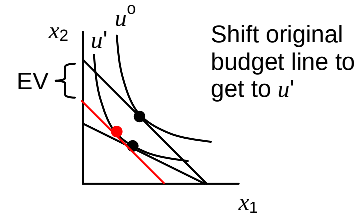
   Graphically - shift new budget line to reach original indifference curve



2. **Equivalent Variation (EV)** - how much consumer is willing to pay to keep price low

   $EV \equiv I - E(\mathbf{P}^o, u') = E(\mathbf{P}', u') - E(\mathbf{P}^o, u')$

   Graphically - shift original budget line to reach new indifference curve

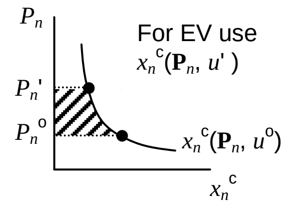


CV vs. EV - answer different questions; relationship depends on whether goods or normal (superior) or inferior; both look at ΔE at fixed utility level with prices changing from $\mathbf{P}^o$ to $\mathbf{P}'$ (CV uses original [higher] utility, uo, and EV uses new [lower] utility, u')

Fundamental Theorem of Integral Calculus -

$$\int_a^b f'(x)dx = f(b) - f(a)$$ = area under the curve from a to b

Note: our graphs are inverted (independent variable on vertical access) so we're looking at area to the left of the curve between a and b



$$CV = \int_{P_n^o}^{P_n'} \frac{\partial E(\mathbf{P}_n, u^o)}{\partial P_n} dP_n = \int_{P_n^o}^{P_n'} x_n^c(\mathbf{P}_n, u^o) dP_n$$

$$EV = \int_{P_n^o}^{P_n'} \frac{\partial E(\mathbf{P}_n, u')}{\partial P_n} dP_n = \int_{P_n^o}^{P_n'} x_n^c(\mathbf{P}_n, u') dP_n$$

How does x_n^c change wrt to utility?... same as x_n^o wrt to I (look at income consumption curve [x_n^o] or utility consumption curve [x_n^c]); three cases:

Normal Good - superior good; EV < CV

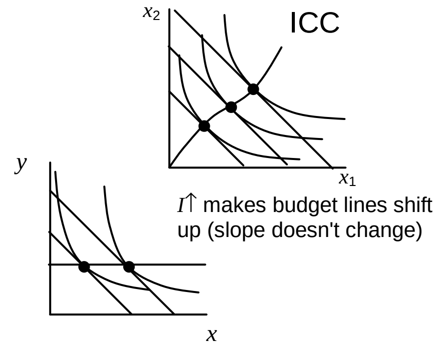
Inferior Good - EV > CV

Neutral - EV = CV is x_n^o doesn't vary with I

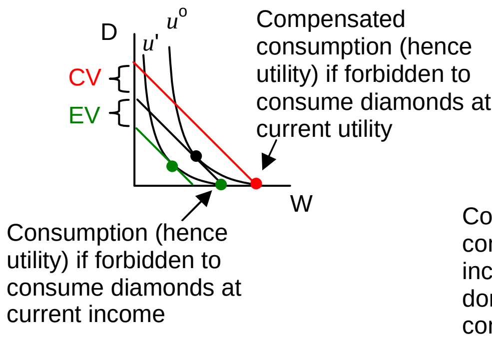
Example - quasilinear utility $u = kx + f(y)$ (from Midterm)

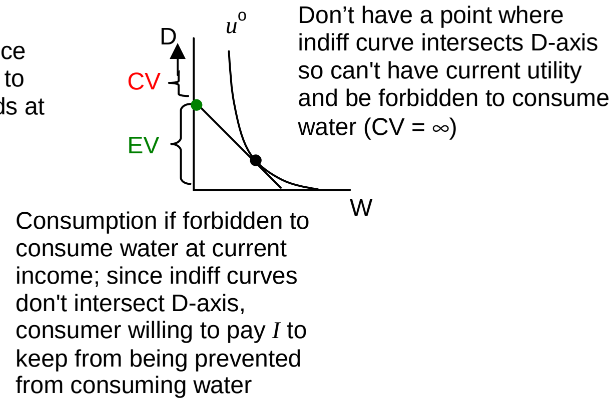
$f(y)$ must be concave for 2nd order condition to hold

indifference curves are horizontal shifts (y doesn't change with income)



Diamond-Water Revisited - diamonds cost more, but $EV_W > EV_D$ and $CV_W > CV_D$





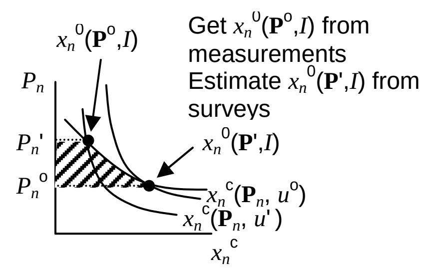
Consumer Surplus - worth above what individual pays for it; EV or CV (usually use EV, but depends on problem being considered); examples:

Government Projects - proposal to build expressway connecting Tampa and Jacksonville; how does government figure out if it's worth it? cost-benefit analysis... problem with uncertainties in costs and benefits since both occur over time, but what we're concerned with right now is estimating the benefit (travel between Jax and Tampa at cheaper price)

Accountants Answer - # trips x time saved x $/hr

Economist Answer - accountant is wrong because assuming number of trips is constant; economist would predict number of trip ↑ because price ↓; should use EV because we're interested in how much consumers are willing to pay

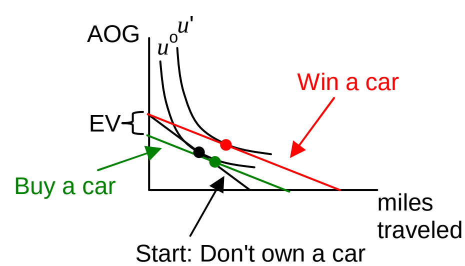
Problem - can't get $\mathbf{x}^c$; usually can only get $\mathbf{x}^o$ at certain points; applied economists usually consider area using $\mathbf{x}^o$ consumer surplus (CS), but it adjusts for income effect; assuming normal good EV < CS < CV



Distributional Effects - also need to consider distributional effects; who benefits more? How do we compare $1 benefit to rich person vs. poor person?

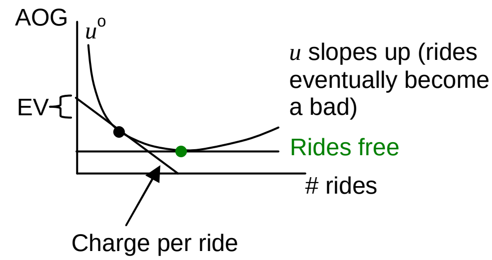
Welfare Analysis of Monopoly - again economists use CS instead of EV or CV

Durables - consume over number of periods (e.g., cars, houses, washing machines); don't actually consume good, but services of the good (e.g., transportation, shelter, washing clothes); these goods don't really fit out model until we consider buying a washer equivalent to gaining ability to



purchase service at lower price; what's a washer worth? EV (maximum consumer willing to pay for the lower price of service)

Disney World - used to pay per ride; now Disney rides are monetarily free (still stand in line), but pay large entry fee; based on graph Disney figured most they could charge with free rides to keep consumers on same indifference curve; notice that at new point, there's less spent on AOG (all other goods); that's a gain for Disney; harder to figure out in reality because people's preferences are different



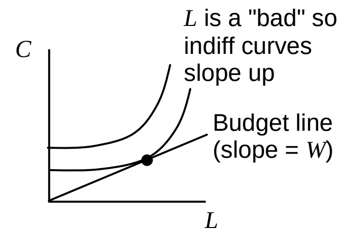
Price Discrimination - used by firms to capture more consumer surplus

Income Not Given

Before, we assumed income was exogenously given, but in real world, people choose income based on how much they work (usually as function of price of labor)

Labor-Leisure Problem - utility is a function of consumption (C) and labor (L);

Bad - labor is a "bad" (utility goes down as you have more labor)



Original Formulation - Max $U(C,L)$ st $P{\cdot}C \le W{\cdot}L$

W = wages (P of labor)

Problem - doesn't look like "standard" problem because indifference curves and budget line slope upwards

Standardize Formulation -

Objective - need to get it downward sloping so use $L = T - R$ (time available minus leisure time); now this is upward sloping with respect to R; we can write it as $\hat{U}(C,R)$

Constraint - again use $L = T - R$; constraint becomes $P{\cdot}C \le W{\cdot}(T - R) \Rightarrow P{\cdot}C + W{\cdot}R \le W{\cdot}T$; so now we see that W is the price of leisure (we "buy " leisure with forgone wages)

Endowment Income - $I(W) = W{\cdot}T$; original income $W{\cdot}L$ is income according to the IRS; endowment income is income to economist (if you worked full time); endowment income is also exogenous and closer to I we used before

Other Changes -

Homogeneity - don't really need to say it's homogenous in I because the $W{\cdot}T$ guarantees it

Ordinary Demand - two options $\hat{C}^o(W,P)$ and $\hat{R}^o(W,P)$ or $C^o(W, P, I(W))$ and $R^o(W, P, I(W))$; the latter is more "standard" because it shows prices (W, P) and income ($I(W)$)

Change in Leisure - look at change in R^o wrt W (based on comparative static's)

$$-\frac{\partial L}{\partial W} = \frac{\partial \hat{R}}{\partial W} = \frac{\partial R^o(W,P,I(W))}{\partial W} = \frac{\partial \tilde{R}^o}{\partial W} + \frac{\partial R}{\partial I}\cdot\frac{\partial I}{\partial W}$$

Note 1: effect of W only on prices is "old" Slutsky equation: $\frac{\partial \tilde{R}^o}{\partial W} = \frac{\partial R^c}{\partial W} - R\frac{\partial R}{\partial I}$

Note 2: R^c is compensated leisure (change income to account for new wage in order to stay at same level of utility; i.e., $W\uparrow \Rightarrow T\downarrow$)

Note 3: $\partial I/\partial W = \partial(T{\cdot}W)/\partial W = T$

$$\frac{\partial R^o}{\partial W} = \frac{\partial R^o}{\partial W} - R\frac{\partial R}{\partial I} + T\frac{\partial R}{\partial I} = \frac{\partial R^o}{\partial W} + (T-R)\frac{\partial R}{\partial I} = \underbrace{\frac{\partial R^o}{\partial W}}_{\text{Substitution Effect}} + \underbrace{L\frac{\partial R}{\partial I}}_{\text{Income Effect}}$$ (modified Slutsky Eqn)

NOTE: sign of income effect for labor is opposite from what we had in Slutsky equation before; consumer owns his own labor so P_L↑ (i.e., W↑) means labor is worth more and consumer is better off

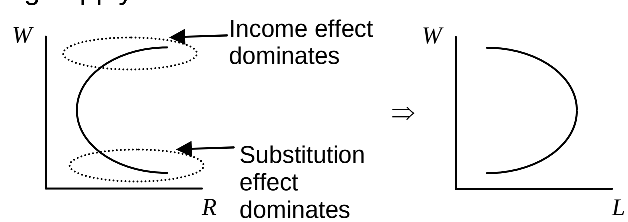
Substitution Effect - $\frac{\partial R^{\text{o}}}{\partial W} < 0$

Income Effect - $L\frac{\partial R}{\partial I} > 0$

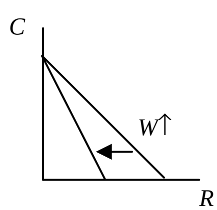
Technically this is ambiguous; > 0 if R is normal good; < 0 if R is inferior good; empirical evidence for R being normal good is "overwhelming"

Interpretation - income and substitution effects are in different directions for change in R (and L) wrt W is ambiguous; in general if L is small, substitution effect dominates so W↑ ⇒ R↓ (and L↑; work more); if L is large, income effect dominates so W↑ ⇒ R↑ (and L↓; work less); leads to backward bending supply curve for labor

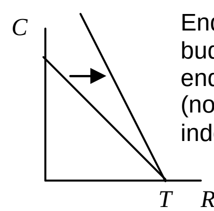
Hard to Observe - could cut L by retiring early rather than changing work week; brings up problems with tax cut... revenue based on whether people will work more or less (which we can't observe)



Modify Constraint (again) - $P{\cdot}C + W{\cdot}R \le W{\cdot}T$; can multiply P by anything because of homogeneity; use 1/W: $(P/W){\cdot}C + R \le T$ ∴ ΔW acts like a cross effect (W↑ is equivalent to P↓... makes budget line steeper, but note the pivot point is different because of endowment effect... explained more in general case)



New budget lines have same slope, but better off with endowment



Endowment Effect budget line pivots on endowment point (note that T is independent of W)

General Case - $\mathbf{P}{\cdot}\mathbf{x} \le \mathbf{P}{\cdot}\boldsymbol{\omega}$;

Endowment Vector (ω) - (omega) is endowment vector (capital goods, land, etc. owned by consumer)

Generalized Slutsky Equation - follow similar argument as with labor-leisure example

$$\hat{\mathbf{x}}^{\text{o}}(\mathbf{P}) \equiv \mathbf{x}^{\text{o}}(\mathbf{P}, I(\mathbf{P})) \text{ and } I(\mathbf{P}) = \mathbf{P}\cdot\boldsymbol{\omega} \Rightarrow \frac{\partial I(\mathbf{P})}{\partial P_i} = \omega_i$$

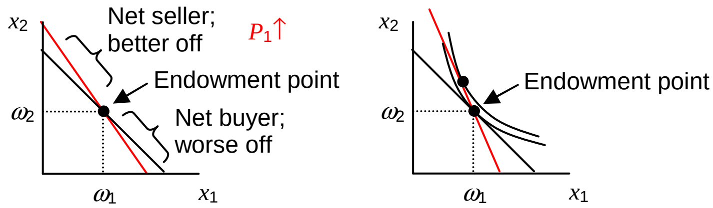
$$\frac{\partial x_j^{\text{o}}}{\partial P_i} = \frac{\partial x_j^{\text{c}}}{\partial P_i} - x_i^{\text{o}}\frac{\partial x_j^{\text{o}}}{\partial I} + \frac{\partial x_j^{\text{o}}}{\partial I}\cdot\frac{\partial I}{\partial P_i} = \frac{\partial x_j^{\text{c}}}{\partial P_i} - x_i^{\text{o}}\frac{\partial x_j^{\text{o}}}{\partial I} + \omega_i\frac{\partial x_j^{\text{o}}}{\partial I} = \boxed{\frac{\partial x_j^{\text{c}}}{\partial P_i} + (\omega_i - x_i^{\text{o}})\frac{\partial x_j^{\text{o}}}{\partial I}}$$

Interpretation - both terms are ambiguous

Substitution Effect - if own effect ($i = j$) it's < 0; if cross effect ($i \ne j$) it's > 0 (Hixian substitutes or < 0 (Hixian compliments); (Recall if there are only two goods, they have to be substitutes)

Income Effect - depends on whether consumer is net buyer ($\omega_I < x_i^{\text{o}}$) or net seller ($\omega_I > x_i^{\text{o}}$) and depends on whether x_j^{o} is normal or inferior good

Endowment Effect - price change rotates budget line on the endowment point; can always consume at endowment point, even if prices change so consumer will only change consumption point if he can do better ($U\uparrow$); in this case, any change will be purely a substitution effect



Revealed Preference

Samuelson - came up with revealed preference; <u>assumed</u> preferences strictly convex so ordinary demand is single point (i.e., choice is strictly better than anything else available); also <u>assumed</u> preferences are constant (only way to draw conclusions from revealed preference)

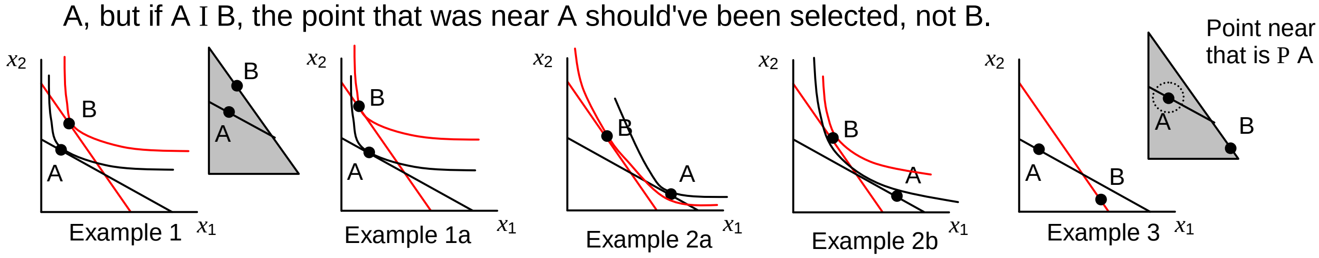
Intuitive View - looking at choices for consumer given a budget set ($B(\mathbf{P},I)$); we don't know preferences or even functional for of demands, just observe several points and try to determine if consumer is rational

Rational - should be able to draw two indifference curves that satisfy rules for rational consumer (1 tangent to each consumption point); this doesn't imply the consumer is rational, just that the choices he made are rational

Example 1 - rational; know B is on higher indifference curve than A because B is chosen when A is available (could also use monotonicity of preferences because B has more of both, but that argument doesn't work in example 1a [graph on right])

Example 2 - rational, but don't know whether B is on higher or lower indifference curve than A because A is not available when B is chosen and B is not available when A is chosen; 2a shows A better than B and 2b shows B better than A; both have valid indifference curves

Example 3 - not rational; can't draw tangent indifference curves without having them intersect; notice that B is chosen when A is available so B RP A; ditto for A (A RP B); can't have both unless A I B which violates local nonsatiation and transitivity; by local nonsatiation ∃ point near A that is P A; this point is available when B is selected over A, but if A I B, the point that was near A should've been selected, not B.



Revealed Preferred (RP) - since we can't observe preferences, all we see is a revealed preference of one bundle over another given a certain budget set that makes both bundles feasible; we write this as A RP B (A is revealed preferred to B)

Weak Axiom of Revealed Preference (WARP) - proposed by Samuelson who argued this is equivalent to preferences satisfying standard properties; can't be done (need SARP); 3 ways to look at it:

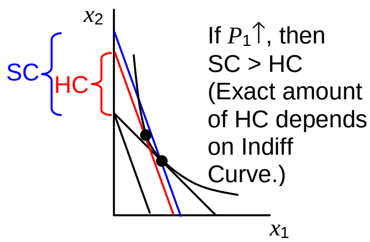
1. If $\mathbf{x}$ RP $\mathbf{y}$ under some budget set, then ∃ no budget set under which $\mathbf{y}$ RP $\mathbf{x}$
2. if $\mathbf{x}$ RP $\mathbf{y}$ under some budget set, and $\mathbf{y}$ is chosen under a second budget set, then $\mathbf{x} \notin$ new budget set (i.e., $\mathbf{x}$ costs more than $\mathbf{y}$ in second budget set)

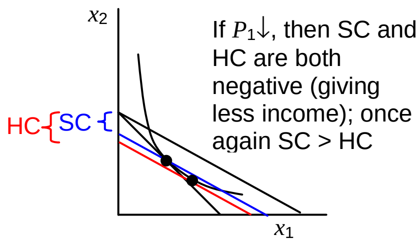
3. If $\mathbf{x}$ is chosen under prices $\mathbf{P}'$ and $\mathbf{P}'\cdot\mathbf{x} \geq \mathbf{P}'\cdot\mathbf{y}$ (i.e., $\mathbf{y}$ is in the budget set) and if $\mathbf{y}$ is chosen under prices $\mathbf{P}''$, then $\mathbf{P}''\cdot\mathbf{y} < \mathbf{P}''\cdot\mathbf{x}$

Slutsky Compensation (SC) - when prices change, give individual more (or less) income so consumer has ability to purchase original bundle (i.e., new budget line and old budget line will intersect at original bundle); **Note:** implicitly assuming ratio of prices is different so budget lines don't have same slope

Compared to Hixian Compensation - SC doesn't say consumer will stick with the old bundle; in fact, consumer will be at least as well off under SC because he can purchase the original bundle (i.e., stay just as happy); the only reason to not purchase the original bundle would be if he's increasing utility beyond where he started; Hixian compensation only adds (subtracts) income to the point tangent to the original indifference curve; the reason we need SC is that we don't know what the indifference curve looks like

Note: Hixian compensation will always be less than Slutsky compensation; the only time they are equal is if there is a kink or corner in the indifference curve such as for perfect complements





Using WARP to prove own substitution effect < 0 (previously proved this with comparative statics and again with expenditure function)

Assume $\mathbf{x}$ is chosen at prices $\mathbf{P}'$

At new prices $\mathbf{P}''$ we adjust income so $\mathbf{x}$ lies on new budget line (Slutsky compensation)

Assume new choice at prices $\mathbf{P}''$ as $\mathbf{y}$ $\therefore$ $\mathbf{P}''\cdot\mathbf{y} = \mathbf{P}''\cdot\mathbf{x}$

Since $\mathbf{y}$ chosen when $\mathbf{x}$ is available we know $\mathbf{y}$ RP $\mathbf{x}$

By using 2nd version of WARP, $\mathbf{P}'\cdot\mathbf{y} > \mathbf{P}'\cdot\mathbf{x}$ (i.e., $\mathbf{y}$ wasn't affordable when $\mathbf{x}$ was chosen)

We have two inequalities; multiply second one by -1: $-\mathbf{P}'\cdot\mathbf{y} < -\mathbf{P}'\cdot\mathbf{x}$

Add this to original equation: $(\mathbf{P}'' - \mathbf{P}')\cdot\mathbf{y} < (\mathbf{P}'' - \mathbf{P}')\cdot\mathbf{x}$

Move everything to left side: $(\mathbf{P}'' - \mathbf{P}')\cdot(\mathbf{y} - \mathbf{x}) < 0$

That is, a vector of price changes times a vector of quantity changes: $\Delta\mathbf{P}\cdot\Delta\mathbf{x} < 0$

Assume $\Delta P_k = 0\ \forall\ k \neq j$ and $\Delta P_j \neq 0$ (i.e., only price of good j changed)

$\therefore\ \Delta\mathbf{P}\cdot\Delta\mathbf{x} = \sum_{i=1}^{n} \Delta P_i \cdot \Delta x_i = \Delta P_j \cdot \Delta x_j < 0$

Since $\Delta\mathbf{P}$j $\neq 0$, we know $(\Delta P_j)^2 > 0$ so we can divide by that without changing the inequality

$\Delta P_j \cdot \Delta x_j < 0 \Rightarrow \dfrac{\Delta P_j \cdot \Delta x_j}{(\Delta P_j)^2} < 0 \Rightarrow \dfrac{\Delta x_j}{\Delta P_j} < 0$

$\therefore$ own substitution effect with Slutsky compensation is negative; **Note:** this is good for a discrete change (any amount), not just an infinitesimal amount (which comes from using derivatives)

Because $\dfrac{\Delta x_j}{\Delta P_j} < 0$, $\lim\limits_{\Delta P_j \to 0} \dfrac{\Delta x_j}{\Delta P_j} = \dfrac{dx_j}{dP_j} \leq 0$

Tying SC to HC - define Slutsky compensated demand as $x_j^{SC}(\mathbf{P}, \mathbf{x}^0)$; use $\mathbf{x}^0$ instead of I because we set I such that $I = \mathbf{P}\cdot\mathbf{x}^0$

Given work done above, we know $\frac{\partial x_j^{SC}}{\partial P_j} \le 0$

Now look at Hixian compensated demand $x_j^C(\mathbf{P}, I)$; we know at $\mathbf{P}^0$ that

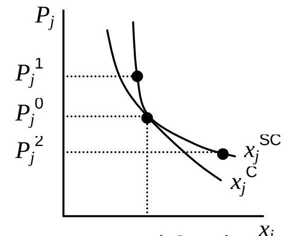
$$x_j^{SC}(\mathbf{P}, \mathbf{x}^0) = x_j^C(\mathbf{P}, I)$$

Assume x_j is normal (superior) good; if we raise P_j, we saw earlier that SC > HC, ∴

$x_j^{SC}(\mathbf{P}, \mathbf{x}^0) > x_j^C(\mathbf{P}, I)$; if we lower P_j, we saw earlier that SC > HC, ∴

$x_j^{SC}(\mathbf{P}, \mathbf{x}^0) > x_j^C(\mathbf{P}, I)$; that means the x_j^{SC} is always above x_j^C and it's tangent at $\mathbf{P}^0$ (i.e., slopes are the same)

$$\therefore \frac{\partial x_j^C(\mathbf{P}^0)}{\partial P_j} = \frac{\partial x_j^{SC}(\mathbf{P}^0)}{\partial P_j} \le 0$$



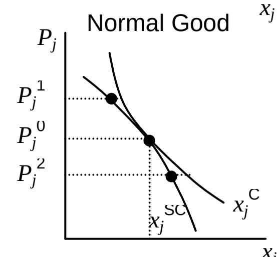
Normal Good

Assume x_j is an inferior good; if we raise P_j, we saw earlier that SC > HC,

∴ $x_j^{SC}(\mathbf{P}, \mathbf{x}^0) < x_j^C(\mathbf{P}, I)$; if we lower P_j, we saw earlier that SC > HC,

∴ $x_j^{SC}(\mathbf{P}, \mathbf{x}^0) < x_j^C(\mathbf{P}, I)$; that means the x_j^{SC} is always below x_j^C and it's tangent at $\mathbf{P}^0$ (i.e., slopes are the same)

$$\therefore \frac{\partial x_j^C(\mathbf{P}^0)}{\partial P_j} = \frac{\partial x_j^{SC}(\mathbf{P}^0)}{\partial P_j} \le 0$$



Inferior Good

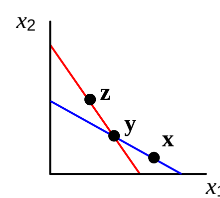
∴ at the limit (small ΔP), Slutsky compensated and Hixian compensated demand curves are the same and both have negative own substitution effect

Ordinary Demand - related to $\mathbf{x}^c$ by Slutsky equation; intersects at $\mathbf{P}^0$, but not tangent

Strong Axiom of Revealed Preference (SARP) - proposed by Houthakker; assumed transitivity holds which allows us to find all properties of preferences

Without Transitivity - can only do pair wise comparisons with WARP; e.g. **x** RP **y** and **y** RP **z** says nothing about **x** and **z** (at most can only figure that **z** costs more than **x** when **x** is chosen <u>and</u> **x** costs more than **z** when **z** is chosen)

Transitivity - can conclude that **x** $\overline{\text{RP}}$ **z**; **z** is indirectly revealed preferred to **z**

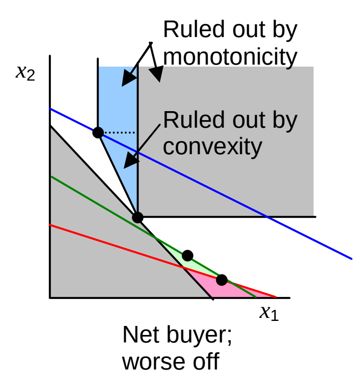


Using Revealed Preference - collect data and try to find indifference curves; arguably with infinite number of revealed bundled, we can get to indifference curve (it has to be in the area we don't shade in)

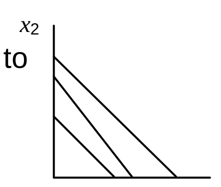
Completes Circle - if we know $\mathbf{x}^o(\mathbf{P}, I)$, we know revealed preference and can get to indifference curves by substituting infinite number of combinations of $\mathbf{P}$ and I (not practical but possible)

Finite Set of Data - fit demand curve to observed data and check properties of these curves; usually <u>do not</u> satisfy all properties... so either people are irrational or we have a problem:

1. **Aggregation** - not data for single individual
2. **Imposed Functional Form** - may not be correct form
3. **Preferences Change** - if they change while data is collected, the data can't be used to derive a single indifference curve



Testing Revealed Preferences - usually can't because have I↑; never allows possibility to violate revealed preference theory if budget lines don't cross



Aggregation

Aggregation - sum demands across different individuals

Results - AD will be homogeneous of degree 0 wrt **P** and I and will satisfy adding up property, but <u>does not</u> have to satisfy other properties

Generalized Slutsky Equation for Individual *k* - $\frac{\partial x_j^k}{\partial P_i} = \frac{\partial x_j^{ck}}{\partial P_i} + (\omega_i^k - x_i^{ok})\frac{\partial x_j^{ok}}{\partial I}$

Substitution Effect - if $i = j$, $\partial xj/\partial Pi < 0$ (own substitution effect); otherwise, term is uncertain (can be < or > 0)

Income Effect - if **P** & I unrelated, $\omega_i = 0$ so $(\omega_i - x_i) < 0$; in labor leisure problem $x_i = 0$ so $(\omega_i - x_i) > 0$; in general term can be < or > 0 based on consumer being net buyer or net seller of commodity x_i; also $\partial x_j/\partial I$ can be < or > 0 depending on good being normal or inferior; this income effect is what makes consumer theory so difficult... the sign is uncertain (in multiple ways)

Adding Generalized Slutsky Eqn - $\sum_{k=1}^{n}\frac{\partial x_j^k}{\partial P_i} = \sum_{k=1}^{n}\frac{\partial x_j^{ck}}{\partial P_i} + \sum_{k=1}^{n}(\omega_i^k - x_i^{ok})\frac{\partial x_j^{ok}}{\partial I}$

Aggregate Income Effect - $\sum_k(\omega_i^k - x_i^{ok}) = 0$ for closed economy because everything sold belonged to someone (supply = demand)

Same $\partial x_j/\partial I$ - if we assume $\partial x_j/\partial I$ is same for all individuals, we can factor it out and income effect goes away

Correlated $\partial x_j/\partial I$ - if we assume $\partial x_j/\partial I$ is positively correlated to $(\omega_i - x_i)$; this means high $\partial x_j/\partial I \Rightarrow$ seller $[\omega_i - x_i > 0]$) & income effect > 0; low $\partial x_j/\partial I \Rightarrow$ buyer $[\omega_i - x_i > 0]$) & income effect < 0; if we have enough different consumers, this will be the case so income effect is uncertain

Additional Assumptions

Make additional assumptions about preferences (or other method for describing consumer) and check of effect on other methods for describing consumer

Utility Separability - consider some goods more closely related to others (e.g., orange juice and grapefruit juice); allows consumer to simplify budget decision by allocated money to different categories of goods independent of each other

Additivity - simplest version of separability; $U(x_1,\ldots,x_n) = \sum_{i=1}^{n}u^i(x_i)$

Homothetic Demand - $\mathbf{x}\ \mathrm{R}\ \mathbf{y} \Rightarrow (a\mathbf{x})\ \mathrm{R}\ (a\mathbf{y})$ for all $a > 0$; if we have two bundles (**x** and **x**') that are indifferent and double quantity of both bundles, the new bundles will also be indifferent to each other (on same indifference curve) ∴ doubling income results in doubling consumption of all goods

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Euler's Theorem - if F is <u>homogeneous of degree t</u> (i.e., $F(a\mathbf{x}) = a^tF(\mathbf{x})$), we can take the total derivative with respect to a and evaluate it at $a = 1$

$\sum_{j=1}^{n}x_j\frac{\partial F}{\partial x_j} = tF(x)$ (left-hand side is chain rule)