Consumer Theory - Preferences & Choice Functions

What's First - arguments for and against presenting consumer theory first or production first

Production - easier; clearer results
Consumer - more fundamental; production relies on consumer; underlies welfare economics

Consumer Theory - look at how to describe consumer choices; many different ways of looking at same consumer; no substantive difference between different ways, but each exists to make results easier to get to mathematically

Assumptions - each method to describe consumers has set of assumptions or properties that makes them equivalent to other methods

Example - preferences and utilities aren't observable, but are used in theoretical models; translate properties from these models into demand which can be observed; collect data on demand to verify these properties on preferences and utilities

Mathematical Economists - eliminate assumptions to see what can still be concluded; not very useful and very technical; won't do in this course

Applied Economics - study implications of additional assumptions; problems arise when new assumptions seem reasonable to one method, but then give nonsensical result in another method

Methods - we'll study these in detail:

- Preferences
- Indifference curves
- Utility functions
- Demands
- Revealed preferences
- Expenditure functions
- Indirect Utility function
- Compensated demand

Preferences

Common Usage - "I prefer apples to oranges"; worthless for economics; says nothing about quantity (e.g., 1 apple vs. any number of oranges?) or circumstance (e.g., other goods available)

Technical Definition - defined only between commodity bundles (vectors)

Weak Preference Ordering ($x \, R \, y$) - $x$ "at least as good as" $y$; also write $x \succeq y$ or $x \succeq y$

$x \, R \, y \iff x \, P \, y$ or $x \, I \, y$

Strict Preference Ordering ($x \, P \, y$) - $x$ is preferred to $y$; also write $x > y$ or $x \succ y$

$x \, P \, y \iff x \, R \, y$ and $\neg(y \, R \, x)$

Indifference Ordering ($x \, I \, y$) - indifferent between $x$ and $y$; also write $x \approx y$

$x \, I \, y \iff x \, R \, y$ and $y \, R \, x$

Commodity Bundles - vector listing of amounts of everything you consume (e.g., all else equal (1 apple, 0 orange) preferred to (0 apple, 1 orange))

Time Dated - preferences change over time too

Average Consumption - avoid time dated problem by looking at average for some period of time (e.g., # apples/week)

Rationality Postulates - consumer who's preferences are complete and transitive; guarantees individual will always be able to make a choice

Rational - technical definition: individual can always make a choice; common usage: sensible... these definitions aren't the same!

Economist Preferences - if you prefer $x$ to $y$, you'd choose $x$ over $y
Philosophers - argue choices ≠ preferences; causes problems because you can't measure preferences

Probabilistic Choice - because of unobservables, sometimes you'll pick x and sometimes you'll pick y; prefer x to y means Pr[choose x over y] > 0.5; stronger preference means greater probability of choosing x over y

5 Assumptions - complete, transitive, continuous, monotonic, & convex
1. **Complete** - given a pair of bundles, individual can make a choice; consumer never says I can't decide; only says one of three things: (i) I prefer A, (ii) I prefer B, (iii) I'm indifferent (don't care); 2 equivalent definitions:
   (a) x R y or y R x; at least one of these two must hold
   (b) x P y or y P x or x I y

Minor Decisions - assumption seems reasonable for minor decisions

Incomplete Preferences - bundles can't be compared because of lack of experience (e.g., job in North Dakota vs. job in Florida for someone who's never seen snow)

2. **Transitivity** - given larger groups of bundles (> 2), a choice is possible
   \[ \forall x, y, z, \ x R y \text{ and } y R z \Rightarrow x R z \]

Violation - if you have prerequisite and conclusion doesn't hold; if you don't have prerequisite, transitivity is vacuously true; Example: x P y, y P z, z P x; given any two of these preferences, individual can make a choice, but given all there, he can't (it's circular)

Weaker Assumptions -

2a. **Quasitransitivity** - transitivity of strict preferences; x P y and y P z ⇒ x P z

Theorem - 2 ⇒ 2a, but 2a \( \not\Rightarrow \) 2

Proof: Consider x I y, y I z, and x P z
   Can rewrite x I y and y I z as y R x and z R y
   Transitivity would suggest z R x, but that's not the case
   This is quasitransitive (vacuously true because it doesn't satisfy the prerequisite)
   \[ \therefore \ 2a \not\Rightarrow \ 2 \]

Consider x P y and y P z and assume transitivity
Assume this violates quasitransitivity (i.e., x P z not true so z R x)
Can rewrite x P y and y P z as x R y and y R z
Now have z R x, x R y, and y R z which violates transitivity
   \[ \therefore \ 2 \Rightarrow 2a \]

2b. **Acyclicity** - not cycling; no patterns; x P y and y P z ⇒ x R z (ruling out z P x)

Theorem - 2a ⇒ 2b, but 2b \( \not\Rightarrow \) 2a

Proof: x P y, y P z and x I z is 2b, but not 2a (not completed in class)

Why Have Weaker Assumptions?

Psychology - there are circumstances where small, insignificant differences pile up and become significant; e.g., rooms with temperatures varying by 0.1 degree; individual would be indifferent between any two adjacent rooms, but may prefer one room over all others; this violates transitivity, but doesn't violate quasitransitivity or acyclicity

Transitivity Realistic?

Spouse Experiment - asked college students to rate potential spouses from list of three based on attractiveness, intelligence, and wealth; only ratings were + (very good), 0 (average), - (very bad); students

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asked to choose between pairs: I vs. II, II vs. III, and III vs. I; roughly 1/3 of the students choose I P II, II P III, and III P I (violates transitivity)

**Why Inconsistent** - picking best 2 of 3 attributes (different dimensions); don’t have quantitative data (e.g., how much wealthier?)

**When Transitive** - transitivity OK to assume if bundles are sufficiently different and there’s quantitative data

**Theorem (Transitivity Builds)** - If \( R \) is transitive or quasi-transitive over every triple of alternatives taken from feasible set \( A \), then \( R \) is transitive over the entire set \( A \); i.e., transitivity for small sets builds to larger sets

**Example** - given \( x R y, y R z, \) and \( z R w \), use transitivity with triples to say \( x R z \), then again to say \( x R w \); just used transitivity of triples to show transitivity of all four

**NOT for Acyclicity** - doesn’t build to larger sets; have to check them all

**Example** - given \( y P x, z P y, w P z, x P w, x I z, y I w \) (see picture)

- take any three bundles and acyclicity is satisfied (e.g., \( x, y, w \))
- \( y P x \) and \( x P w \) \( \Rightarrow \) \( y I w \) (or \( y R w \))... unique choice is \( y \); if you expand to all four bundles there is a cycle

**Theorem (Rationality 1)** - \( C(A,R) \) is nonempty for any finite feasible set \( A \) if and only if \( R \) is complete and acyclic; (transitive and quasi-transitive are sufficient, but not necessary)

**Proof:**

Assume \( \exists \) pair of alternatives \( x, y \) with \( \neg(x R y) \) and \( \neg(y R x) \) (i.e., not complete) or there is a set \( \{x, y\} \) of 2 or more alternatives on which \( R \) has a cycle

\[ C(\{x, y\}, R) = \emptyset \] and \( C(\{x, y\}, \alpha, R) = \emptyset \) (This proves \( \neg B \Rightarrow \neg A \)... see aside below)

Assume \( C(A,R) = \emptyset \) for some set \( A \)

If \( A = \{x, y\} \), then \( \neg(x R y) \) and \( \neg(y R x) \)... if this wasn’t the case \( C(A,R) \neq \emptyset \)

If \( A \) has more than 2 alternatives, a cycle must exists

- e.g., \( A = \{x, y, z\} \); if \( R \) not complete, obviously \( C = \emptyset \) so assume \( R \) is complete
- Consider \( x \)... it’s not in choice set \( :\) \( \neg(x R y) \) and \( \neg(y R z) \)... that means \( y P x \)
- or \( (z P x) \)... assume the first one
- Consider \( y \)... it’s not in choice set \( :\) \( z P y \)
- Consider \( z \)... it’s not in choice set \( :\) \( \neg(z R x) \)... that means \( x P z \)
- This forms a cycle: \( x P z, z P y, \) and \( y P x \)
- \( :\) either \( R \) is not complete or there is a cycle (This proves \( \neg A \Rightarrow \neg B \))

**Practical** - although all we need is acyclicity, we’ll work with transitivity instead for two reasons: (1) it’s easier to test, (2) also guarantees \( C(A,R) \neq \emptyset \) for all \( A \) with imposed structure like a budget set (not just finite \( A \))

3. **\( R \) is Continuous** - mostly technical with little economic value

\[ R^*=\{y: y R x\} \]... set of all bundles at least as good as \( x \)

\[ R^=\{y: x R y\} \]... set of all bundles that \( x \) is at least as good as

**Definition 1** - \( \forall x R^* (x) \) and \( R^* (x) \) are closed sets

**Definition 2** - \( \forall x R^= (x) \) and \( R^= (x) \) are open sets

**Indifference Curves** - indifference curve is boundary between \( R^* (x) \) and \( R^= (x) \); continuous \( R \Rightarrow \) indifference curve is well behaved continuous function (i.e., no gaps)
Lexicographic Preferences - preferences similar to dictionary (alphabetical order) were there are no tradeoffs (so azzzz always comes before baaaa); extreme example of preferences that aren't continuous

Example - \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \) (this generalizes to \( n \) dimensions)

\[ x \mathcal{P} y \iff x_1 > y_1 \quad \text{or} \quad (x_1 = y_1 \quad \text{and} \quad x_2 > y_2) \]

\( x \mathcal{P} y \) (i.e., \( \mathbb{R}^2 \)) is shaded area plus line and dot

\( y \mathcal{P} x \) (i.e., \( \mathbb{R}^2 \)) is white area plus dotted line and dot

Can have series all in \( \mathbb{R}^2 \), but limit is in \( \mathbb{R}^\leq \). \( R \) not continuous

Examples - single issue voter; Fear Factor where contestant will not eat worms for any price (no tradeoffs); "Rather push a Chevy than drive a Ford" bumper sticker

Economic Interpretation - \( R \) is continuous means small changes in individual bundles will have a small effect on the individual's well being; this is not the case with lexicographic preferences

Theorem (Utility Representation 1) - if \( R \) is complete, transitive, and continuous, then \( \exists \) a continuous utility representation... very hard to prove

Theorem (Rationality 2) - \( C(A,R) \neq \emptyset \) for any compact set \( A \) if \( R \) is complete, transitive and continuous

Proof:

\[ C(A,R) = \{ x \in A : \forall y \in A \} = \{ x \in A : U(x) \geq U(y) \ \forall y \in A \} = \{ x \in A : x \text{ solves max } U(y) \text{ over } y \in A \} \]

which is guaranteed to have a solution if \( U \) is continuous and \( A \) is compact

4. Listed in order from weakest to strongest (opposite of property 2); \( d \Rightarrow c \Rightarrow b \Rightarrow a \);

rough translation: "more is better"

4a. Nonsatiation - for any bundle \( x \), there exists a bundle \( y \) with \( y \mathcal{P} x \) (i.e., there is no satiation or bliss point; you can always find something better)

4b. Local Nonsatiation - for any bundle \( x \), there exists a bundle \( y \) nearby with \( y \mathcal{P} x \); rules out "thick" indifference curves

Nearby - "in the neighborhood"; no matter how you define distance, the distance between \( x \) and \( y \) is \( < \varepsilon \)

Meaning - slight modification of bundle will make individual happier; don't know how much better (that's cardinal), just know it's better

Implication - if \( A \) is compact and \( R \) is complete, transitive, continuous, & has local nonsatiation, then \( C(A,R) \) is a subset of the boundary of \( A \)

4c. (Weak) Monotonicity - \( x > y \Rightarrow x \mathcal{P} y \)

4d. (Strict) Monotonicity - \( x \geq y \Rightarrow x \mathcal{P} y \)

Definitions - confusing because different authors use different symbols

1. \( > \) (or \( >> \)) - \( x_i > y_i \ \forall \ i = 1,2,\ldots,n \)

2. \( \geq \) (or \( \geq \)) - \( x_i > y_i \ \forall \ i \) and \( x_j \geq y_j \) for some \( j \)

3. \( \geq \) - \( x_i \geq y_i \ \forall \ i = 1,2,\ldots,n \)

Not much difference between 1 & 2; 1 is for weak monotonicity (doesn't include any bundles with same quantity as \( y \)); 2 is for strict monotonicity (only need at least 1 commodity with more than \( y \)); 3 doesn't work for monotonicity because if \( y \) is included, you can't have \( x \mathcal{P} y \)

Implication - monotonicity \( \Rightarrow \) local nonsatiation
**Perfect Compliments** - $U(x_1, x_2) = \min(x_1, x_2)$; special case where weak monotonicity (4c) holds, but strict monotonicity (4d) doesn't

**Example** - $x_1 = \text{left shoe}; x_2 = \text{right shoe}$

**No Monotonicity** - some things are "bads" so less is better (e.g., exams, pollution, labor); this can be handled by defining a new commodity that's the opposite (e.g., leisure instead of labor) so monotonicity really means adding more has same effect (not just "more is better"); other commodities don't satisfy monotonicity either way because sometimes more is better and sometimes less is better (e.g., temperature)

**Economic Meaning** - if any commodity in the bundle satisfies monotonicity, the choice must be on the "outer" boundary (e.g., choice will be on the budget line... that means you spend all your money)

**Theorem (Utility Representation 2)** - if $R$ is complete, transitive, continuous, and monotonic (weak or strict) then $\exists$ a continuous and monotonic utility representation...

**Proof** (of $\exists U(x)$ and monotonicity using 2 commodities... generalizes to more):

Assign $U$ to each bundle on a 45° line such that $U(a, a) = a$

Since $R$ is monotonic 45° line is monotonic

Given assumptions about $R$, $\forall$ bundles $x$ not on the 45° line $\exists$

a unique bundle on the 45° line such that $U(x) = U(a, a) = a$,

That means $x \succeq (a, a)$.

Thus: the 45° line is a monotonic utility representation

Proving that $\exists$ unique bundle on 45° line:

Define $A \equiv R^>(x) \cap 45°$ line and $B \equiv R^<(x) \cap 45°$ line

$R^>(x)$ and $R^<(x)$ are closed by continuity assumption

45° line is closed by definition

If you move up from $x$ (or left if it's on the other side of the line) and then move up the 45° line, by monotonicity of $R$, $\exists$ points preferred to $x$;

similarly, if you move up from $x$ and then move down the 45° line, $\exists$ points inferior to $x$; $\therefore$ A and B are nonempty and closed

Since $R$ is complete, $A \cup B = 45°$ line

$A \cap B$ is nonempty $\therefore \exists$ points on the 45° line that are indifferent to $x$

**Theorem** - If set $T$ is closed, then $T^c$ is open

**Theorem** - Given 2 closed sets with union equal to everything, then intersection is nonempty

**Proof**: if $A \cap B$ were empty, then $A = B^c$, but both $A$ and $B$ are closed; that a contradicts previous theorem so $A \cap B \neq \emptyset$

Assume there are multiple points $z$ and $w \in A \cap B$ with $z \neq w$

Both $z$ and $w$ are on 45° line so either $z > w$ or $w > z$ (i.e., one of them has more of everything); order not important so pick $z > w$

By monotonicity $z \succ w$ Knowing $z \succ w$ and $w \sim x$ (because $w \in A \cap B$), then $z \succ x$ by transitivity

That contradicts $z \in A \cap B$ which says $z \preceq x$

$\therefore$ there is a unique bundle on the 45° line that is indifferent to $x$

Note that 45° line was used for convenience, any line with positive slope could be used so there is actually an infinite number of utility representations

**No $U$ for Lexicographic** - with lexicographic preferences every bundle has to be assigned a unique $U$; since there are an infinite number of values for $C_2$ for each of an infinite number of values for $C_1$, there is no way to use a utility
representation... we'd run out of numbers (there are various levels of infinity and this one is bigger than the infinite number of numbers)

5. **Convexity** - various definitions, make a difference when paired with other assumptions;
   differences not important right now; i ⇔ ii
   (i) $x R y \Rightarrow [\lambda x + (1 - \lambda)y] R \forall \lambda \in (0,1)$
   (ii) $R^2(y)$ is convex set (i.e., $z R y \& x R y \Rightarrow [\lambda x + (1 - \lambda)y] R y$)
   (iii) $x R y$ and $x \neq y \Rightarrow [\lambda x + (1 - \lambda)y] P \forall \lambda \in (0,1)$... this is **strict** convexity

**Infinite Bundles** - convexity assumes # of alternatives (i.e., $A$) is infinite

**Hard to Test** - difficult to confirm convexity empirically

**Theorem (Choice 1)** - if $A$ is compact and convex and $R$ is complete, transitive, continuous, monotonic (weak or strict) and convex then the choice function $C(A,R)$ is convex

**Proof:**
Pick $x$ and $y \in C(A,R)$

$[\lambda x + (1 - \lambda)y] \in A$ (i.e., feasible) for $\lambda \in (0,1)$ because $A$ is convex
$x R z \& \forall z \in A$ and $y R z \& \forall z \in A$ because $x \& y \in C(A,R)$
$[\lambda x + (1 - \lambda)y] R z$ because $R$ is convex (using definition ii)
$\therefore [\lambda x + (1 - \lambda)y] \in C(A,R)$ for $\lambda \in (0,1)$

**Theorem (Choice 2)** - change previous theorem to say $R$ is strictly convex, then $C(A,R)$ is unique choice (only one bundle)

**Proof:**
Pick $x$ and $y \in C(A,R)$

That means $x I y$ which can be written $x R y$ and $y R x$
$[\lambda x + (1 - \lambda)y] P x$ because $R$ is strictly convex
But that means $x$ can't be $\in C(A,R)$ because there's a better bundle

**Reasonable Assumption?** - convexity basically means "balanced bundles are better"

**Specific Time** - may not be true at specific time, but is true for longer periods;
   e.g., can have chicken or steak for dinner doesn't mean you'll prefer a combination, but if you're talking about 100 dinners, you'll probably prefer a combination of chicken and steak rather than having only one of them 100 times.

**Drinking** - mixing drinks results in worse hangovers so people prefer 1 bottle of bourbon or 1 bottle of scotch rather than a half bottle of each

**Risk Aversion** - people buy insurance to reduce risk so consumption is the same with or without an accident

**Risk Loving** - opposite case so preferences aren't convex;
   e.g. gambling
Choice Function

Choice Function, \( C(A,R) \) - gives choice set \( (C) \) from set of bundles \( (A) \) based on preference relationship \( (R) \); Formally: \( C(A,R) \equiv \{ x \in A: x \ R \ y \ \forall \ y \in A \} \)

Examples:
\[
C([x,y], \{ x \ P \ y \}) = \{ x \}
\]
\[
C([x,y], \{ x \ I \ y \}) = \{ x, y \}
\]

Theorem - If \( R \) is complete and transitive, then \( C(A,R) \neq \emptyset \)

Problem - \( C(A,R) \) isn't observable because we can't observe preferences \( (R) \)

Observable Choice Function, \( C(A) \) - different function used in book; given observed choices, we can try to rationalize them into a preference ordering \( (R) \)

Example:
\[
\hat{C}([x,y]) = \{ x \} \Rightarrow x \ R \ y
\]
\[
\hat{C}([x,y,z]) = \{ z \} \Rightarrow z \ R \ x \ and \ z \ R \ y
\]
\[
\hat{C}([y,z]) = \{ y \} \Rightarrow y \ R \ z
\]
This can't be rationalized because \( z \ R \ y \) and \( y \ R \ z \) means \( y \ I \ z \), but if that were the case, then \( \hat{C}([y,z]) = \{ y, z \} \) which isn't what was observed; if there's only one data point, this can slide, but would need multiple trials to see if the choices can actually be rationalized... assuming preferences don't change over time

Example: Demand; prices determine feasible set; we don't know preferences, but we observe choices to derive demand

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Aside - Do people really do this stuff? No, but they act as if they do (paraphrased from Milton Friedman)

Baseball Analogy - player ends up in right spot even without knowing Newtonian physics and equations of motion (this is good argument to support consumer theory)

Test Conclusions - if conclusions are OK, we don't need to test assumptions; this drove lots of other economists wild because many times conclusions \( \leftrightarrow \) assumptions (same)

Proving \( A \iff B \) - Three options:
1. Show \( A \implies B \) and \( B \implies A \)
2. Show \( A \implies B \) and \( \neg A \implies \neg B \)
3. Show \( \neg B \implies \neg A \) and \( \neg A \implies \neg B \)

Stronger vs. Weaker Statements - if \( A \implies B \) and \( C \implies B \), whichever antecedent \( (A \ or \ C) \) is less restrictive makes the stronger statement