Describing Games

Economics - two areas
- **Optimization**
- **Equilibrium** - two areas
  - **Competitive** - don't care about what competitors are doing; participants only need to know their own technology (firms) or preferences (consumers) and the market price
  - **Interaction** - have to worry about other players; e.g., Coke vs. Pepsi, schools competing for graduate students

Optimization - maximize objective subject to constraint(s); one time event

Game - set of simultaneous, interrelated optimizations; choice of 1 player affects optimization of the other

Background Info

Interdependence - one person's behavior affects another person's well-being, either positively or negatively

Strategic Setting - situations of interdependence; in order for one person to decide how best to behave, he must consider how others around him choose their actions

Purpose of Game Theory - 2 views
- **Normative** - help participants know what to do; "here's how you should play this game"
- **Positive** - develop an understanding of how people actually behave; predictive theory; this view is used more for economics and social sciences
  - **Limited Rationality** - trying new models with limitations on rationality to get model predictions to better reflect real world outcomes (mainly focused on limited memory)

Game - situation in which 2 or more adversaries match wits; inherently entail interdependence; usually have sets of rules that must be followed by the players
- **Constant (Zero) Sum Game** - if one participant gains, the other loses by same amount; will always have "efficient" outcome because sum of payoffs is always the same; unrealistic; introduced by <u>Von Neumann</u> & <u>Morgenstern</u>
- **Non-Constant Sum Game** - possible for both parties to gain (or lose); e.g., in labor strike both sides lose; brings up question of efficiency
- **Non-cooperative Game** - participants don't work together; each player decides on his own, independent of the other people present in the strategic environment; <u>Nash</u> focused on non-constant sum, non-cooperative games
- **Cooperative Game** - look at what a coalition can do, how it will form, and how it will divide profits; no all that useful because there are too many equilibria
  - **Coalition** - 2 or more players join to improve their payoffs at the expense of other players
- **Complete Information** - participants know everything there is to know about the game (who makes what decisions and when); focuses on *structure* of the game
- **Incomplete (Private) Information** - player knows more about something in the game than another player
- **Perfect Information** - players knows everything that happened before (i.e., aware of previous decisions by other players); equivalent to saying all information sets have only 1 node or saying it's a sequential game; focuses on *decisions* in the game
  - **Perfect Recall** - player remembers his own choices
- **Imperfect Information** - player doesn’t know what choice opponent made; equivalent to having a simultaneous choice or saying at least one information set has 2 or more nodes

Common Knowledge - each player knows the other has complete info
1-Shot vs. Infinitely Repeated - results depend on whether there is an infinite time horizon
Chess Example - chess is a 1-shot constant sum, non-cooperative game with complete, perfect information (except for limits on skill, calculation, and mistakes)

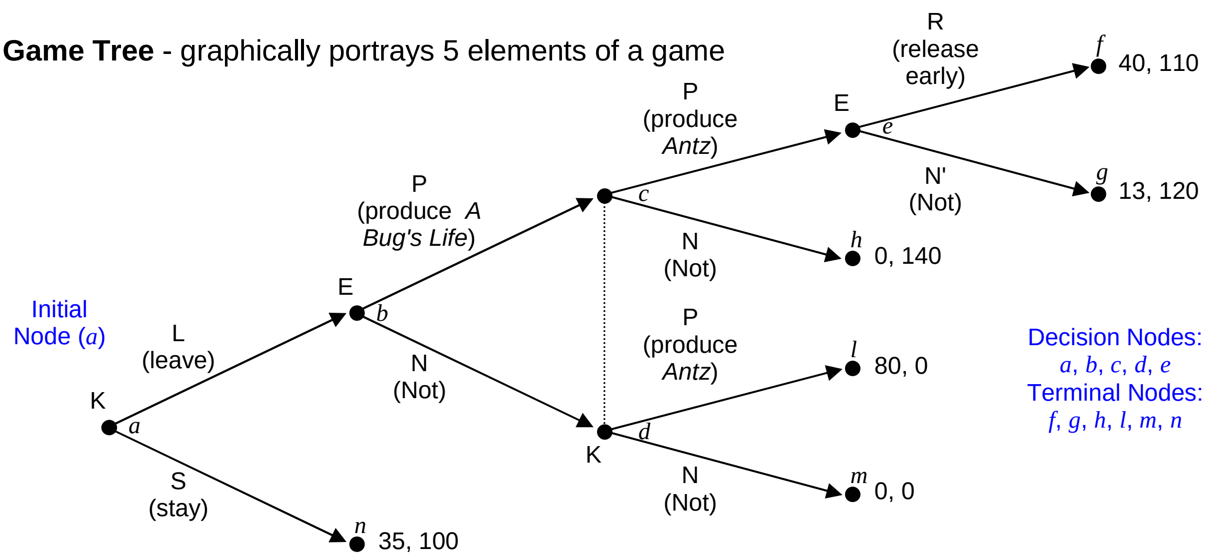
Information - note that what's available to the modeler may not be the same as the players; psychology of players or technical aspects of firms may not be known to the modeler, but may be somewhat known by players

Elements of a Game -
- **Players** - need a list of everyone involved; 2 types
  - **Strategic Player** - makes choices
  - **Nature** - no objectives or payoffs; makes random moves
- **Possible Actions** - complete description of what players can do; usually conditioned on where they are in the game
- **Information** - description of what players know at each decision point (perfect vs. imperfect info, perfect vs. imperfect recall, beliefs about other players, etc.)
- **Outcomes** - results of every possible combination of player actions
- **Preferences** - players preferences over outcomes
  - **Ordinal** - just shows order of preference
  - **Cardinal** - shows order of preference and assigns numerical values to know how much more one outcome is preferred; required for using nature because there will be probability distributions

Strategy - set of instructions on how to play the game

Extensive Form

Game Tree - graphically portrays 5 elements of a game



Node - represents a place where something happens in the game
- **Decision Node** - a player makes a decision at that place in the game
- **Initial Node** - every extensive-form game has exactly one initial node
- **Terminal Node** - places where the game ends; represent outcomes of the game; each terminal node corresponds to a unique path through the tree
  - **Payoffs** - listed as a vector at each terminal node; entries correspond to player order (e.g., from node *n* above, K gets 35 and E gets 100); could also use utilities

Label Them - each node is assigned to a player by putting the player number (or name) next to the node

Player 0 - nature; other players assigned numbers (1, 2, etc.)

Branch - indicates various actions that players can choose at a node

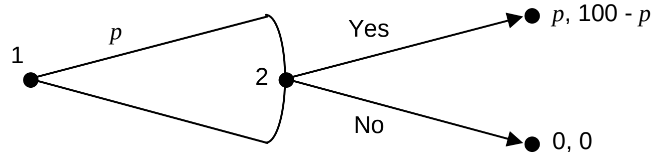
Label Them - write out description of action taken; if you want to abbreviate it make sure you use a unique identifier; note in the example Eisner (player E) has N and N' to distinguish between his two "Not" alternatives; Katzenberg (player K) however, has the same N because nodes *c* and *d* are in the same information set (simultaneous move)

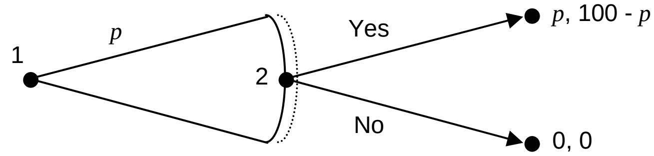
Information Set - what a player knows at a decision node; every node is in one information set, although one information set can contain multiple nodes; only one decision is made at each information set

Sequential Move - player knows what opponent did prior to making his decision

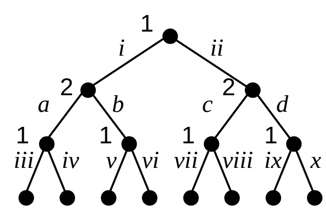
Simultaneous Move - player doesn't know what opponent did prior to making his decision so the decision nodes are in the same information set; represented by connecting nodes with a dotted line; **Note:** nodes representing a simultaneous decision must have the same possible actions (see nodes *c* & *d* above)

Infinite Number of Actions - represented as range; example: <u>ultimatum bargaining</u>; player 1 offers one time take-it-or-leave-it offer of anything from 0 to p dollars to sell a painting; player 2 gets a chance to accept or reject the offer; the painting is worth nothing to player 1 and $100 to player 2





Strategy - complete contingent plan for a player in the game; full specification of a player's behavior which describes the actions that the player would take at each of his possible decision points; entries in brackets denote which decision the player should make based on the opponent's previous decision (described in more detail in next section)



Example - player 1 has 8 strategies: {(*i*,[*iii*,*v*]), (*i*,[*iii*,*vi*]), (*i*,[*iv*,*v*]), (*i*,[*iv*,*vi*]), (*ii*,[*vii*,*ix*]), (*ii*,[*vii*,*x*]), (*ii*,[*viii*,*ix*]), (*ii*,[*viii*,*x*])}; player 2 has 4 strategies: {([*a*,*c*]), ([*a*,*d*]), ([*b*,*c*]), ([*b*,*d*])}

Book Version - doesn't eliminate strategies that can be ruled out such as (*i*,[*vii*,*ix*]) so player 1 has 16 strategies

Not Observable - we observe single iteration of a game at a time; that only reveals part of a player's strategy; can't observe the complete plan

"Simple Game" - tic-tac-toe; # of strategies for player 1 is between $9(7^8)(5^{8\cdot 6})$ and $9(7^8)(5^{8\cdot 6})(3^{8\cdot 6\cdot 4})$; for player 2 it's between $8^9(6^{9\cdot 7})$ and $8^9(6^{9\cdot 7})(4^{9\cdot 7\cdot 5})(2^{9\cdot 7\cdot 5\cdot 3})$; these numbers can be reduced if you take advantage of the symmetry of the game, but the point is it's not a complicated game to play, but it's definitely complicated to model; a person can quickly figure out how to play to a tie every time without using extensive form

Normal (Strategic) Form

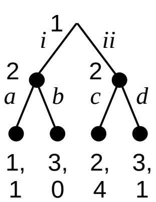
Lists all strategies available to player; shows payoffs for each combination of strategies

Set Notation - normal form consists of set of players, strategy spaces for the players, and payoff functions for the players; only use set notation if too many players (or infinite strategies)

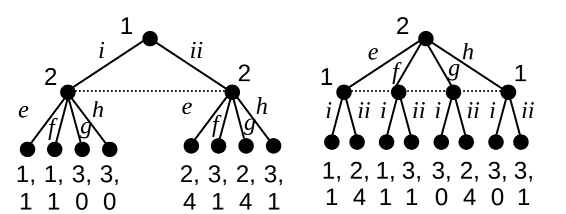
- $\boldsymbol{I}$ (or S in text) - set of players (1, 2, ..., n)
- $\boldsymbol{S}$ (or T in text)- set of strategies S^1 x S^2 x ... x S^n (Cartesian product; e.g., S^1 = {A,B} and S^2 = {X,Y}, $S = S^1$ x S^2 = {(A,X), (A,Y), (B,X), B,Y)})
  - **Strategy Space, S^i** - the set of all possible strategies for player i (e.g., S^1 = {A,B})
  - **Specific Strategy, s_i** - a specific strategy for player i, $s_i \in S^i$ (e.g., s_i = A)
  - **Opponents' Strategies, $s_{\sim i}$** - strategies played by player i's opponents (e.g., $s_{\sim i}$ = (X,D), where S^2 = {X,Y} and S^3 = {C,D}); Note: can use ~i or -i
  - **Strategy Profile** - vector of strategies, one for each player that fully describes how the game is played and is associated with a payoff vector (e.g., (A,X,D) means player 1 plays A, player 2 plays X, and player 3 plays D)
  - **Independence** - s_i is assumed to be independent of what other players do (i.e., $s_{\sim i}$); dependencies are already built in to the game (e.g., cattleman vs. farmer; cattleman has several options when there is a fence or there isn't a fence [the farmer's strategies]; the actual strategy the cattleman picks may depends on the farmer's strategy, but the strategy space is unchnaged)
- $\boldsymbol{U}$ (or P in text)- set of payoff functions (u_1, u_2, ..., u_n)
  - **Payoff Function, u_i** - function with domain in set of strategy profiles (S) and whose range is the real numbers ($u_i : S \rightarrow \mathbf{R}$); sometimes written as function of strategies $u_i(s_1, s_2,..., s_n)$ or $u_i(s_i,s_{\sim i})$

Bimatrix Game - for two player game with finite number of strategies, use matrix to list 1 player's strategies by row and the other player's by column; cells contain payoffs for each player resulting from strategies (hence name bimatrix for pairs of numbers)

Link to Extensive Form - strategic form models players that simultaneously and independently selecting complete contingent plans (strategies) for an extensive form game; there is only one strategic form for an extensive form game, but reverse isn't true (see below)



Player 1	Player 2: ac (e)	ad (f)	bc (g)	bd (h)
i	1, 1	1, 1	3. 0	3, 0
ii	2, 4	3, 1	2, 4	3, 1

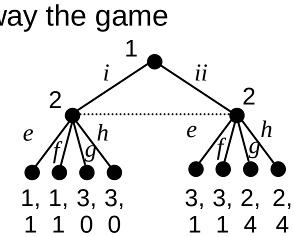


Extensive form leads to this strategic form

But this strategic form could also come from these extensive forms

Difference? - if game only has simultaneous and independent moves, strategic and extensive forms are identical; some theorists argue there is a difference, but others say there are a series of transformation s that can link all the different extensive forms that create a single strategic form; debate centers on whether these transformations change the way the game is played (i.e., are the extensive forms equivalent)

- **Redundant Strategy** - alternatives that have same payoffs; these can be added or deleted as one of the transformations discussed above; in example shown here e & f are redundant and so are g & h



3 Player - write a matrix for each of player 3's strategies; player 3 picks the matrix to be played on and players 1 & 2 play on that matrix

4 Player - write a page of matrices; one page for each of player 4's strategies; gets difficult to visualize and not very useful

Classic Normal-Form Games

Can gain great insights from simple 2x2 games

Prisoners' Dilemma - two suspects are suspected of having committed a major crime, but the prosecutor only has enough evidence to convict on a lesser offense (1 year max); prosecutor needs confession (C) in order to convict for longer sentence; if one prisoner confesses, he gets a "good deal" (either 0 times or 4 years if both confess); note that payoffs equal jail time (a bad) so objective is to minimize the payoff; from perspective of an individual prisoner, it's always best to confess (dominant strategy), but if both prisoners don't confess they're better off; there's an inefficient outcome from prisoner's point of view

Player 1 \ Player 2	C	N
C	4, 4	0, 9
N	9, 0	1, 1

- **Mechanism Design** - try to set up payoffs to induce people to behave a certain way
- **Powerful Payoffs** - doesn't matter if prisoners are in separate rooms or even if they talk to each other; basic problem still exists: even if they agree to not confess, their incentives will be contrary to the agreement and they are more likely to confess than not
- **Other Examples** - donations to common-use good (free-rider problem); firms colluding
- **Solving Dilemma** - organized crime essentially changes the payoffs by punishing those who confess; trying to negate the mechanism design
- **Fundamental Insights in Economics** - only 2
  - **Invisible Hand** - in surprising ways, individuals looking for own best interest (maximizing own utility), creates efficient outcome (Adam Smith)
  - **Opposite Result** - circumstances like prisoner's dilemma where inefficiency results when people look at own best interest

Coordination Game - both players obtain same positive payoff if they select the same strategy, otherwise they get nothing; have multiple equilibria in which neither player has an incentive to change strategies

Player 1 \ Player 2	R	L
R	1, 1	0, 0
L	0, 0	1, 1

- **Island Example** - two drivers on opposite ends of same road have choice to drive on left or right side of road
- **Problem** - how do you get to the equilibria?
- **Communication** - players can discuss which side they will drive on; talk only works if it coincides with incentives (which is why it doesn't work in prisoners' dilemma)
- **Role of Government** - (one of many) solves coordination problem by supplying communication (tells people what side of the road to drive on)

Pareto Coordination - same thing by both players prefer to coordinate on a particular strategy

Player 1 \ Player 2	R	L
R	2, 2	0, 0
L	0, 0	1, 1

Battle of the Sexes - two friends prefer to do something together but each likes one activity more than the other (here player 1 prefers boxing and

Player 1 \ Player 2	Bal	Box
Bal	1, 2	-1, -1
Box	0, 0	2, 1

player 2 prefers ballet); players have to make decision independently and simultaneously because they can't communicate

Distributional Consideration - just like coordination game, there are two equilibria, but this solution isn't as simple as arbitrarily picking one of them because payoffs are different

Matching Pennies - two players simultaneously and independently select heads (H) or tails (T) by uncovering a penny in his hand; if selections match, player 2 gives his penny to player 1; otherwise, player 1 gives his penny to player 2

Two Representations - can look at change in pennies help (top matrix) or total pennies at end of round (bottom matrix); result is the same

No Equilibrium - there isn't a cell where both players are content (at least 1 has an incentive to move to another cell)

Result - best strategy is mixed strategy; players must randomize decision so opponent won't know what the other is doing

Another Example - rock, paper, scissors game

Player 2

Player 1	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Player 2

Player 1	H	T
H	2, 0	0, 2
T	0, 2	2, 0

Player 2

Player 1	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0