EAVESDROPPING AND JAMMING COMMUNICATION NETWORKS

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ABSTRACT. Eavesdropping and jamming communication networks is an important part of any military engagement. However, until recently there has been very limited research in this area. Two recent papers by Commander et al. [2, 3] addressing this problem represent the current state-of-the-art in the study of the WIRELESS NETWORK JAMMING PROBLEM. In this chapter, we survey the problem and highlight the results from the aforementioned papers. Finally, we indicate future areas of research and mention several extensions which are currently under investigation.

1. INTRODUCTION

In any military engagement, disrupting the communication mechanism of one's enemy is an important strategic maneuver. Depending on the circumstances the goal may be to intercept the communication, neutralize the communication network, or both. These problems can be modeled in the same manner. Without the loss of generality, we will refer the problem of determining the optimal placement and quantity of eavesdropping or jamming devices to either intercept or destroy the communication as the WIRELESS NETWORK JAMMING PROBLEM (WNJP) and our descriptions will be in this context. Hence jamming devices can also be thought of as eavesdropping devices.

Research on optimization problems in telecommunications is abundant [8]. However, until recently there has been very limited research on the use of optimization techniques for jamming wireless networks. Two recent papers by Commander et al. [2, 3] addressing this problem represent the current state-of-the-art in the study of the WIRELESS NETWORK JAMMING PROBLEM. In this chapter, we survey the problem and highlight the results from the aforementioned papers. Finally, we indicate future areas of research and mention several extensions which are currently under investigation.

The organization of the paper is as follows. Section 2 contains several formulations of the WNJP. The models considered are formulated with the assumption that the locations of the set of communication devices to be jammed is known. In Section 3, we will review several recent results on jamming wireless networks when no information about the topology of underlying network is assumed. In this case, bounds are derived for the optimal number of jamming devices required to cover an area in the plane. Finally in Section 4, conclusions and future directions of research will be addressed.

2. DETERMINISTIC FORMULATIONS

Before continuing, we will introduce some of the idiosyncrasies, symbols, and notations we will employ throughout this paper. Denote a graph $G = (V, E)$ as a pair consisting of a set of vertices V, and a set of edges E. All graphs in this paper are assumed to be undirected and unweighted. We use the symbol "$b := a$" to mean "the expression a defines the (new) symbol b" in the sense of King [7]. Of course, this could be conveniently extended so that a statement like "$(1-\epsilon)/2 := 7$" means "define the symbol ϵ so that $(1-\epsilon)/2 = 7$ holds." Finally, we will use *italics* for emphasis and SMALL CAPS for problem names. Any other locally used terms and symbols will be defined in the sections in which they appear.

In [3], the authors provide several formulations for the WNJP. Some assumptions made about the network nodes and the jamming devices are as follows. The jamming devices and network nodes are assumed to have omnidirectional antennas and operate as both transmitters and receivers. In other words, given a graph $G = (V, E)$, we can represent the enemy communication devices as the vertices of the graph. An undirected edge would connect two nodes if they are within a certain communication threshold.

2.1. **Coverage Based Formulations.** Given a set $\mathcal{M} = \{1, 2, \ldots, m\}$ of communication nodes to be jammed, our goal is to find a set of locations for placing jamming devices in order to suppress the functionality of the network. We assume that the *jamming effectiveness* of device j is a function $d : \mathbb{R} \to \mathbb{R}$, where d is a decreasing function of the distance from the device to the node being jammed. For the problem at hand, we are considering radio transmitting nodes, and correspondingly, jamming devices which emit electromagnetic waves. Thus the jamming effectiveness of a device depends on the power of its electromagnetic emission, which is assumed to be inversely proportional to the squared distance from the device to the node begin jammed. That is,

$$d_{ij} := \frac{\lambda}{r^2(i, j)},$$

where $\lambda \in \mathbb{R}$ is a constant, and $r(i, j)$ represent the distance between node i and jammer location j. Without the loss of generality, we can set $\lambda = 1$.

Define the cumulative amount of jamming energy received at node $i \in \mathcal{M}$ as

$$Q_i := \sum_{j=1}^{n} d_{ij} = \sum_{j=1}^{n} \frac{1}{r^2(i, j)},$$

where n is the number of jamming devices placed. Then, we can formulate the WIRELESS NETWORK JAMMING PROBLEM (WNJP) as the minimization of the number of jamming devices placed, subject to a set of covering constraints:

$$\textbf{(WNJP)} \quad \text{Minimize} \quad n \tag{1}$$

$$\text{s.t.} \quad Q_i \geq C_i, \quad i = 1, 2, \ldots, m. \tag{2}$$

The solution to this problem provides the optimal number of jammers needed to ensure a certain jamming threshold at every node. As mentioned in [3], a continuous optimization approach where one is seeking the optimal placement coordinates $(x_j, y_j), j = 1, 2, \ldots, n$ for jamming devices given the coordinates $(X_i, Y_i), i = 1, 2, \ldots, m$, of network nodes, leads to highly non-convex formulations. For example, consider the covering constraint for some node $i \in \mathcal{M}$,

$$\sum_{j=1}^{n} \frac{1}{(x_j - X_i)^2 + (y_j - Y_i)^2} \geq C_i.$$

It is easy to verify that this constraint is non-convex.

To overcome the non-convexity of the above formulation, we now develop some integer programming models for the problem. Suppose now that along with the set of communication nodes $\mathcal{M} = \{1, 2, \ldots, m\}$, there is a fixed set $\mathcal{N} = \{1, 2, \ldots, n\}$ of possible locations to place the jamming devices. This assumption is valid, because in reality the set of possible placement locations may be limited. Define the decision variable x_j as

$$x_j := \begin{cases} 1, & \text{if device installed at location } j \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Then if C_i and d_{ij} are as defined above, we can formulate the OPTIMAL NETWORK COVERING (ONC) formulation of the WNJP as

$$\textbf{(ONC)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{4}$$

s.t.

$$\sum_{j=1}^{n} d_{ij} x_j \geq C_i, \quad i = 1, 2, \ldots, m, \tag{5}$$

$$x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n. \tag{6}$$

Here our objective is to minimize the number of jamming devices while achieving some minimum level of jamming at each node. The coefficients c_j in (4) represent the costs of installing a jamming device at location j. In a battlefield scenario, placing a jamming device in a direct proximity of a network node may be theoretically possible; however, such a placement might be undesirable to do security considerations. In this case, the location considered would have a higher placement cost than a safer location [3]. If there are no preferences for device locations, then

$$c_j = 1, \quad j = 1, 2, \ldots, n.$$

Notice that ONC is formulated as a MULTIDIMENSIONAL KNAPSACK PROBLEM which is well-known to be $\mathcal{NP}$-hard [4].

In a philosophical sense, the objective of the WNJP is not to simply jam the communication nodes, but rather to destroy the *functionality* of the underlying communication network. Therefore, we need to develop a method for characterizing the connectivity of the network nodes. In the next section, we provide an alternate formulation based on constraining the number of nodes any given vertex can communicate with. The intuition is that by limiting the *connectivity index* of the nodes, we can place the jamming devices in such a way that disconnects the network into several small components. This will prohibit global information sharing and will effectively dismantle the network.

2.2. **Connectivity Based Formulation.** Given a graph $G = (V, E)$, the *connectivity index* of a node is defined as the number of nodes reachable from that vertex (see Figure 1 for examples). In this section, we develop a formulation for the WNJP based on constraining connectivity indices of the network nodes. We will assume that the topology of the network is known. That is, we know that there is a set of communication nodes $M = \{1, 2, \ldots, m\}$

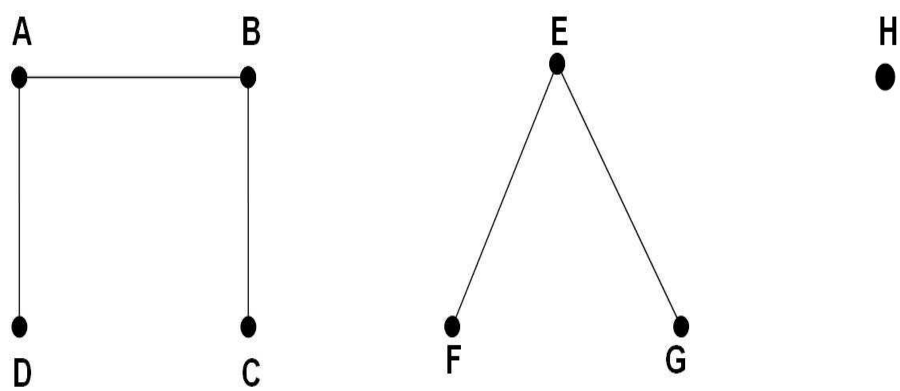


FIGURE 1. Connectivity Index of nodes A,B,C,D is 3. Connectivity Index of E,F,G is 2. Connectivity Index of H is 0.

to be jammed and a set of possible jammer locations $N = \{1, 2, \ldots, n\}$. Let $S_i := \sum_{j=1}^{n} d_{ij} x_j$ denote the cumulative level of jamming at node i. Then as before, node $i \in \mathcal{M}$ is said to be jammed if S_i exceeds some minimum threshold value C_i. We say that a communication link between nodes i and j is severed if at least one of the nodes is jammed. Further, let $y : V \times V \to \{0, 1\}$ be a surjection where $y_{ij} := 1$ if there exists a path from node i to node j in the jammed network. Lastly, define $z : V \to \{0, 1\}$ to be a surjective mapping where z_i returns 1 if node i is **not** jammed.

The objective of the CONNECTIVITY INDEX PROBLEM (CIP) formulation of the WNJP is to minimize total jamming cost subject to a constraint that the connectivity index of each node does not exceed some pre-described level L. The corresponding optimization problem is the following:

$$\textbf{(CIP)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j \tag{7}$$

s.t.

$$\sum_{j \neq i} y_{ij} \leq L, \ \forall\, i, j \in \mathcal{M} \tag{8}$$

$$M(1 - z_i) \geq S_i - C_i \geq -M z_i, \ \forall\, i \in \mathcal{M} \tag{9}$$

$$x_j \in \{0, 1\}, \ \forall\, j \in \mathcal{N} \tag{10}$$

$$z_i \in \{0, 1\} \ \forall\, i \in \mathcal{M}, \tag{11}$$

$$y_{ij} \in \{0, 1\} \ \forall\, i, j \in \mathcal{M}, \tag{12}$$

where M is some large constant. Constraint (8) is the connectivity constraint, which ensures that there are less than L nodes reachable from any vertex. Further, (9) implies that those nodes $i \in \mathcal{M}$ which receive a cumulative level of energy in excess of the threshold level C_i are considered jammed. Finally, (10)-(12) define the domains of the decision variables used.

Now, let $v : V \times V \to \{0, 1\}$ and $v' : V \times V \to \{0, 1\}$ be defined as follows:

$$v_{ij} = \begin{cases} 1, & \text{if } (i, j) \in E, \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

and

$$v'_{ij} = \begin{cases} 1, & \text{if } (i, j) \text{ exists in the jammed network,} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Then, as seen in [3], an equivalent integer programming formulation is given by

$$\textbf{(CIP-1)} \quad \text{Minimize} \quad \sum_{j=1}^{n} c_j x_j, \tag{15}$$

s.t.

$$y_{ij} \geq v'_{ij}, \ \forall\, i, j \in \mathcal{M}, \tag{16}$$

$$y_{ij} \geq y_{ik} y_{kj}, \ k \neq i, j; \ \forall\, i, j \in \mathcal{M}, \tag{17}$$

$$v'_{ij} \geq v_{ij} z_j z_i, \ i \neq j; \ \forall\, i, j \in \mathcal{M}, \tag{18}$$

$$\sum_{j=1}^{m} y_{ij} \leq L, \ j \neq i, \ \forall\, i \in \mathcal{M}, \tag{19}$$

$$M(1 - z_i) \geq S_i - C_i \geq -M z_i, \ \forall\, i \in \mathcal{M}, \tag{20}$$

$$z_i \in \{0, 1\}, \ \forall\, i \in \mathcal{M}, \tag{21}$$

$$x_j \in \{0, 1\}, \ \forall\, j \in \mathcal{N}, \qquad y_{ij} \in \{0, 1\} \ \forall\, i, j \in \mathcal{M}, \tag{22}$$

$$v_{ij} \in \{0, 1\}, \ \forall\, i, j \in \mathcal{M}, \qquad v'_{ij} \in \{0, 1\}, \ \forall\, i, j \in \mathcal{M}. \tag{23}$$

We present the following lemma from [3] without proof, which establishes the equivalency between formulations CIP and CIP-1.

Lemma 1. *If* CIP *has an optimal solution then,* CIP-1 *has an optimal solution. Further, any optimal solution x^* of the optimization problem* CIP-1 *is an optimal solution of* CIP.

Now by applying some basic linearization techniques, we can formulate the linear 0-1 program, CIP-2 as

$$
\begin{aligned}
\textbf{(CIP-2)} \quad \text{Minimize} \quad & \sum_{j=1}^{n} c_j x_j && (24)\\
\text{s.t.} \quad & && \\
& y_{ij} \geq v_{ij}^{'},\ \forall\, i,j = 1,\ldots,\mathcal{M}, && (25)\\
& y_{ij} \geq y_{ik} + y_{kj} - 1,\ k \neq i,j;\ \forall\, i,j \in \mathcal{M}, && (26)\\
& v_{ij}^{'} \geq v_{ij} + z_j + z_i - 2,\ i \neq j;\ \forall\, i,j \in \mathcal{M}, && (27)\\
& \sum_{j=1}^{I} y_{ij} \leq L,\ j \neq i,\ \forall\, i \in \mathcal{M}, && (28)\\
& M(1 - z_i) \geq S_i - C_i \geq -Mz_i,\ \forall\, i \in \mathcal{M}, && (29)\\
& z_i \in \{0,1\},\ \forall\, i \in \mathcal{M}, && (30)\\
x_j \in \{0,1\},\ \forall\, j \in \mathcal{N}, \quad & y_{ij} \in \{0,1\}\ \forall\, i,j \in \mathcal{M}, && (31)\\
v_{ij} \in \{0,1\},\ \forall\, i,j \in \mathcal{M}, \quad & v_{ij}' \in \{0,1\},\ \forall\, i,j \in \mathcal{M}. && (32)
\end{aligned}
$$

Similar to the previous lemma, the following establishes the equivalency between CIP-1 and CIP-2.

Lemma 2. *If* CIP-1 *has an optimal solution then* CIP-2 *has an optimal solution. Any optimal solution x^* of* CIP-2 *is an optimal solution of* CIP-1.

We have as a result of the above lemmata the following theorem which states that the optimal solution to the linearized integer program CIP-2 is an optimal solution to the original connectivity index problem CIP. The proof of which follows directly from **Lemma 1** and **Lemma 2** above.

Theorem 1. *If* CIP *has an optimal solution then* CIP-2 *has an optimal solution. Any optimal solution of* CIP-2 *is an optimal solution of* CIP.

In addition to the above formulations, in [3] the authors present coverage and connectivity formulations with the addition of percentile risk constraints. Prevalent in financial engineering applications and stochastic optimization problems, risk measures can also be applied in deterministic setups in order to control the loss associated with the objective function or set of constraints. In particular, suppose it is determined that in order to neutralize a network in a given instance of the WNJP, that only a percentage of the nodes must to be jammed. For instances such as these, percentile constraints can be applied which can result in near optimal solutions while providing a significant reduction in cost.

In [3] the authors consider the Value-at-Risk (VaR) [5] and Conditional Value-at-Risk (CVar) [10] measures and present formulations of the ONC and CIP with each. Case studies indicate that models incorporating the CVaR constraints provide solutions which are comparable to the models using VaR type constraints and can be solved up to two orders of magnitude faster.

3. Jamming Under Complete Uncertainty

In this section, we highlight the contributions of Commander et al. from [2]. Considered in this paper was the case of jamming a communication network when no a priori information was assumed other than a general region (i.e. a map grid) known to contain the network. The authors provide upper and lower bounds on the optimal number of jamming devices required to cover the region when the jammers are placed at the nodes of a uniform grid covering the area of interest.

If we ignore the cumulative effect of the jamming devices, then the problem reduces to determining the optimal covering of an area in the plane with uniform circles. This covering problem was solved in 1936 by Kershner [6]. In this section, we will review the recent work from [2] which shows that accounting for the cumulative effect of all the devices can lead to significant decreases in costs, i.e. the required number of jamming devices needed to destroy the network [2].

Again, we assume that energy received at a point X, from a jamming device decreases reciprocally with the squared distance from a device. In particular we have the following definitions about the considered scenario.

Definition 1. *A point (communication node) X is said to be jammed or covered if the cumulative energy received from all jamming devices exceeds some threshold value E:*

$$\sum_i \frac{\lambda}{r^2(X,i)} \geq E, \tag{33}$$

where $\lambda \in \mathbb{R}$ and $r(X,i)$ represents the distance from X to jamming device i. This condition can be rewritten as:

$$\sum_i \frac{1}{r^2(X,i)} \geq \frac{1}{L^2}, \tag{34}$$

where $L = \sqrt{\frac{\lambda}{E}}$.

The latter inequality implies that a jamming device covers any point inside a circle of radius L.

Definition 2. *A connection (arc) between two communication nodes is considered blocked if any of the two nodes is covered.*

The intuition behind the approach is as follows. Consider a square region in the plane with side length $a \in \mathbb{R}$ which is known to contain the network to be jammed. Then, in order to place the minimum number of jamming devices on a uniform grid over the region, the grid step size, R, should be as large as possible and still jam every point in the region. Without the loss of generality, it is assumed that if the length of a square side a is not a multiple of R, then we cover a larger square with a side of length $R([\frac{a}{R}] + 1)$. Figure 2 provides a graphical representation of this scenario.

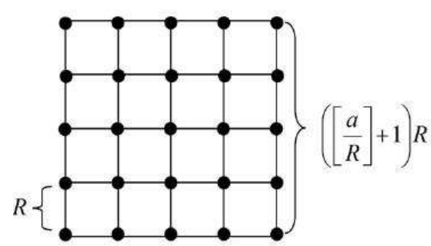


FIGURE 2. Uniform grid with jamming devices.

At the very basis of the theorems in which the bounds are derived is the result of a lemma which establishes that a point located in a corner square of the bounding area will receive the least amount of cumulative energy from all the jamming devices.

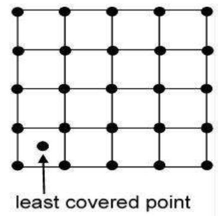


FIGURE 3. The least covered point is shown in the lower left grid cell.

Lemma 3. *For any covering of a square with a uniform grid, a point which receives the least amount of jamming energy lies inside a corner grid cell (see Figure 3).*

This is intuitively clear as with the nature of wireless broadcasting. In the same manner, we would expect that the points in the center of the region would receive the most energy since the maximum distance to any other point is minimum. With this lemma it is clear that if a grid step size guarantees coverage of the corner cells of the grid, then all points in the area will be jammed. With this established, a lower bound on the optimal grid step size can be established as follows. In all formulated theorems, we consider covering a square with side length a.

Theorem 2. *The unique solution of the equation*

$$\frac{1}{2R^2}\left(\pi \ln(\frac{a}{R}+1)+\pi-3\right)=\frac{1}{L^2} \tag{35}$$

is a lower bound $\underline{R}$ for the optimal grid step size R^*.

Thus, a uniform grid with step size $\underline{R}$ jams any point P inside a corner cell. According to Lemma 3, the grid jams the least covered point in the square implying that the whole square is jammed. Furthermore, since the function $f(R)=\frac{1}{2R^2}(\pi\ln(\frac{a}{R}+1)+\pi-3)$ is monotonic, the solution to the equation $f(R)=\frac{1}{L^2}$ is unique, and can be easily determined by a numerical procedure such as a binary search. Therefore, using (35), we can obtain a step size $\underline{R}$ such that the corresponding uniform grid covers the entire square. Further, the number of jamming devices in the grid does not exceed

$$N_1=\left(\frac{a}{\underline{R}}+2\right)^2. \tag{36}$$

The result from Kershner [6] using only circles, i.e not accounting for the cumulative effect of the jamming devices is that in the limit, the minimum number of circles required to cover the considered square region having area a^2 is

$$N_2=\frac{2a^2}{3\sqrt{3L^2}}. \tag{37}$$

In [2] it was shown that $\lim_{a\to\infty}\frac{N_2}{N_1}=\infty$, which further makes clear the benefit of the wireless broadcast advantage.

Similarly to the derivation of the lower bound on the optimal grid step size, an upper bound $\bar{R}$ can be computed and is given by the following theorem.

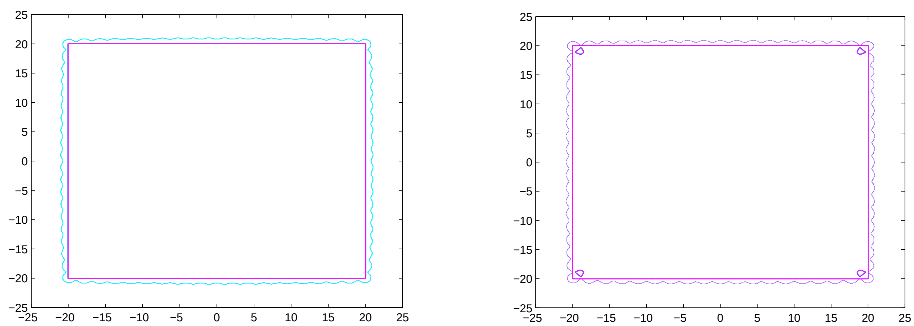
Theorem 3. *The unique solution of the equation*

$$\frac{1}{R^2}\left(\frac{\pi}{2}\ln\left(\frac{2a}{R}+1\right)-\frac{1}{6(\frac{a}{R}+1)}+\frac{\pi}{2}+\frac{19}{3}\right)=\frac{1}{L^2} \tag{38}$$

is an upper bound $\overline{R}$ of the optimal grid step size R^*.

The result of the grid step $\overline{R}$ is that all points except those located in the corner cells are jammed.

A numerical example demonstrating the effectiveness of the bounds was given in [2]. In Figure 4 we are covering a 40×40 square and the required jamming level at each point is 3.0 units. In part (a), we see the coverage associated with the required number of devices from the lower bound of Theorem 2. In this case, $20^2 = 400$ jamming devices are used to cover the area. The scallop shell outside the bounding box indicates the boundary of the coverage from the jamming devices. In part (b), we see the coverage corresponding to the placement of the jamming devices on a uniform grid according to the upper bound of Theorem 3. Here, the required number of devices is $19^2 = 361$. Notice the holes located at the four corners of the region indicating that these points are uncovered. This validates the theoretical results obtained in Theorem 2 and Theorem 3 [2].



(a) (b)

FIGURE 4. (a) The coverage of when jamming devices are placed according to the lower bound from Theorem 2. The total number of jamming devices required is $20^2 = 400$. (b) We see the coverage associated with the result obtained from Theorem 3. In this case, $19^2 = 361$ devices are placed. Notice the corner points are not jammed.

One final result from [2] was the following convergence result indicating that the derived upper and lower bounds are indeed tight within a constant.

Theorem 4.

$$\lim_{a \to \infty} \frac{\overline{R}}{\underline{R}} = 1, \tag{39}$$

where $\overline{R}$ *and* $\underline{R}$ *are bounds obtained from equations (35) and (38), correspondingly. Moreover, the following inequality holds:*

$$1 \leq \frac{\overline{R}}{\underline{R}} \leq \sqrt{1 + \frac{c}{\ln(a)}}, \tag{40}$$

for constants $M \in \mathbb{R}, c \in \mathbb{R}$.

4. CONCLUDING REMARKS AND FUTURE RESEARCH

Using optimization approaches to jam wireless communication networks is a novel approach which was only recently introduced [2, 3]. There is still a great deal of work to be done on this problem which will help military strategists ensure the best level of performance against a hostile force. Below we briefly outline several extensions and areas in which our current efforts are focused.

4.1. **Alternative Formulations.** A generalization of the node coverage formulation including uncertainties in the number of communication nodes and their coordinates might be considered. For the connectivity index problem, there might exist uncertainties in the number of network nodes, their locations, and the probability that a node will recover a jammed link. Furthermore, there are several alternative formulations which can be formulated and considered. Some possibilities include a formulation problem based on maximum network flows [1]. That is, consider the communication graph $G = (V, E)$ with a set U of arc capacities, a source node $s \in V$, and a sink node $t \in V$. Each node in the graph acts as both a server and client with the exception of s which only transmits and t which only receives data. Suppose further that each receiver is equipped with a k-sectored antenna. A sectored antenna is a set of directional antennas that can cover all directions but can isolate the sectors. Thus, an arc can be jammed only by those jamming devices that are located in the same sector as the transmitter. Then the objective of the MAXIMUM NETWORK FLOWS formulation is to find locations of jamming devices such that the expected maximum flow on the network is minimized. We are considering this formulation and with the incorporation of various uncertainties in the arc capacities, the number and coordinates of the communication nodes, type of sectored antennas, and the probability of nodes recovering from a jammer.

4.2. **Heuristics.** The inherent complexity of the aforementioned formulations motivates the need for efficient heuristics to solve real-world instances within reasonable computing times. Currently, we are working on a randomized local search heuristic for the case of jamming under complete uncertainty. Preliminary results are promising, and we hope to report our progress at next year's conference. Another heuristic currently being tested is a Greedy Randomized Adaptive Search Procedure (GRASP) [9] for the OPTIMAL NETWORK COVERING formulation. Finally, we are designing a combinatorial algorithm for the CONNECTIVITY INDEX PROBLEM formulation. Other endeavors involving heuristic development and design along with the implementation of advanced cutting plane techniques would indeed be helpful.

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