BROADCAST SCHEDULING PROBLEM, BSP

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1. SYNONYMS

The BROADCAST SCHEDULING PROBLEM is also referred to as the TDMA MESSAGE SCHEDULING PROBLEM [6].

2. INTRODUCTION

Wireless mesh networks (WMNs) have become an important means of communication in recent years. In these networks, a shared radio channel is used in conjunction with a packet switching protocol to provide high-speed communication between many potentially mobile users. The stations in the network act as transmitters and receivers, and are thus capable of utilizing a multi-hop transmission procedure. The advantage of this is that several stations can be used as relays to forward messages to the intended recipient. This allows beyond line of sight communication between stations which are geographically disbursed and potentially mobile [2].

Mesh networks have increased in popularity in recent years and the number of applications is steadily increasing [25]. As mentioned in [1], WMNs allow users to integrate various networks, such as Wi-Fi, the internet and cellular systems. WMNs can also be utilized in a military setting in which tactical datalinks network various communication, intelligence, and weapon systems allowing for streamlined communication between several different entities [6]. For a survey of wireless mesh networks, the reader is referred to [1].

In WMNs, the critical problem involves efficiently utilizing the available bandwidth to provide collision free message transmissions. Unfettered transmission by the network stations over the shared channel will lead to message collisions. Therefore, some medium access control (MAC) scheme should be employed to schedule message transmissions so as to avoid message collisions. The time division multiple access (TDMA) protocol is a MAC scheme introduced by Kleinrock in 1987 which was shown to provide collision free broadcast schedules [19]. In a TDMA network, time is divided into frames with each frame consisting of a number of unit length slots in which the messages are scheduled. Stations scheduled in the same slot broadcast simultaneously. Thus, the goal is to schedule as many stations as possible in the same slot so long as there are no message collisions.

When considering the broadcast scheduling problem on TDMA networks, there are two optimization problems which must be addressed [31]. The first involves finding the minimum frame length, or the number of slots required to schedule all stations at least once. The second problem is that of maximizing the number of stations scheduled within each slot, thus maximizing the throughput. Both of these problems however, are known to be $\mathcal{NP}$-hard [2]. Therefore, efficient heuristics are typically used to quickly provide high quality solutions to real-world instances.

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2.1. **Organization.** The organization of this article is as follows. In the following section, we formally define the problem statement and provide a mathematical programming formulation. We also examine the computational complexity the problem. In Section 4, we review several solution techniques which appear in the literature. We provide some concluding remarks in Section 5 and indicate directions of future research. Finally, a list of cross references is provided in Section 6.

2.2. **Idiosyncrasies.** We will now briefly introduce some of the symbols and notations we will employ throughout this paper. Denote a graph $G = (V, E)$ as a pair consisting of a set of vertices V, and a set of edges E. All graphs in this paper are assumed to be undirected and unweighted. We use the symbol "$b := a$" to mean "the expression a defines the (new) symbol b" in the sense of King [18]. Of course, this could be conveniently extended so that a statement like "$(1-\epsilon)/2 := 7$" means "define the symbol ϵ so that $(1-\epsilon)/2 = 7$ holds." Finally, we will use *italics* for emphasis and SMALL CAPS for problem names. Any other locally used terms and symbols will be defined in the sections in which they appear.

3. FORMULATION

A TDMA network can be conveniently described as a graph $G = (V, E)$ where the vertex set V represents the stations and the set of edges E represents the set of communication links between adjacent stations. There are two types of message collisions which must be avoided when scheduling messages in TDMA networks. The first, called a *direct collision* occurs between *one-hop neighboring stations*, or those stations $i, j \in V$ such that $(i, j) \in E$. One-hop neighbors which broadcast during the same slot cause a direct collision. Further, if $(i, j) \notin E$, but $(i, k) \in E$ and $(j, k) \in E$, then i and j are called *two-hop neighbors*. Two-hop neighbors transmitting in the same slot cause a so-called *hidden collision* [2].

Assume that there are M slots per frame. Further, assume that packets are sent at the beginning of each time slot and are received in the same slot in which they are sent. Let $x : M \times V \mapsto \{0, 1\}$, be a surjection defined by

$$x_{mn} \quad := \quad \begin{cases} 1, & \text{if station } n \text{ scheduled in slot } m, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Also, let $c : E \mapsto \{0, 1\}$ return 1 if i and j are one-hop neighbors, i.e., if $(i, j) \in E$ and $i \neq j$.

Using the aforementioned definitions and assumptions, we can now formulate the BROADCAST SCHEDULING PROBLEM (BSP) on TDMA networks as the following multiobjective optimization problem:

$$\text{Minimize } M$$

$$\text{Maximize } \sum_{i=1}^{M} \sum_{j=1}^{|V|} x_{ij}$$

subject to:

$$\sum_{m=1}^{M} x_{mn} \geq 1, \qquad \forall\, n \in V, \tag{2}$$

$$c_{ij} + x_{mi} + x_{mj} \leq 2, \qquad \forall\, i, j \in V, i \neq j, m = 1, \ldots, M, \tag{3}$$

$$c_{ik}x_{mi} + c_{kj}x_{mj} \leq 1, \qquad \forall\, i, j, k \in V, i \neq j, j \neq k, k \neq i, m = 1, \ldots, M, \tag{4}$$

$$x_{mn} \in \{0, 1\}, \qquad \forall\, n \in V, m = 1, \ldots, M, \tag{5}$$

$$M \in \mathbb{Z}^+. \tag{6}$$

The objective provides a minimum frame length with maximum bandwidth utilization, while constraint (2) ensures that all stations broadcast at least once. Constraints (3) and (4) prevent direct and hidden collisions, respectively. Constraints (5) and (6) define the proper domain of the decision variables.

Suppose that we relax the BSP and only the consider the first objective function. This is referred to as the FRAME LENGTH MINIMIZATION PROBLEM (FLMP) and is given by the following integer program: $\min\{M : (2) - (6)\}$. Clearly any feasible solution to this problem is feasible for BSP. Now, consider a graph $G' = (V, E')$ where V follows from the original communication graph G, but whose edge set is given by $E' = E \cup \{(i, j) : i, j \text{ are two-hop neighbors}\}$. Then using this augmented graph, we can formulate the following theorem due to Butenko et al. [2].

Theorem 1. *The* FRAME LENGTH MINIMIZATION PROBLEM *on $G = (V, E)$ is equivalent to finding an optimal coloring of the vertices of $G'(V, E')$.*

Proof. Recall that in order for a message schedule to be feasible, all stations must broadcast at least once and no collisions occur, either hidden or direct. Notice now that E' contains both one-hop and two-hop neighbors, and in any feasible solution, neither of these can transmit in the same slot. Thus, there is a one-to-one correlation between time slots in G and vertex colors in G'. Hence, a minimum coloring of the vertices of G' provides the minimum required slots needed for a collision free broadcast schedule on G. □

After one has successfully solved the FLMP by solving the corresponding GRAPH COLORING PROBLEM, an optimal frame length M^* is attained. With this, the THROUGHPUT MAXIMIZATION PROBLEM (TMP) given as follows $\max\{\sum_{i=1}^{M^*} \sum_{j=1}^{|V|} x_{ij} : (2) - (6)\}$ can be solved, where M is replaced by M^* in $(2) - (6)$. A direct result of Theorem 1 is that finding an optimal frame length for a general instance of the BSP is $\mathcal{NP}$-hard [11]. The reader is referred to the paper by Butenko et al. [2] for the complete proof. Also, in [8], the TMP was also shown to be $\mathcal{NP}$-hard [8]. Thus it is unlikely that a polynomial algorithm exists for finding an optimal broadcast schedule for an instance of the BSP [11]. It is interesting to note however, that if we ignore constraint (4) which prevents two-hop neighbors from transmitting simultaneously, then the resulting problem is in $\mathcal{P}$, and a polynomial time algorithm is provided in [13].

Due to the computational complexity of the BSP, several heuristics have been applied and appear throughout the literature [2, 3, 6, 28, 31]. In the following section, we highlight several of these methods and examine their effectiveness when applied to large-scale instances.

4. METHODS

In this section, we review many of the heuristics which have been applied to the BSP. We analyze the techniques used and compare their relative performance as reported in [6]. The particular algorithms we examine are as follows:

- Sequential vertex coloring [31];
- Mixed neural-genetic algorithm [27];
- Greedy randomized adaptive search procedures (GRASP) [2, 3];
- A multi-start combinatorial algorithm [6].

We note here that none of the heuristics which we describe in this section attempt to solve the BSP by using the typical multiobjective optimization approach, in which one combines the multiple objectives into one scalar objective whose optimal value is a Pareto optimal solution to the original problem. Instead all of the methods decouple the objectives and handle each independently. This is done because for instances of the BSP, frame length minimization usually takes precedence over the utilization maximization problem [28, 31, 27].

4.1. Sequential Vertex Coloring.

Yeo et al. propose a two-phase approach based on sequential vertex coloring (SVC). The first phase computes an approximate solution for the FLMP. Then using the computed frame length, the TMP is considered in the second phase. Specific details are as follows.

4.1.1. *Frame Length Minimization.*

For this phase, the FRAME LENGTH MINIMIZATION PROBLEM is considered and an approximate solution is computed by solving a graph coloring problem in the augmented graph. A sequential vertex ordering approach is used whereby the stations are first ordered in descending order of the number of one-hop and two-hop neighbors. The first vertex is colored and the list of the other $N-1$ vertices are scanned downward. The remaining vertices are colored with the smallest color which has not already been assigned to one of its one-hop neighboring station. The process is continued until all vertices have been colored.

4.1.2. *Throughput Maximization.*

To solve the TMP in the frame length computed in phase 1, an ordering method of the sequential vertex coloring algorithm is applied. The stations are now ordered in ascending order of the the number of one-hop and two-hop neighbors. The first ordered station is then assigned to any slots in which it can simultaneously broadcast with the previously assigned stations. This process is repeated for every station in the ordered list.

4.2. Mixed Neural-Genetic Algorithm.

As with the coloring heursitic presented described above, Salcedo-Sanz et al. [27] introduced a two-phase heuristic based on combining both Hopfield neural networks [15] and genetic algorithms as in [29]. As with the vertex coloring algorithm, phase one considers the FLMP and phase two attempts to maximize the throughput.

4.2.1. *Frame Length Minimization.*

In order to solve the FRAME LENGTH MINIMIZATION PROBLEM, a discrete-time binary Hopfield neural network (HNN) is used. As described in [27], the HNN can be represented as a graph whose vertices are the neurons (stations) and whose edges represent the direct collisions. The neurons are updated one at a time after a randomized initialization until the system converges. For specific implementation details, the reader should see [27].

4.2.2. *Utilization Maximization.* In this phase, a genetic algorithm [12] is used to maximize the throughput within the frame length that was determined in phase one. Genetic algorithms (GAs) get their names from the biological process which they mimic. Motivated by Darwin's Theory of Natural Selection [7], these algorithms evolve a *population* of solutions, called *individuals*, over several *generations* until the best solution is eventually reached. Each component of an individual is called a *allele*. Individuals in the population mate through a process called *crossover*, and new solutions having traits, i.e. alleles of both parents are produced. In successive generations, only those solutions having the best *fitness* are carried to the next generation in a process which mimics the fundamental principle of natural selection, *survival of the fittest* [12]. Again, the reader should reference [27] for implementation specific information.

4.3. **Greedy Randomized Adaptive Search Procedures (GRASP).** GRASP [9] is a multi-start metaheuristic that has been used with great success to provide solutions for several difficult combinatorial optimization problems [10], including SATISFIABILITY [24], QUADRATIC ASSIGNMENT [21, 23], and most recently the COOPERATIVE COMMUNICATION PROBLEM ON AD-HOC NETWORKS [4, 5].

GRASP is a two-phase procedure which generates solutions through the controlled use of random sampling, greedy selection, and local search. For a given problem Π, let F be the set of feasible solutions for Π. Each solution $X \in F$ is composed of k discrete components $a_1, \ldots, a_k$. GRASP constructs a sequence $\{X\}_i$ of solutions for Π, such that each $X_i \in F$. The algorithm returns the best solution found after all iterations.

4.3.1. *Construction Phase.* The construction phase for the GRASP constructs a solution iteratively from a partial broadcast schedule which is initially empty. The stations are first sorted in descending order of the number of one-hop and two-hop neighbors. Next, a so-called *Restricted Candidate List* (RCL) is created and consists of the stations which may broadcast simultaneously with the stations previously assigned to the current slot. From this RCL a station is randomly chosen and assigned. A new RCL is created and another station is randomly selected. This process continues the RCL is empty, at which time the slot number is incremented and the procedure is repeated recursively for the subgraph induced by the set of all vertices whose corresponding stations have not yet been assigned to a time slot.

4.3.2. *Local Search.* The local search phase used is a swap-based procedure which is adapted from a similar method for graph coloring implemented by Laguna and Martí in [20]. First, the two slots with the fewest number of scheduled transmissions are combined and the total number of slots is now given as $k = m - 1$, where m is the frame length of the schedule computed in the construction phase. Denote the new broadcast schedule as $\{x_{m',n}, m' = 1, \ldots, k, n = 1, \ldots, N\}$. Now, let the function $f(x) = \sum_{i=1}^{k} E(m'_i)$, where $E(m'_i)$ is the set of collisions in slot m'_i. $f(x)$ is then minimized by the application of a local search procedure as follows.

A colliding station in the combined slot is chosen randomly and every attempt is made to swap this station with another from the remaining $k - 1$ slots. After a swap is made, $f(x)$ is re-evaluated. If $f(x)$ has a lower value than before the swap, the swap is kept and the process repeated with the remaining colliding stations. If after every attempt to swap a colliding station the result is unimproved, a new colliding station is chosen and the swap routine is attempted. This continues until either a successful swap is made or for some specified number of iterations. If a solution is improved such that $f(x) = 0$, then the frame length has been successfully decreased by one slot. The value of k is then decremented and the process is repeated.If the procedure ends with $f(x) > 0$, then no improved solution was found.

4.4. **Multi-start Combinatorial Algorithm.** To our knowledge, the most recent heuristic for the BSP is a hybrid multi-start method by Commander and Pardalos [6]. This heuristic combines a graph coloring heuristic with a randomized local search to provide high-quality solutions for large-scale instances on the problem. As with the previously described method, this heuristic is also a two-phase approach. The reader should see [6] for pseudo-code and other implementation specific details.

4.4.1. *Frame length minimization.* First a greedy randomized construction heuristic was used to determine the value for M. As a result of Theorem 1, the method is based on the construction phase of the Greedy Randomized Adaptive Search Procedure (GRASP) [26] for coloring sparse graphs proposed by Laguna and Martí in [20]. This particular method was chosen because it is able to quickly provide excellent solutions for the frame length. That being said, any other coloring heuristic would provide a value for M such as the Sequential Vertex Coloring method described above. However, the randomized approach of the selected method allows the search space to be more thoroughly investigated. This is due to the fact that different optimal colorings will yield different solutions in the second phase.

4.4.2. *Throughput maximization.* the solution from the first phase will not provide an optimal throughput in general, because each station will only be scheduled to transmit once in the frame. Therefore, a randomized local improvement method is used to schedule each station as many times as possible in the frame. This method locally optimizes each slot by considering the set of nodes which may transmit with the currently scheduled slot. A node from this set is randomly selected and the process repeats until no other stations may broadcast in the current slot. The next slot is then considered and the process is repeated until the solution is locally optimal.

4.5. **Computational Effectiveness.** In [6], the authors performed an extensive computational experiment comparing the effectiveness of the aforementioned heuristics. They tested all of the algorithms on a common platform and reported solutions for 63 instances ranging from 15 to 100 stations with varying densities. In addition, they implemented the integer programming model from Section 3 using the Xpress-MP™ optimization suite from Dash Optimization [17]. Xpress-MP contains an implementation of the simplex method [14], and uses a branch and bound algorithm [30] together with advanced cutting-plane techniques [16, 22].

For each instance tested, the combinatorial algorithm of [6] is superior to the other heuristics mentioned. For all 63 instances tested, the method found solutions at least as good as any of the other algorithms from the literature for all of the networks, outperforming them on 56 cases. The performance of the GRASP [2] and the Mixed Neural-Genetic Algorithm [27] were comparable, with GRASP performing slightly better on average. The weakest of the methods was the Sequential Vertex Coloring [31] algorithm. For specific numerical results, see [6].

5. CONCLUSION

In this article, we introduced the BROADCAST SCHEDULING PROBLEM on TDMA networks. The BSP is an important problem that occurs in wireless mesh networks regarding efficiently scheduling collision free broadcasts for the network stations. We formally defined the problem, examined the computational complexity, and discussed several algorithms which have been applied to the BSP, all with competitive results.

We conclude with a few words on possible directions of future research. In addition to the ones described, other metaheuristics could be considered and approximation algorithms developed. Also, a heuristic exploration of cutting plane algorithms on the IP formulation would be an interesting alternative. Another alternative would be to consider instances of the problem in which the stations are part of a mobile ad-hoc network. In this case, the topology of the network would change as the stations change position. This could potentially cause significant difficulties in determining the evolving sets of one-hop and two-hop neighbors. There is no doubt that as technology advances and research on ad-hoc networks increases, so too will applications of the BSP which will require advanced solution techniques [25].

6. CROSS REFERENCES

See also: **Frequency assignment problem; Genetic algorithms; Greedy randomized adaptive search procedures, GRASP; Graph coloring, GC; Multi-objective integer linear programming, MOILP; Optimization problems in unit-disk graphs; Simulated annealing, SA.**

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