"A Model of Entrepreneurs, Venture Capital, And Endogenous Growth"

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This paper combines existing models of Schumpetarian endogenous growth and financial sector with Knightian entrepreneurs to create a theoretical model on how entrepreneurs and venture capitalists affect the rate of economic growth.

Introduction

In the 1990's the United States experienced its longest period of economic growth in its history. The technological revolution that spearheaded the growth was funded (for the most part) by financial intermediaries known as venture capital firms. The hypothesis is that investments by venture capital firms cause innovation, which translates into economic growth. Thus far only two papers have attempted to empirically explain the relationship between economic growth and venture capital. Neither paper explicitly uses an existing theoretical model of growth as the basis for their analysis.

The first paper by Hellman and Puri (2000) uses a random sample of firms in San Jose, California. They found that venture capital firms primarily invest in companies that bring a new product to market rather than those that replicate or imitate existing products. Of the innovators, they are more likely to bring a product to market and to bring a product to market quicker if they have received backing from a venture capital firm. While the authors conclude that venture capital results in innovation, the model suffers from problems of causality. They do not adjust for the possibility that "more innovative firms select venture capital for financing, rather than venture capital causing firms to be more innovative" (Gompers and Lerner, 2001b p. 165). The second paper takes great pains to address this issue with a completely different data set. However, even when adjusting for causality, Kortum and Lerner (2000) find similar results. They exploit an exogenous policy shift in 1979 to compare the number of patents filed to the amount of venture capital funding. At each level of specification they conclude that venture capital funding is highly correlated with growth. The difficulty with this model is the imperfection of the patents variable as it applies to innovation. Merely because a patent is filed does not mean it results in a new product. Therefore their most conservative estimate that a

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venture capital dollar is seven times more effective in spurring innovation than other R&D sources is highly suspicious.

Neither article refers to a growth theory as the foundation for their analysis yet the implication is that venture capital spurs innovation via entrepreneurs as in an Schumpeterian endogenous growth model. The theoretical underpinnings of the hypothesis were last studied in 1993 when King and Levine published an article on the finance sector and economic growth specifically citing the venture capital industry as an example. Since 1993 the venture capital industry has grown from $4.8 billion to $81.4 billion in 2000 (Gompers and Lerner 2001a, p. 73). Given the increased size and relative importance of the industry and albeit limited empirics, we feel it is important to reevaluate the King and Levine model. This paper revisits King and Levine's original model using product rather than process innovations as suggested by the papers cited above. We avoid comment on the relationship between the finance sector and economic development and instead focus our attention on how venture capital funding will affect the rate of innovation and therefore the rate of economic growth.

To do this we begin our paper with a review of the Schumpetarian growth literature as modeled by Grossman and Helpman (1991). The second part of our paper restates the finance component of the King and Levine model for use with product--not process innovations. The third section establishes the equilibrium conditions needed for the fourth section. There we use comparative statics and general equilibrium analysis to show the value of venture capital firms and entrepreneurs in terms of innovation. In the final section we offer our conclusions and recommendations for future research.

1. The Basic Growth Model

1.1. Consumers

The guiding force behind this model, as with all Schumpetarian growth models, is the consumer utility function. In the basic model we assume the economy is made up of individuals endowed with *N* units of labor. We further assume that population does not grow or at least that there is a maximum of *N* units of labor available each time period. Each individual has an identical utility function as shown in (1). The utility function used in this model is intertemporal and identical to that used in Grossman and Helpman (1991).

(1) $U = \int_0^\infty e^{-\rho t} \ln[D(t)]dt$

The variable ρ represents the consumer's intertemporal discount rate and has a value greater than zero. Consumers choose between consumption and savings in such a way as to maximize their lifetime utility.

The variable *ln[D(t)]* is the consumer subutility function and represents consumption of goods by each consumer over time. The subutility is the sum of the log of all goods consumed at time t from industry j. The subscript j is an index of industries such that $j \in [0,1]$.

$$(2)\ \ln D(t) = \int_0^1 \log[\sum_m q_m(j)x_{mt}(j)]dj$$

Quality levels of products are designated by the subscript m. Most Schumpetarian growth literature uses λ^m to designate quality steps where λ represents the quality multiplier from one product generation to the next. Here, as in Grossman and Helpman, it is appropriate to think of $q_m(j) = \lambda^m$. Note that quality increments are not indexed by time. Advances in quality occur randomly with a Poisson distribution.

Within each industry j, differing qualities may substitute imperfectly. However once we adjust for quality differences, each product within the same industry substitute perfectly. The variable $q_m(j)$ is the means by which we are able to compare products in the same industry but different quality levels i.e., $q_m(j) = \lambda q_{m-1}(j)$. The lowest quality product available ($m = 0$) at time $t = 0$ is indexed to a value of 1 so that $q_0(j) = 1$. We will return to the changes in quality later in the paper.

As Grossman and Helpman point out, an alternative interpretation to the instantaneous household utility function is to treat $D(t)$ as the final output of a single product as a result of intermediate inputs. This interpretation is similar to that presented by King and Levine however the former interpretation will be used in this paper.

Consumers may choose to either spend or save their income. We assume that companies produce the exact amount of goods demanded. Thus, the product of price and quantity will equal expenditures for each time period, $E(t) = p_{mt}(j)x_{mt}(j)$. Transforming this into a different form we obtain,

$$(3)\ x_{mt}(j) = \frac{E(t)}{p_{mt}(j)}$$

1.2. Producers

Goods consumed by individuals are produced by j different industries. Each jth industry represents one commodity being produced. In setting the framework for the more advanced model we assume that at time $t = 0$, every industry is perfectly competitive. Profits for every active firm are zero and price is equal to marginal cost. In the tradition of Grossman and Helpman, we assume that the only "primary" factor input is labor and thus wage (w) is the marginal cost of production so $p_{mt} = w$. We further assume that the wage rate is constant across all industries and time periods.

As quality increments change it is feasible that production functions and production technologies will change, either increasing or decreasing costs and creating competition and product differentiation. Here, we assume that production costs remain unchanged for all industries and all qualities. Given the setup, the marginal productivity of labor is equal to one.

1.3. New Products and Firms

So far, we have introduced the concept of changes in quality but have yet to discuss the incentives for these changes. Standard consumer utility theory assumes that preferences are monotonic--that more consumption is preferred to less. This model does not allow for changes in income so the only way for consumers to increase their level of utility is to purchase better goods. Thus the demand for innovation is implicit within the model's consumer utility function. For better goods to be produced, research and development (R&D) must occur. The supply of innovative activity and R&D is slightly more complex than the demand side so we will first discuss innovation separate from the model before adding this component.

1.3.1 Innovation in Isolation: One Quality Improvement

Assume that there are two profit maximizing firms, a leader and follower, in industry *i*. The leader has access to the technology needed to increase the quality of good *i* one step up the quality ladder to $q_1(i)$. The other firm, called the follower, may only produce the product at the level $q_0(i)$. Recall that the difference between the two qualities is reflected by λ^t. The competition between the leader and follower in a Bertrand setting results in a non-cooperative game such that each firm prices the good at the marginal cost of production, wage.[1] Plugging this price into equation (3) and the consumer utility function we obtain:

(4) $p = \lambda w$

The intuition behind the price level is simple. If the state of the art product were priced above λw, the price would be higher than demand because the price adjustment given the quality is overstated resulting in no sales of the new product. A price set lower than λw would mean excess demand and would violate the profit maximization assumption. Therefore the leader prices the new product exactly at λw and takes market share from the follower.[2] Formally, the consumer is indifferent between the two products as they are equivalent after the quality adjustment. For simplicity sake we assume that consumers will always desire the state of the art product in this instance.

Since the technology is quality enhancing, λ is strictly greater than 1. Plugging equation (4) into equation (3), we arrive at the new profitability level below,

(5) $\pi = \mathrm{E}\left[1 - \frac{1}{\lambda}\right] > 0$.

The strictly positive profit level represents the monopoly power of new technologies. As with all Schumpeterian growth models, it is the temporary monopoly profits that lead firms to adopt technology that will improve the quality of the products produced.

[1] Recall from section 1.2 we assumed new technology would not affect the production function of an existing product line. Therefore marginal cost remains at the same amount, *w*. See Grossman and Helpman, p. 91.

[2] Grossman and Helpman (p. 90) provides a graph that may aid in the intuition.

Therefore innovation is not good will on behalf of producers for consumers but is a direct result of the profit maximization assumption.

1.3.2 Innovation in Isolation: Multiple Quality Improvements

Now assume that the market leader is endowed with technology that will improve quality to λ^2. The prevailing market price then becomes $\lambda^2 w$ and thus profits of

$$(6)\quad \pi = \mathrm{E}\left[1-\frac{1}{\lambda^2}\right] > 0\,.$$

As Grossman and Helpman point out, had the firm not applied the latest technology they would have earned profits as shown in (5). With the second quality step they receive the monopoly profits from the second quality step but lose the profits from the first quality improvement. The real value of the latest technology is therefore (6) less (5), or

$$(7)\quad \pi = \frac{\mathrm{E}}{\lambda}\left[1-\frac{1}{\lambda}\right].$$

It is clear the profits including both the first and second quality steps are strictly less than what they would have been had the company chosen not to use the latest technology. Further, the company now owes dividends to two cohorts of investors, one for each quality step initiated. While the profits after applying the second innovation are higher than just with the first, they are not high enough to pay two sets of investors. Thus the firm would not choose to innovate beyond one step due to the profit maximization assumption and its debt obligation to its investors. Note well that this equation would be the same for every firm, regardless of industry.

1.3.3 Model with Leader-Follower Component

In the model where technology is the result of R&D and not an endowment the outcome is the same. The leader will not choose to invest in R&D for the purposes of innovation because it will cause profits to be lower than if they stayed with the current state of the art technology.

We assume now that rather than perfectly competitive industries, each industry has a leader which produces at the marginal cost of wage as the result of a Bertrand non-cooperative game. The leaders for each industry are using state-of-the-art technology and are monopolists in their respective industries. Since we are still at $t = 0$ where $\lambda = 1$, profits are still zero but there is no incentive for the existing firms to innovate beyond the current quality.

With the basic model established, we recall that consumers still demand quality improvements even though the existing firms have no incentive to provide them. To meet the excess demand and enjoy temporary monopoly profits, companies outside the current set of leaders must innovate and begin producing the next quality level of products. The entrepreneur's incentive to innovate is the potential to earn profits as described in (6).

2. Entrepreneurs and the Finance Sector

In this section we follow the model established by King and Levine who formulated their model by combining Frank Knight's insights on entrepreneurs and Joseph Schumpeter's opinions on the finance sector. We avoid the details on their (often contradictory) opinions here but a good discussion on Schumpeter and Knight may be found in either Van Praag (1999) or Brouwer (2000). The model presented below follows the structure of a principal-agent model. Thus we use the typical information economics method and refer to the financial intermediary as 'she' and to the entrepreneur as 'he'. Unlike King and Levine we specify the type of financial intermediary as the venture capitalist.[3] It is possible that other forms of financial intermediary may perform the roles as outlined in this paper however we choose to focus our attention on the venture capitalist given the growth of the venture capital industry in the United States in the past two decades. We use the term entrepreneur and innovator interchangeably throughout the rest of the paper.

2.1. Financial Intermediaries and the Entrepreneur

Entrepreneurs have a unique set of personality traits that distinguish them from regular individuals. Both Schumpeter and Knight agree that there are few people in the economy that have the traits to be a successful entrepreneur. Here, we let α represent the probability that any one individual has the characteristics to be an entrepreneur. We also assume that the individual does not know he is or cannot convincingly represent himself as having those traits. At a cost of wf the financial intermediary is able to identify whether the individual has the characteristics of a successful entrepreneur. Where f denotes the number of hours needed to rate the entrepreneur. Without the rating the entrepreneur would be unable to obtain funding for his project.

In equilibrium the maximum value a venture capitalist is willing to invest on a rating is the expected residual value of the individual, αq. Consider the residual value as the value of an entrepreneurial venture once startup costs and repayment of financing debt is taken into account. Therefore the cost wf must be offset by an equally valuable payoff as below.

(8) $\alpha q = wf$

Now that the entrepreneur is rated he may seek funding from a third party to start a new business. This funding is used to pay for R&D that will hopefully lead to innovation. Assume that R&D uses e units of labor both by the entrepreneur and by their employees for a total cost of we. The entrepreneur obtains initial funding from a venture capital firm equal to the cost of initial R&D[4]. The venture capitalist has the money to loan because

[3] King and Levine refer to the venture capitalist as a suggestion for their financial intermediary but choose not to "specify the precise form of contracts and institutions that provide these services." (1993, p 521)

[4] The model places a limitation on where an entrepreneur may obtain start up capital. Technically a loan could be taken from a bank or other lender where the cost is we plus a fixed rate of interest. However,

she has pooled the assets of small individual consumers for investment. In return for her investment, the venture capitalist takes τ percent of the stock market value of the new firm. Consider the stock market value *v* to be profits as described in equation (6). This is an obvious relationship since the firm must pay dividends to their shareholders equal to the amount of profit the company earns. The structure of the model is such that each equivalent quality step results in the same amount of profit regardless of industry. Therefore there is no incentive by the venture capitalist to favor one industry over the other. The benefit of this structure is that she is now able to diversify risk across all industries, which helps to ensure a reasonable rate of return on her investment.

At this point we clarify that merely because an individual is an entrepreneur does not mean that they are going to be successful. Successful innovation has a probability ϕ which has a Poisson distribution. It may be helpful to think of ϕ as the joint probability of α and some arbitrary probability of success. The repayment and sunk costs are subtracted from the expected discounted profits so that the residual value of the entrepreneur is,

(9) $q = (1-\tau)\phi\rho v - we$.

Returning to the residual value of the entrepreneur we can see that expected discounted profits, less repayments to the venture capitalist, less sunk startup costs is the maximum value the venture capitalist may spend on rating an individual with probability α of being an entrepreneur. In equilibrium, the cost must equal the benefit so using equations (8) and (9) we see that,

(10) $\phi\rho v = a(\tau)w$,

where $a(\tau) = (f/\alpha + e)/(1 - \tau)$. Recall that f/α and e represent the labor expenditures for rating and initial investment, respectively. Once a firm innovates, the equation may be rewritten as $v = a(\tau)w$.

So far we have seen three of the four roles a financial sector fills as described by Schumpeter and outlined by King and Levine. First we see that they rate an individual to see if he has the characteristics to be an entrepreneur. Second, they act as primary investor in new firms created by entrepreneurs they identify. They coordinate investment resources by pooling funds saved by consumers as part of their maximization routine. The structure of the model makes the financial intermediaries role as diversifier of risk easy. Each role is performed by the venture capitalist, a financial intermediary that is integral part of the financial system. The final role of the finance sector is to effectively value a company so that investors may make informed and rational decisions on where to invest their money. This is done via the stock market.

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given the randomness of innovation this is a risky proposition. If the entrepreneur fails to innovate, they still owe money to the bank. In this instance, the entrepreneur pays out a percentage of profits only if they are successful.

2.2. Stock Market

When an entrepreneur successfully innovates, the profits of the previous leader disappear. The ever-present possibility that profits may disappear must be included in any estimation of stock market value. To further complicate things, several entrepreneurs may be competing to become the next leader in a respective industry. As did King and Levine, we assume that the probability of the next innovation being discovered is proportional to the number of entrepreneurs competing in the industry. If ϕ represents the probability that one entrepreneur will successfully innovate then $\xi\phi = \Phi$ is the probability that the next innovation will occur by one of ξ competing entrepreneurs.

For a consumer to continue to be indifferent between holding a stock through a change in time δt, the future stock market value must be equal to the present stock market value. If the future value of a stock were high, consumer savings would flow from venture capital to equities of established firms. Likewise, if the future value of stocks were low, consumers would sell stocks and invest in venture capital funds. When taking into account the possibility for an existing firm to be replaced by a successful entrepreneur the expected future value of the current leader is $(1-\Phi)pt + \delta tvt + \delta t$. In the current time period the value of the firm is v_t less the dividends paid to investors. Setting the future and current stock market value equal we get an equilibrium of,

(11) $(1-\Phi)\rho t + \delta tvt + \delta t = v_t - d_t$.

Given this equilibrium and the equal expected profits for every industry, each individual consumer finds it useful to diversify his or her risks across all industries and each stage of development—stocks from established firms or dividends from a successful entrepreneur.

3. Equilibrium Assumptions

In constructing the model we have already established three equilibrium conditions. Recall from the growth section that we assumed $E(t) = p_{mt}(j)x_{mt}(j)$ so that production supply equals consumer demand. In the financial section we had several equilibrium assumptions. The first is (10) where cost to fund is equal to the benefit of funding. The expected net gain is zero. The other condition is the assumption that rational stock market valuations take into account the possibility that those profits would be lost if an entrepreneur is successful in innovating to the next level of the quality ladder. To analyze the model in a general equilibrium setting with comparative statics, further assumptions must be made. This section will complete the set of equilibriums for the model. The next section will use them to evaluate the model.

Using equation (3) and the assertion that consumption is equal to expenditures we establish the first additional equilibrium condition. By taking logs and differentiating the equation with respect to time, we see that the growth rate is the same for consumption and expenditures. From Dinopoulos (1996) and Grossman and Helpman we obtain (12) and see that $r = \rho$.

$$(12)\ \frac{\dot{E}}{E} = \frac{\dot{C}}{C} = r - \rho = 0$$

Our next equilibrium assumption relates to consumer choice between investment options. Once a firm has successfully innovated the equilibrium condition requires dividends paid to consumers must be the same as the rate of return on comparable assets. If this were not true, money to be invested by consumers would flow to the asset that promised the larger rate of return. From this we can see that $\pi + \dot{v} - \phi v = rv$. In equilibrium, we know that $\partial v / \partial t = 0$. So the equation may be rewritten as

$$(13)\ \pi - \phi v = rv.$$

Our final equilibrium condition is derived from the limits of the labor resource in the model. The production function allowed for labor as the only factor input and is limited by the total population and the labor units used by the financial sector. We assumed that there are N units of labor in the economy. We further assume that labor is used efficiently so that the market is in equilibrium, therefore

$$(14)\ n + a(0)\xi = N$$

The first part of the sum represents all the employees of established firms in all j industries. The second part is the combined labor force from rating and start-up costs multiplied by the number of entrepreneurs in the economy.

4. Equilibrium and Comparative Statics

In this section we calculate the equilibrium condition based on the supply and demand for labor in the finance sector. We begin with the labor supply and then estimate the demand side equation. We calculate the equilibrium condition by setting supply equal to demand. The final part of this section is used for comparative statics.

We begin by combining equations (3) and (4) and obtain a new equation for x. Using wage as the numeraire and recalling that the marginal productivity of labor in this economy is equal to one we see that $n = E/\lambda$. Plugging this equation into (14) and restating ξ as a ratio between ϕ and Φ we obtain the first part of the equilibrium equation:

$$(15)\ a(\tau) = \left[N - \frac{E}{\lambda^m} \right] \frac{\phi}{\Phi}.$$

The next half of the equilibrium is slightly more difficult. We restate (13) in terms of v and then recall the equilibrium condition $v = a(\tau)$. We set these two equations equal to one another and make further substitutions. From (12) we substitute ρ for r and include the definition for profits.

$$(16)\quad a(\tau) = \frac{E\left[\dfrac{\lambda^m - 1}{\lambda^m}\right]}{\Phi + \rho}.$$

Setting (15) and (16) equal to one another and expressing the equilibrium result in terms of Φ we obtain (17). From this equation we can see that an increase in population would result in an increased amount of innovation. This is apparent given the increase in population size would also result in an increase in the number of individuals with entrepreneurial characteristics. Additional entrepreneurs performing innovative research will result in a higher probability of successful innovation. Similarly, we can see that an increase in ρ will lead to increased probability of innovation. This is true because as consumers save more, given the indifference between types of investment, there is more money to fund entrepreneurial endeavors. Increased spending by consumers yields the opposite result. More money spent rather than invested leads to decreased probability of innovation. The remaining exogenous variable is λ^m. Here the outcome is ambiguous. An increase in m affects both the denominator and numerator. Depending on the size of ρ, the outcome could be an increase in innovation or a decrease.

$$(17)\quad \Phi = \frac{\phi\rho[N - E/\lambda^m]}{E((\lambda^m - 1)/\lambda^m) - \phi N - \phi E/\lambda^m}$$

5. Conclusion

In this paper we have constructed a model based on the works of Grossman and Helpman and King and Levine. Within the general equilibrium setting we have shown how the entrepreneur and financial intermediary, in tandem, create and improve innovation and economic growth. Better financial services improve efficiencies of innovation. That innovation leads to increased economic growth as defined by Schumpetarian growth theory.

The model presented may be improved in several ways. First it could be modeled with an efficiency wage rather than one wage rate across all industries. In addition it would be interesting to model the effects of an individual choosing to become an entrepreneur over a production job based on incentives rather than natural attributes. Second, removing the fixed effects of zero population growth would make the model more dynamic. Third, the assumption on perfect foresight of entrepreneurial success is difficult to justify. The speculation by venture capital firms in the 1990's can be pointed to as a contradiction. Further, the study of shakeouts and the evidence from that research should lead one to question this simplifying assumption. Additional research in the overshooting of investment by venture capital firms in high growth industries would be beneficial both in parallel and as an addition to this model. Ultimately, as with any theory, the model should be tested empirically with necessary adjustments for causality.

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