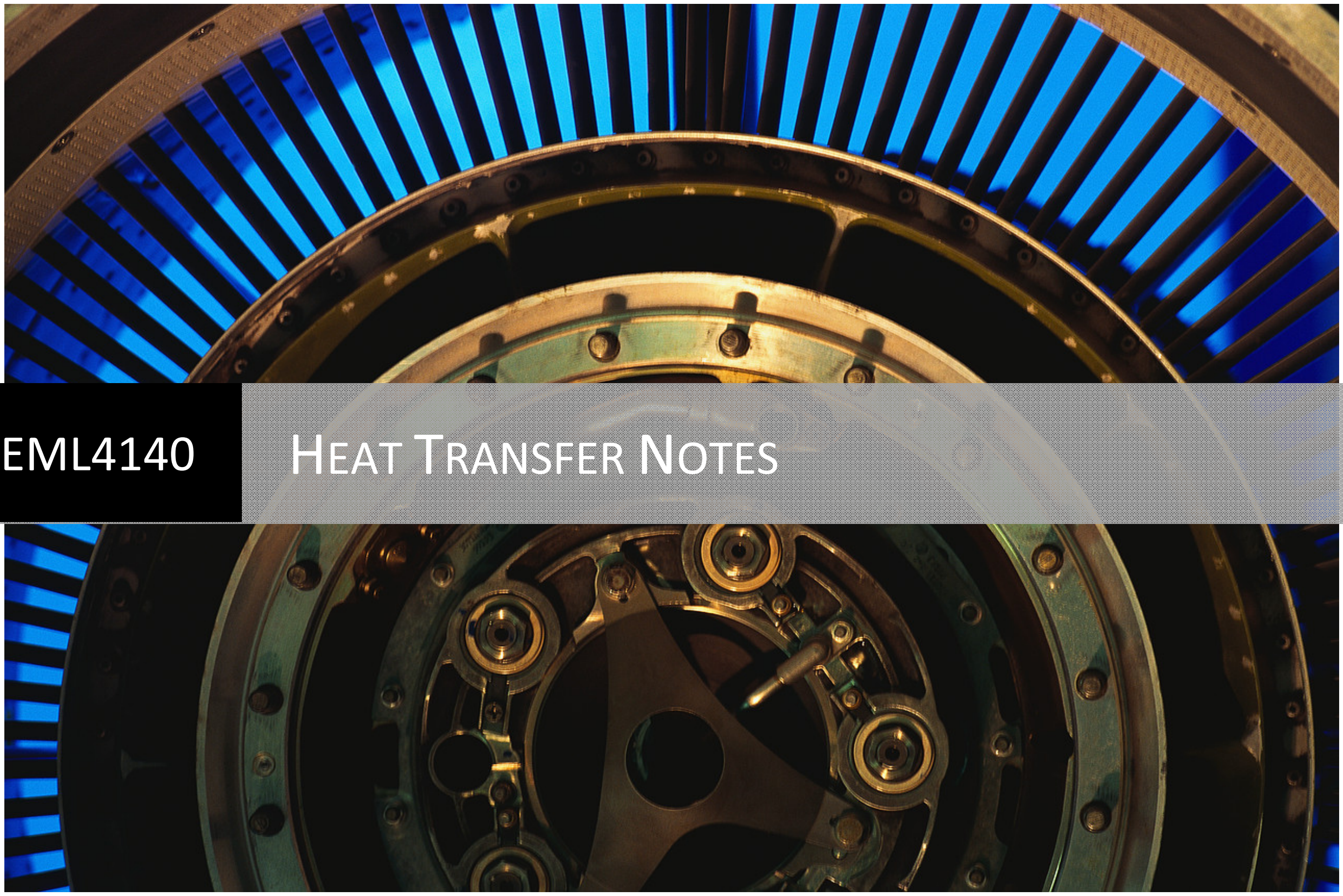
University of Florida EML4140 §5964 Spring 2008



EML4140

Heat Transfer Notes

Prof. Subrata Roy

NEB 100, MWF5
11:45 - 12:35

http://www.mae.ufl.edu/courses/spring2008/eml4140

EML 4140 Heat Transfer – Spring 2008

Instructor: Dr. Subrata Roy
Associate Professor, Department of Mechanical and Aerospace Engineering
Office: 336 MAE-B
Phone: 392-9823: E-mail: roy@ufl.edu

Office Hours:
M W 1:30 to 2:30 PM
Generally briefly available after class, and by appointment at other times.
No assistance with homework after office hours on the day that homework is due.
No homework assistance by e-mail.

Teaching Assistants:
Ankush Bhatia (ankushbhatia@ufl.edu),
Patricia Soupy Dalyander (salex@ufl.edu)
Office hours: M T W R F 9-11 AM
Room 126 MAE-C

Required Text: Fundamentals of Heat and Mass Transfer,
F.P. Incropera and D.P. DeWitt,
6th Edition (John Wiley & Sons, 2007)

Course Website: http://www.mae.ufl.edu/courses/spring2008/eml4140

Prereq: CGS 2425, EML 3100, EGM 4313

Meeting Time & Place: M W F, 5th Period in NEB Rm. 100

Course Objectives and Outcomes: This course provides a intermediate level coverage of thermal transport processes via conduction, convection, and radiation heat transfer. This course stresses fundamental engineering science principles applied to engineering thermal analysis. Students will learn to apply the conservation of energy to control volumes and express the conservation of energy through mathematical formulations, including both steady state and transient analyses, with emphasis on the fundamental physics and underlying mathematics associated with heat transfer. Upon completion of this course, students are expected to understand basic heat transfer problem formulation and solution techniques, coupled with a strong foundation and appreciation for the physics of heat transfer.

Program Objectives and Outcomes: EML 4140 supports several educational objectives enumerated in the Mission Statement of the Department of Mechanical and Aerospace Engineering. Specific objectives supported by this course include: 1) To understand and perform engineering analyses in the area of thermal systems, 2) To comprehend quantitative, analytical, and experimental methods, 3) To acquire the knowledge base, confidence, and mental discipline for self-education and a lifetime of learning.

Catalog Description: Credits: 3; Steady state and transient analysis of conduction and radiation heat transfer in stationary media. Heat transfer in fluid systems, including forced and free convection.

Contribution of course to meeting the ABET professional component:

4A. EML 4140 supports several program outcomes enumerated in the Mission Statement of the Department of Mechanical and Aerospace Engineering. Specific ME program outcomes supported by this course include:

(1) Using knowledge of chemistry and calculus based physics with depth in at least one of them (ME Program Outcome M1);

(2) Using knowledge of advanced mathematics through multivariate calculus and differential equations (ME Program Outcome M2);

(3) Being able to work professionally in the thermal systems area (ME Program Outcome M4).

4B. Mathematical Sciences (15%), Physical Sciences (15%), Engineering Sciences (70%).

Relationship of course to ABET program outcomes:

This course achieves the following ABET outcomes. Note that the outcome number corresponds to the respective ABET outcomes (**a**) through (**k**).

(a) Apply knowledge of mathematics, science, and engineering: Outcome (a), method of assessment is specially selected problems on three exams and homework.

(e) Identify, formulate, and solve engineering problems: Outcome (e), method of assessment is specially selected problems on three exams and homework.

(i) Recognize the need for, and engage in life long learning: Outcome (i), method of assessment is several critiques of research papers in the field of Heat Transfer and critiquing professional seminars in Heat Transfer.

(k) Use the techniques, skills, and modern engineering tools necessary for engineering practice: Outcome (k), method of assessment is specially selected problems on three exams and homework.

Course Outline:

First unit: 5 Weeks

1. Introduction to heat transfer and rate laws
2. Fourier's Law and heat diffusion equation
3. Rate equations and conservation of energy
4. Introduction to conduction
5. One-dimensional steady-state conduction (planar and cylindrical)
6. Contact resistance and thermal circuits
7. Heat transfer from extended surfaces
8. Two-dimensional steady state heat transfer: Finite difference method
9. Energy Balance method for nodal equations and boundary nodes
10. Transient conduction, lumped capacitance method
11. Transient conduction, exact solutions and Heisler Charts

Reading material:

1. Chapters 1 and 2
2. Chapter 3 (omit 3.2)
3. Chapter 4 (omit 4.2, 4.3, 4.5)
4. Chapter 5 (omit 5.8, 5.9)

Second unit: 5 Weeks

1. Introduction to convective transport processes
2. Introduction to boundary layers
3. Convective transport equations in differential form
4. Dimensionless variables and Reynolds analogy
5. Effects of turbulence
6. Introduction to external flow heat transfer
7. External flow heat transfer correlations
8. Introduction to internal flow heat transfer
9. Internal flow heat transfer coefficient and correlations
10. Introduction to natural convection
11. Introduction to phase change heat transfer

Reading material:

1. Chapter 6 (omit 6.7 and 6.8)
2. Chapter 7 (omit 7.5 through 7.8)
3. Chapter 8 (omit 8.6 through 8.9)
4. Chapter 9 (9.1 - 9.3, 9.5, 9.6.1)
5. Chapter 10 (10.1 - 10.3, 10.6)

Third unit: 4 Weeks

1. Introduction to radiation heat transfer exchange
2. Geometry, radiation intensity, emissive power
3. Irradiation and radiosity
4. Blackbody radiation exchange
5. Band emission

6. Emissivity, reflectivity, absorptivity, transmissivity
7. Kirchoff's Laws
8. Radiation view factors
9. Net radiation exchange among surfaces
10. Black body surfaces
11. Gray-Diffuse surfaces

Reading material:

1. Chapter 12
2. Chapter 13 (omit 13.4 and 13.5)

Homework Schedule: (due in one week at beginning of the class lecture)

Pretest (not included in the final grade)

1. January 16 – 1.11, 1.18, 1.31, 1.39, 1.46, 1.68 (due Jan 23)
2. January 23 – 2.1, 2.2, 2.4, 2.11, 2.26, 3.9, 3.15 (due Jan 30)
3. January 30 – 3.27, 3.41, 3.52, 3.79, 3.103, 3.112 (due Feb 6)
4. February 6 – 3.130, 3.134, 4.32, 4.38, 4.45, 4.51 (due Feb 13)
   Not for submission problems (5.5, 5.8, 5.11, 5.24, 5.40)
5. February 20 – TBA (due Feb 27)
6. February 27 – TBA (due Mar 5)
7. March 5 – TBA (due Mar 11)
8. March 19 – TBA (due Mar 26)
   Not for submission problems (TBA)
9. April 2 – TBA (due Apr 9)
10. April 9 – TBA (due Apr 16)
    Not for submission problems (TBA)

Answers to End-of-Chapter Problems

Selected answers

Examination Schedule:

1. Exam I on Monday, February 11 (Summary)
2. Exam II on Monday, March 24 (Summary)
3. Exam III on Monday, April 21 (Summary)
4. **Final Exam on Wednesday, April 30 from 5:30 PM to 7:30 PM (Fall'07 Final Grade Summary)**
5. All exams are closed book. For the first 3 exams, you may bring in two sheets of paper (8.5" x 11.5") with whatever notes you want on both the front and back. On the final exam, you may bring six sheets of paper with notes on the front and back. (Sample problems are highlighted in red in HWs above. Also study any conceptual questions/graphs. This is just a non-binding guide line.)
   You should bring a calculator for all exams.

Course Grading:

1. Grading basis:

Homework	10%	
3 1-hour Exams	60%	(20% each)
Project	10%	
Final Exam	20%	
Total	100%	

2. Homework: Ten homework assignments total. Homework is due at beginning of the lecture on the assigned due date. Randomly selected problem(s) will be graded. Final homework grade equals the lesser of 100% or (Σ HW_i)/9, for i =1 to 10. **Show all work, clearly mark answers, and be neat.**

3. Group Project: A list of challenge problems will be assigned in the 11th week. A group may have between 3-5 students from the class. A typed final project report is due at the beginning of the last class lecture. Projects will be evaluated as a group task. The following sections are expected in the final report: An *Abstract* describing the summary of the report, an *Introduction* of the project problem, a description of the *Approach*, all *Design Calculations* including figures and photographs, a section describing *Concluding Remarks*, all *References* including internet websites, and a paragraph describing the *Learning Process* from this project. The report should not exceed 8 printed pages.

4. Final Exam will be comprehensive.

5. Grading Scale: The standard deviation of your four test scores may be added to your lowest test score. Alternatively, I may give you a chance to redo tests (except the final) for earning back a percentage of what you missed. This will be done at my discretion. A 10-point scale (i.e. A>90%, B>80%, etc.) will be used as a baseline for final grades. An additional curve may be applied, as determined by the overall final grade distribution of the class. Grades of B+, C+, etc. will be determined at my discretion and are not based on a 5-point scale (e.g. 85%).

Class Policies:

1. Regular class attendance is expected and encouraged. Each student is responsible for all of the material presented in class and in the reading assignments. Exams will emphasize treatment of material covered in lectures.

2. All homework assignments and projects are to be turned in at the beginning of the designated class period. In general, no late assignments will be accepted or makeup exams given. Exceptions will be made for a valid excuse consistent with University Policy. Exceptions may also be made if deemed appropriate, but please contact me ahead of time.

3. SOME collaboration is allowable on homework, but each student is responsible for

performing the bulk of his or her own homework assignment. The copying of solutions from the Solutions Manual (or copies from) is considered cheating, and is not allowed.

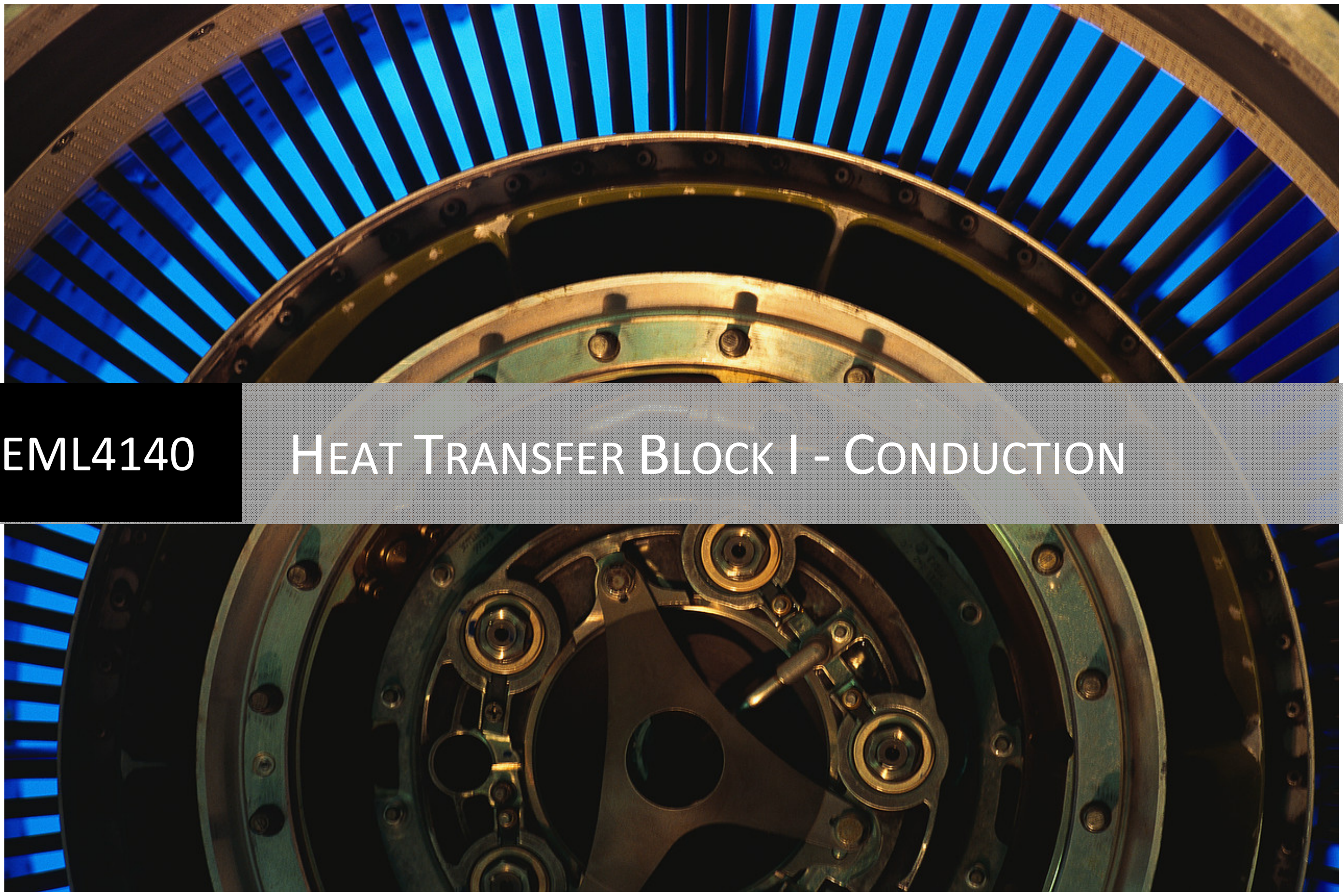
4. **NO collaboration is allowed on exams**.

5. Honesty Policy – All students admitted to the University of Florida have signed a statement of academic honesty committing themselves to be honest in all academic work and understanding that failure to comply with this commitment will result in disciplinary action. This statement is a reminder to uphold your obligation as a UF student and to be honest in all work submitted and exams taken in this course and all others.

6. Accommodation for Students with Disabilities – Students Requesting classroom accommodation must first register with the Dean of Students Office. That office will provide the student with documentation that he/she must provide to the course instructor when requesting accommodation. This process must be completed in advance.

7. UF Counseling Services – Resources are available on-campus for students having personal problems or lacking clear career and academic goals. The resources include: University Counseling Center, 301 Peabody Hall, 392-1575; SHCC Mental Health, Student Health Care Center, 392-1171; Center for Sexual Assault/Abuse Recovery and Education (CARE), Student Health Care Center, 392-1161. Career Resource Center, Reitz Union, 392-1601, for career development assistance and counseling.

8. Software Use – All faculty, staff and student of the University are required and expected to obey the laws and legal agreements governing software use. Failure to do so can lead to monetary damages and/or criminal penalties for the individual violator. Because such violations are also against University policies and rules, disciplinary action will be taken as appropriate. We, the members of the University of Florida community, pledge to uphold ourselves and our peers to the highest standards of honesty and integrity.



EML4140 Heat Transfer Block I - Conduction

http://www.mae.ufl.edu/courses/spring2008/eml4140

Heat Transfer: Physical Origins and Rate Equations

Chapter One
Sections 1.1 and 1.2

- What is heat transfer?

  Heat transfer is thermal energy in transit due to a temperature difference.

- What is thermal energy?

  Thermal energy is associated with the translation, rotation, vibration and electronic states of the atoms and molecules that comprise matter. It represents the cumulative effect of microscopic activities and is directly linked to the temperature of matter.

Heat Transfer and Thermal Energy (cont.)

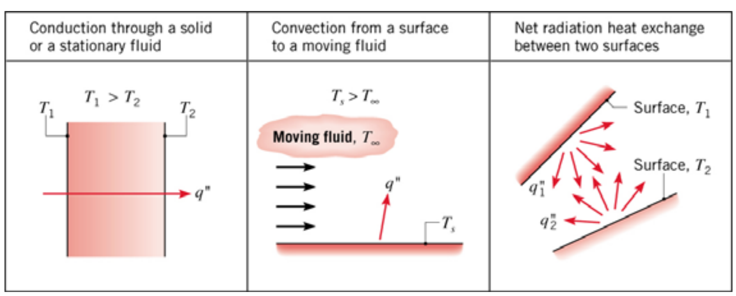
DO NOT confuse or interchange the meanings of Thermal Energy, Temperature and Heat Transfer

Quantity	Meaning	Symbol	Units
Thermal Energy[+]	Energy associated with microscopic behavior of matter	U or u	J or J/kg
Temperature	A means of indirectly assessing the amount of thermal energy stored in matter	T	K or °C
Heat Transfer	Thermal energy transport due to temperature gradients		
Heat	Amount of thermal energy transferred over a time interval $\Delta t > 0$	Q	J
Heat Rate	Thermal energy transfer per unit time	q	W
Heat Flux	Thermal energy transfer per unit time and surface area	q''	W/m^2

+
$U \rightarrow$ Thermal energy of system
$u \rightarrow$ Thermal energy per unit mass of system

Modes of Heat Transfer

Modes of Heat Transfer



Conduction: Heat transfer in a solid or a stationary fluid (gas or liquid) due to the random motion of its constituent atoms, molecules and /or electrons.

Convection: Heat transfer due to the combined influence of bulk and random motion for fluid flow over a surface.

Radiation: Energy that is emitted by matter due to changes in the electron configurations of its atoms or molecules and is transported as electromagnetic waves (or photons).

- Conduction and convection require the presence of temperature variations in a material medium.
- Although radiation originates from matter, its transport does not require a material medium and occurs most efficiently in a vacuum.

Heat Transfer Rates

Conduction:

General (vector) form of Fourier's Law:

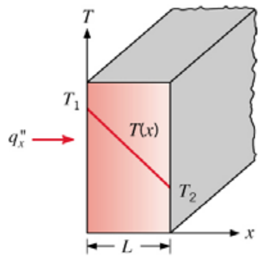
$$\vec{q}'' = -k\vec{\nabla} T$$

Heat flux W/m^2

Thermal conductivity $W/m \cdot K$

Temperature gradient °C/m or K/m

Application to one-dimensional, steady conduction across a plane wall of constant thermal conductivity:



$$q''_x = -k\frac{dT}{dx} = -k\frac{T_2 - T_1}{L}$$

$$q''_x = k\frac{T_1 - T_2}{L} \qquad (1.2)$$

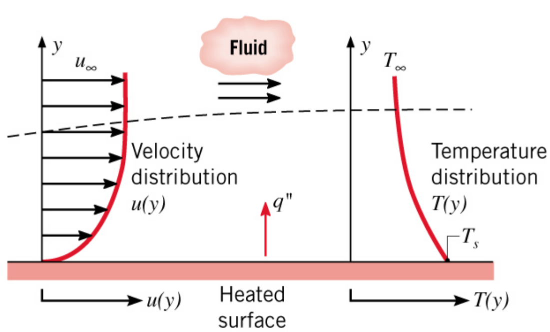
Heat rate (W):

$$q_x = q''_x \cdot A$$

Heat Transfer Rates

Convection

Relation of convection to flow over a surface and development of velocity and thermal boundary layers:



Newton's law of cooling:

$$q'' = h\left(T_s - T_\infty\right) \qquad (1.3a)$$

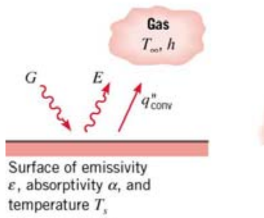
h : Convection heat transfer coefficient $(W/m^2 \cdot K)$

Heat Transfer Rates: Radiation

Heat Transfer Rates

Radiation

Heat transfer at a gas/surface interface involves radiation emission from the surface and may also involve the absorption of radiation incident from the surroundings (irradiation, G), as well as convection $\left(\text{if } T_s \neq T_\infty\right)$.



(a)

Energy outflow due to emission:

$E = \varepsilon E_b = \varepsilon \sigma T_s^4$ (1.5)

E : Emissive power $\left(\text{W/m}^2\right)$

ε : Surface emissivity $\left(0 \leq \varepsilon \leq 1\right)$

E_b : Emissive power of a blackbody (the perfect emitter)

σ : Stefan-Boltzmann constant $\left(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\right)$

Energy absorption due to irradiation:

$G_{abs} = \alpha G$

G_{abs} :Absorbed incident radiation(W/m^2)

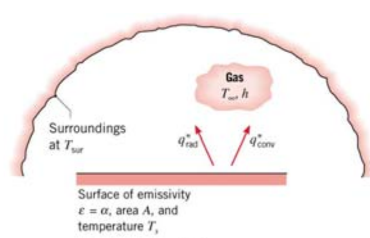
α : Surface absorptivity $\left(0 \leq \alpha \leq 1\right)$

G : Irradiation $\left(\text{W/m}^2\right)$

Heat Transfer Rates: Radiation (cont.)

Heat Transfer Rates

Irradiation: Special case of surface exposed to large surroundings of uniform temperature, T_{sur}



(b)

$G = G_{sur} = \sigma T_{sur}^4$

If $\alpha = \varepsilon$, the net radiation heat flux from the surface due to exchange with the surroundings is:

$q''_{rad} = \varepsilon E_b\left(T_s\right) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)$ (1.7)

Heat Transfer Rates: Radiation (cont.)

Heat Transfer Rates

Alternatively,

$$q''_{rad} = h_r \left(T_s - T_{sur}\right) \tag{1.8}$$

h_r: Radiation heat transfer coefficient $\left(\mathrm{W/m^2 \cdot K}\right)$

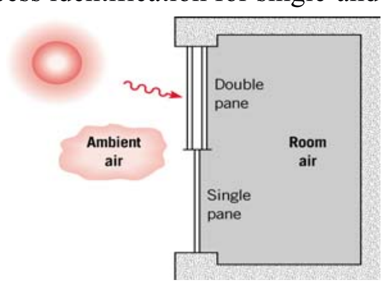
$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \tag{1.9}$$

For combined convection and radiation,

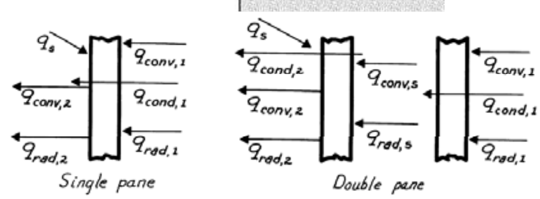
$$q'' = q''_{conv} + q''_{rad} = h\left(T_s - T_\infty\right) + h_r\left(T_s - T_{sur}\right) \tag{1.10}$$

Process Identification

Problem 1.73(a): Process identification for single-and double-pane windows



Schematic:

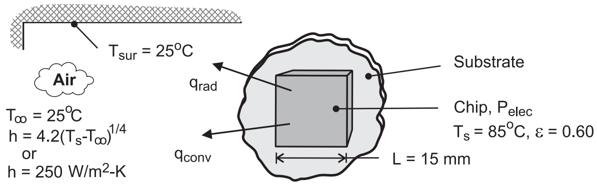


$q_{conv,1}$	Convection from room air to inner surface of first pane
$q_{rad,1}$	Net radiation exchange between room walls and inner surface of first pane
$q_{cond,1}$	Conduction through first pane
$q_{conv,s}$	Convection across airspace between panes
$q_{rad,s}$	Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace)
$q_{cond,2}$	Conduction through a second pane
$q_{conv,2}$	Convection from outer surface of single (or second) pane to ambient air
$q_{rad,2}$	Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground
q_S	Incident solar radiation during day; fraction transmitted to room is smaller for double pane

Problem: Electronic Cooling

Problem 1.31: Power dissipation from chips operating at a surface temperature of 85°C and in an enclosure whose walls and air are at 25°C for (a) free convection and (b) forced convection.

Schematic:



Assumptions: (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

Analysis:

$$P_{elec} = q_{conv} + q_{rad} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{sur}^4)$$
$$A = L^2 = (0.015\text{m})^2 = 2.25\times10^{-4}\text{m}^2$$

(a) If heat transfer is by natural convection,

$$q_{conv} = CA(T_s - T_\infty)^{5/4} = 4.2\text{W/m}^2\cdot\text{K}^{5/4}(2.25\times10^{-4}\text{m}^2)(60\text{K})^{5/4} = 0.158\text{W}$$
$$q_{rad} = 0.60(2.25\times10^{-4}\text{m}^2)5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4(358^4\text{-}298^4)\text{K}^4 = 0.065\text{W}$$
$$P_{elec} = 0.158\text{W}+0.065\text{W}=0.223\text{W}$$

(b) If heat transfer is by forced convection,

$$q_{conv} = hA(T_s - T_\infty) = 250\text{W/m}^2\cdot\text{K}^4(2.25\times10^{-4}\text{m}^2)(60\text{K}) = 3.375\text{W}$$
$$P_{elec} = 3.375\text{W}+0.065\text{W}=3.44\text{W}$$

Fourier's Law and the Heat Equation

Chapter Two

Fourier's Law

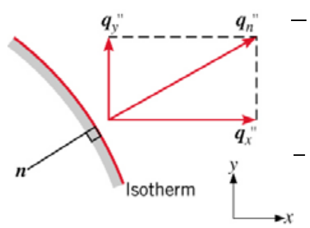
- A rate equation that allows determination of the conduction heat flux from knowledge of the temperature distribution in a medium

- Its most general (vector) form for multidimensional conduction is:

$$\vec{q}'' = -k\,\vec{\nabla} T$$

Implications:

– Heat transfer is in the direction of decreasing temperature (basis for minus sign).



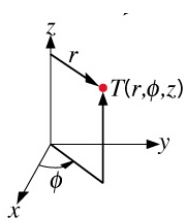
– Fourier's Law serves to define the thermal conductivity of the medium $\left(k \equiv -\vec{q}''/\vec{\nabla} T\right)$

– Direction of heat transfer is perpendicular to lines of constant temperature (isotherms).

– Heat flux vector may be resolved into orthogonal components.

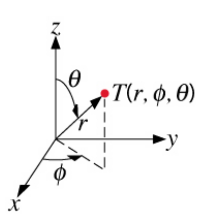
- Cartesian Coordinates: $T(x, y, z)$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial x}}_{q_x''}\vec{i} \underbrace{-k\frac{\partial T}{\partial y}}_{q_y''}\vec{j} \underbrace{-k\frac{\partial T}{\partial z}}_{q_z''}\vec{k} \qquad (2.3)$$



- Cylindrical Coordinates: $T(r, \phi, z)$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}}_{q_r''}\vec{i} \underbrace{-k\frac{\partial T}{r\partial \phi}}_{q_\phi''}\vec{j} \underbrace{-k\frac{\partial T}{\partial z}}_{q_z''}\vec{k} \qquad (2.22)$$



- Spherical Coordinates: $T(r, \phi, \theta)$

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}}_{q_r''}\vec{i} \underbrace{-k\frac{\partial T}{r\partial \theta}}_{q_\theta''}\vec{j} \underbrace{-k\frac{\partial T}{r\sin\theta\,\partial \phi}}_{q_\phi''}\vec{k} \qquad (2.25)$$

- In angular coordinates $(\phi \text{ or } \phi, \theta)$, the temperature gradient is still based on temperature change over a length scale and hence has units of °C/m and not °C/deg.
- Heat rate for one-dimensional, radial conduction in a cylinder or sphere:
  - Cylinder

$$q_r = A_r q_r'' = 2\pi r L q_r''$$

or,

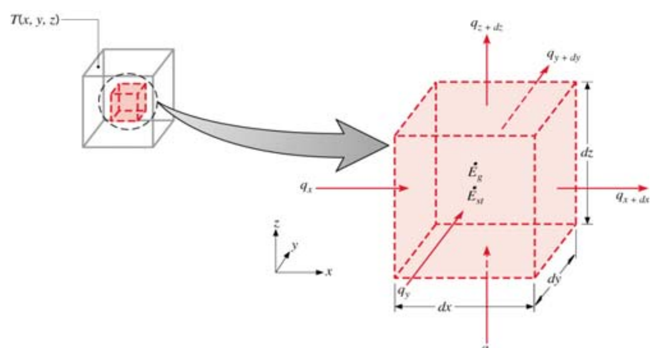
$$q_r' = A_r' q_r'' = 2\pi r q_r''$$

  - Sphere

$$q_r = A_r q_r'' = 4\pi r^2 q_r''$$

The Heat Equation

- A differential equation whose solution provides the temperature distribution in a stationary medium.
- Based on applying conservation of energy to a differential control volume through which energy transfer is exclusively by conduction.
- Cartesian Coordinates:



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.17)$$

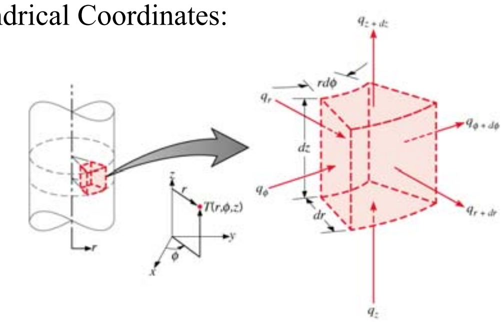
Net transfer of thermal energy into the control volume (inflow-outflow)

Thermal energy generation

Change in thermal energy storage

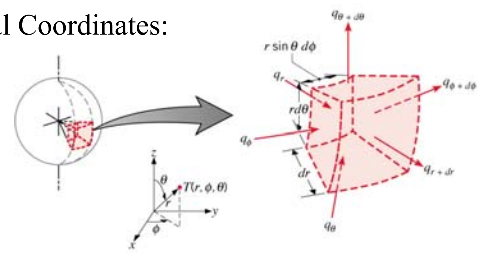
Heat Equation (Radial Systems)

- Cylindrical Coordinates:



$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \qquad (2.24)$$

- Spherical Coordinates:



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \qquad (2.27)$$

Heat Equation (Special Case)

- One-Dimensional Conduction in a Planar Medium with Constant Properties and No Generation

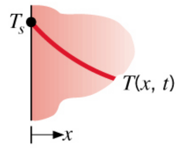
$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\alpha \equiv \frac{k}{\rho c_p} \rightarrow \text{thermal diffusivity of the medium}$$

Boundary and Initial Conditions

- For transient conduction, heat equation is first order in time, requiring specification of an initial temperature distribution: $T(x,t)_{t=0} = T(x,0)$
- Since heat equation is second order in space, two boundary conditions must be specified. Some common cases:

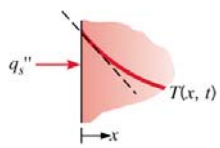
Constant Surface Temperature:



$$T(0,t) = T_s$$

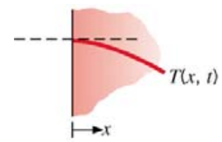
Constant Heat Flux:

Applied Flux



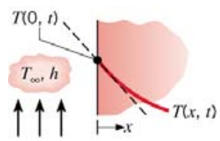
$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = q_s''$$

Insulated Surface



$$\frac{\partial T}{\partial x}\Big|_{x=0} = 0$$

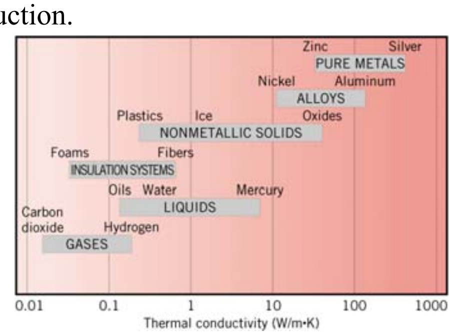
Convection



$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = h\left[T_\infty - T(0,t)\right]$$

Thermophysical Properties

Thermal Conductivity: A measure of a material's ability to transfer thermal energy by conduction.



Thermal Diffusivity: A measure of a material's ability to respond to changes in its thermal environment.

Property Tables:

Solids: Tables A.1 – A.3
Gases: Table A.4
Liquids: Tables A.5 – A.7

Micro- and Nanoscale Effects

- Conduction may be viewed as a consequence of energy carrier (electron or phonon) motion.

- For the solid state:

$$k = \frac{1}{3} C \bar{c} \lambda_{mfp} \qquad (2.7)$$

energy carrier specific heat per unit volume. (C)

average energy carrier velocity, $\bar{c} < \infty$.

mean free path → average distance traveled by an energy carrier before a collision. (λ_{mfp})

- Energy carriers also collide with physical boundaries, affecting their propagation.

  ➢ External boundaries of a film of material

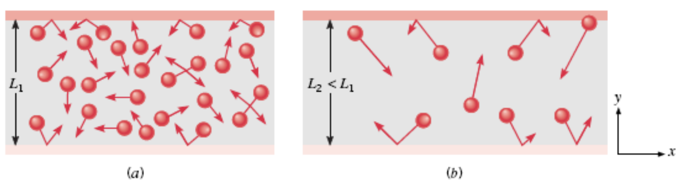


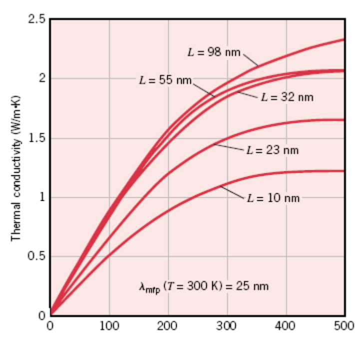
FIGURE 2.6 Electron or phonon trajectories in (a) a relatively thick film and (b) a relatively thin film with boundary effects.

For $L / \lambda_{mfp} < 1$,

$$k_x / k = 1 - 2\lambda_{mfp} / (3\pi L) \qquad (2.9a)$$

$$k_y / k = 1 - \lambda_{mfp} / (3L) \qquad (2.9b)$$

  ➢ Grain boundaries within a solid



Measured thermal conductivity of a ceramic material vs. grain size, L. λ_{mfp} at $T \approx 300\text{K} = 25\text{nm}$.

- Fourier's law does not accurately describe the finite energy carrier propagation velocity. This limitation is not important except in problems involving extremely small time scales.

Typical Methodology of a Conduction Analysis

- Consider possible microscale or nanoscale effects in problems involving very small physical dimensions or very rapid changes in heating or cooling rates.

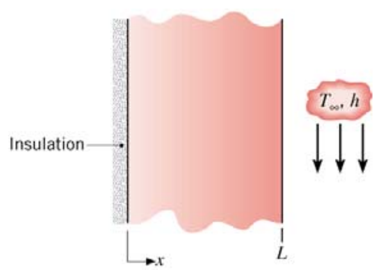
- Solve appropriate form of heat equation to obtain the temperature distribution.

- Knowing the temperature distribution, apply Fourier's Law to obtain the heat flux at any time, location and direction of interest.

- Applications:

  Chapter 3: One-Dimensional, Steady-State Conduction
  Chapter 4: Two-Dimensional, Steady-State Conduction
  Chapter 5: Transient Conduction

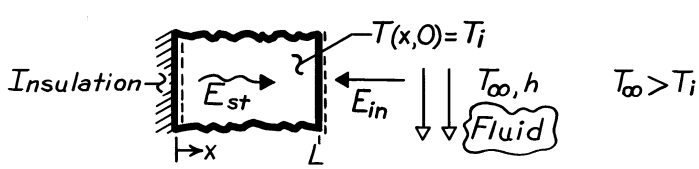
Problem 2.46 Thermal response of a plane wall to convection heat transfer.



KNOWN: Plane wall, initially at a uniform temperature, is suddenly exposed to convective heating.

FIND: (a) Differential equation and initial and boundary conditions which may be used to find the temperature distribution, T(x,t); (b) Sketch T(x,t) for the following conditions: initial ($t \leq 0$), steady-state ($t \rightarrow \infty$), and two intermediate times; (c) Sketch heat fluxes as a function of time at the two surfaces; (d) Expression for total energy transferred to wall per unit volume (J/m^3).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal heat generation.

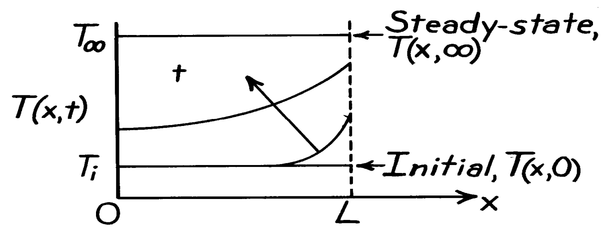
ANALYSIS: (a) For one-dimensional conduction with constant properties, the heat equation has the form,

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

and the conditions are:

$$\begin{cases} \text{Initial, } t \le 0: & T(x,0) = T_i & \text{uniform temperature} \\ \text{Boundaries:} \quad x=0 & \partial T/\partial x|_0 = 0 & \text{adiabatic surface} \\ \qquad x=L & -k\partial T/\partial x|_L = h\left[T(L,t) - T_\infty\right] & \text{surface convection} \end{cases}$$

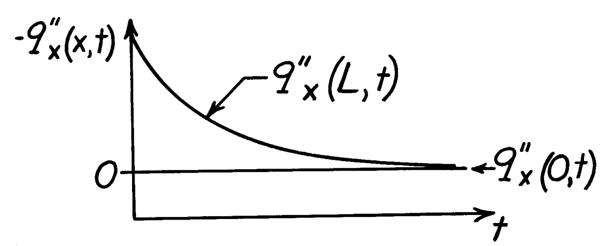
(b) The temperature distributions are shown on the sketch.



Note that the gradient at x = 0 is always zero, since this boundary is adiabatic. Note also that the gradient at x = L decreases with time.

Problem: Thermal Response (Cont.)

c) The heat flux, $q''_x(x,t)$, as a function of time, is shown on the sketch for the surfaces x = 0 and x = L.



d) The total energy transferred to the wall may be expressed as

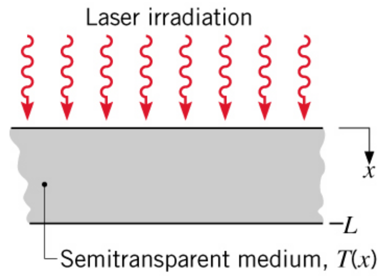
$$E_{in} = \int_0^\infty q''_{conv} A_s dt$$

$$E_{in} = hA_s \int_0^\infty \left(T_\infty - T(L,t)\right)dt$$

Dividing both sides by A_sL, the energy transferred per unit volume is

$$\frac{E_{in}}{V} = \frac{h}{L}\int_0^\infty \left[T_\infty - T(L,t)\right]dt \qquad \left[J/m^3\right]$$

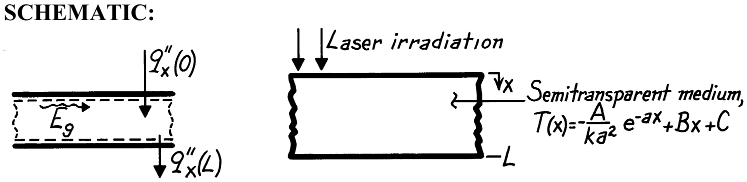
Problem 2.28 Surface heat fluxes, heat generation and total rate of radiation absorption in an irradiated semi-transparent material with a prescribed temperature distribution.



KNOWN: Temperature distribution in a semi-transparent medium subjected to radiative flux.

FIND: (a) Expressions for the heat flux at the front and rear surfaces, (b) The heat generation rate $\dot{q}(x)$, and (c) Expression for absorbed radiation per unit surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in medium, (3) Constant properties, (4) All laser irradiation is absorbed and can be characterized by an internal volumetric heat generation term $\dot{q}(x)$.

ANALYSIS: (a) Knowing the temperature distribution, the surface heat fluxes are found using Fourier's law,

$$q''_x = -k\left[\frac{dT}{dx}\right] = -k\left[-\frac{A}{ka^2}(-a)e^{-ax} + B\right]$$

Front Surface, x=0: $$q''_x(0) = -k\left[+\frac{A}{ka}\cdot 1 + B\right] = -\left[\frac{A}{a} + kB\right]$$ <

Rear Surface, x=L: $$q''_x(L) = -k\left[+\frac{A}{ka}e^{-aL} + B\right] = -\left[\frac{A}{a}e^{-aL} + kB\right].$$ <

(b) The heat diffusion equation for the medium is

$$\frac{d}{dx}\left(\frac{dT}{dx}\right) + \frac{\dot{q}}{k} = 0 \quad \text{or} \quad \dot{q} = -k\frac{d}{dx}\left(\frac{dT}{dx}\right)$$

$$\dot{q}(x) = -k\frac{d}{dx}\left[+\frac{A}{ka}e^{-ax} + B\right] = Ae^{-ax}.$$

(c) Performing an energy balance on the medium,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

On a unit area basis

$$\dot{E}''_g = -\dot{E}''_{in} + \dot{E}''_{out} = -q''_x(0) + q''_x(L) = +\frac{A}{a}\left(1 - e^{-aL}\right).$$

Alternatively, evaluate $\dot{E}''_g$ by integration over the volume of the medium,

$$\dot{E}''_g = \int_0^L \dot{q}(x)dx = \int_0^L Ae^{-ax}dx = -\frac{A}{a}\left[e^{-ax}\right]_0^L = \frac{A}{a}\left(1 - e^{-aL}\right).$$

One-Dimensional, Steady-State Conduction without Thermal Energy Generation

Chapter Three

Sections 3.1 through 3.4

Methodology of a Conduction Analysis

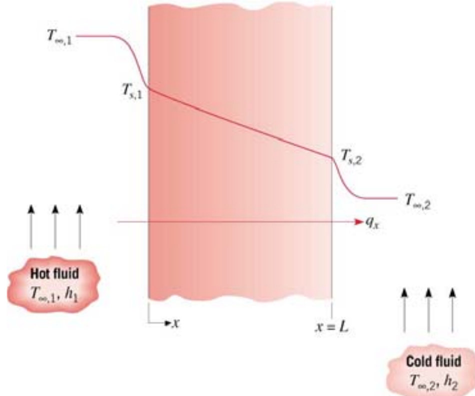
- Specify appropriate form of the heat equation.
- Solve for the temperature distribution.
- Apply Fourier's law to determine the heat flux.

Simplest Case: One-Dimensional, Steady-State Conduction with No Thermal Energy Generation.

- Common Geometries:
  - The Plane Wall: Described in rectangular (x) coordinate. Area perpendicular to direction of heat transfer is constant (independent of x).
  - The Tube Wall: Radial conduction through tube wall.
  - The Spherical Shell: Radial conduction through shell wall.

The Plane Wall

- Consider a plane wall between two fluids of different temperature:



- Heat Equation:

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \qquad (3.1)$$

- Implications:

  Heat flux $\left(q''_x\right)$ is independent of x.

  Heat rate $\left(q_x\right)$ is independent of x.

- Boundary Conditions: $T(0) = T_{s,1}, \quad T(L) = T_{s,2}$

- Temperature Distribution for Constant k:

$$T(x) = T_{s,1} + \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} \qquad (3.3)$$

Plane Wall (cont.)

- Heat Flux and Heat Rate:

$$q''_x = -k\frac{dT}{dx} = \frac{k}{L}\left(T_{s,1} - T_{s,2}\right) \quad (3.5)$$

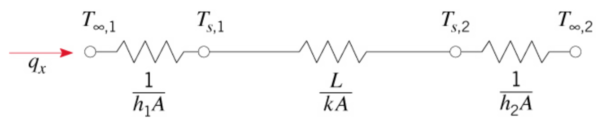
$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right) \quad (3.4)$$

- Thermal Resistances $\left(R_t = \frac{\Delta T}{q}\right)$ and Thermal Circuits:

Conduction in a plane wall: $R_{t,cond} = \frac{L}{kA}$ (3.6)

Convection: $R_{t,conv} = \frac{1}{hA}$ (3.9)

Thermal circuit for plane wall with adjoining fluids:



$$R_{tot} = \frac{1}{h_1A} + \frac{L}{kA} + \frac{1}{h_2A} \quad (3.12)$$

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \quad (3.11)$$

Plane Wall (cont.)

- Thermal Resistance for Unit Surface Area:

$$R''_{t,cond} = \frac{L}{k} \qquad R''_{t,conv} = \frac{1}{h}$$

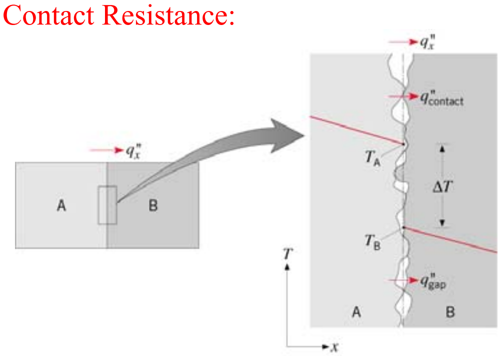
Units: $R_t \leftrightarrow \text{K/W}$ $\qquad R''_t \leftrightarrow \text{m}^2 \cdot \text{K/W}$

- Radiation Resistance:

$$R_{t,rad} = \frac{1}{h_rA} \qquad R''_{t,rad} = \frac{1}{h_r}$$

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) \quad (1.9)$$

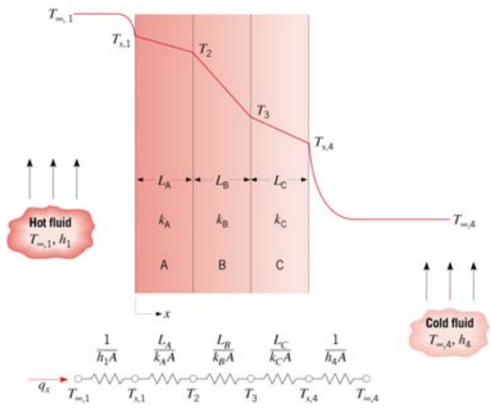
- Contact Resistance:



$$R''_{t,c} = \frac{T_A - T_B}{q''_x} \qquad \boldsymbol{R_{t,c} = \frac{R''_{t,c}}{A_c}}$$

Values depend on: Materials A and B, surface finishes, interstitial conditions, and contact pressure (Tables 3.1 and 3.2)

Plane Wall (cont.)



- Composite Wall with Negligible Contact Resistance:

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R_t} \qquad (3.14)$$

$$\sum R_t = R_{tot} = \frac{1}{A}\left[\frac{1}{h_1} + \frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C} + \frac{1}{h_4}\right] = \frac{R''_{tot}}{A}$$

- Overall Heat Transfer Coefficient (U) :

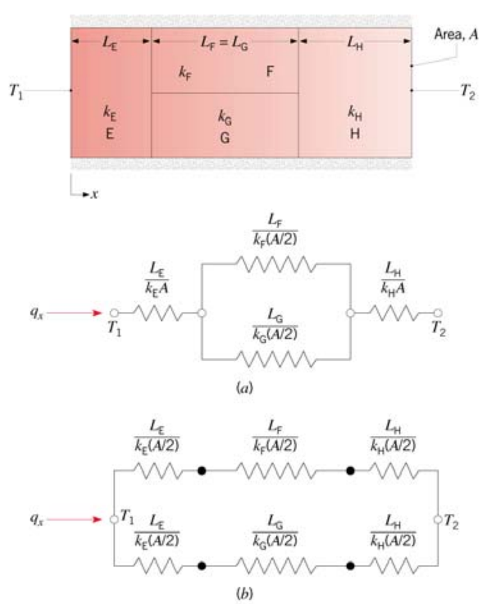
  A modified form of Newton's Law of Cooling to encompass multiple resistances to heat transfer.

$$q_x = UA\Delta T_{overall} \qquad (3.17)$$

$$R_{tot} = \frac{1}{UA} \qquad (3.19)$$

Plane Wall (cont.)

- Series – Parallel Composite Wall:

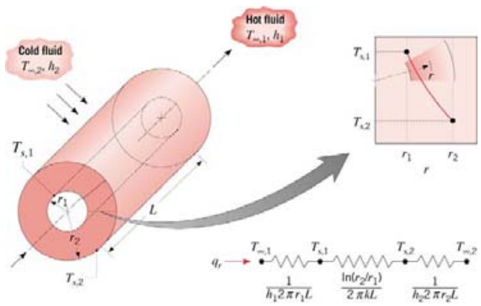


- Note departure from one-dimensional conditions for $k_F \neq k_G$.

- Circuits based on assumption of isothermal surfaces normal to x direction or adiabatic surfaces parallel to x direction provide approximations for q_x .

Tube Wall

The Tube Wall



- Heat Equation:

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0 \qquad (3.23)$$

What does the form of the heat equation tell us about the variation of q_r with r in the wall?

Is the foregoing conclusion consistent with the energy conservation requirement?

How does q''_r vary with r?

- Temperature Distribution for Constant k :

$$T(r)=\frac{T_{s,1}-T_{s,2}}{\ln(r_1/r_2)}\ln\left(\frac{r}{r_2}\right)+T_{s,2} \qquad (3.26)$$

Tube Wall (Cont.)

- Heat Flux and Heat Rate:

$$q''_r=-k\frac{dT}{dr}=\frac{k}{r\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right)$$

$$q'_r=2\pi r q''_r=\frac{2\pi k}{\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right)$$

$$q_r=2\pi r L q''_r=\frac{2\pi L k}{\ln(r_2/r_1)}\left(T_{s,1}-T_{s,2}\right) \qquad (3.27)$$

- Conduction Resistance:

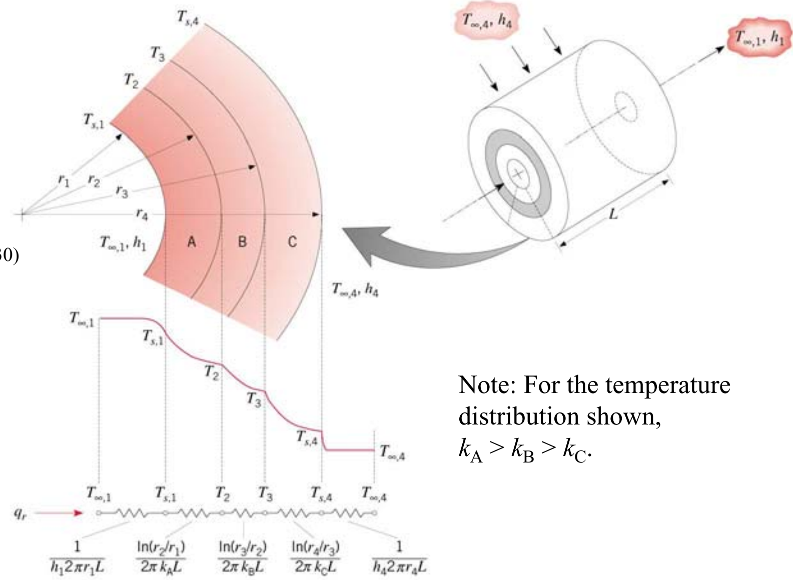
$$R_{t,cond}=\frac{\ln(r_2/r_1)}{2\pi L k} \qquad \text{Units} \leftrightarrow \text{K/W} \qquad (3.28)$$

$$R'_{t,cond}=\frac{\ln(r_2/r_1)}{2\pi k} \qquad \text{Units} \leftrightarrow \text{m}\cdot\text{K/W}$$

Why doesn't a surface area appear in the expressions for the thermal resistance?

- Composite Wall with Negligible Contact Resistance

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA\left(T_{\infty,1} - T_{\infty,4}\right) \qquad (3.30)$$



Note: For the temperature distribution shown, $k_A > k_B > k_C$.

Note that

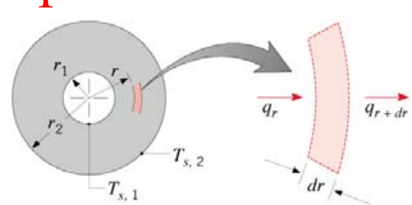
$$UA = R_{tot}^{-1}$$

is a constant independent of radius.

But, U itself is tied to specification of an interface.

$$U_i = \left(A_i R_{tot}\right)^{-1} \qquad (3.32)$$

Spherical Shell



- Heat Equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

What does the form of the heat equation tell us about the variation of q_r with r? Is this result consistent with conservation of energy?

How does q''_r vary with r?

- Temperature Distribution for Constant k:

$$T(r) = T_{s,1} - \left(T_{s,1} - T_{s,2}\right)\frac{1 - \left(r_1/r\right)}{1 - \left(r_1/r_2\right)}$$

- Heat flux, Heat Rate and Thermal Resistance:

$$q_r'' = -k\frac{dT}{dr} = \frac{k}{r^2\left[(1/r_1)-(1/r_2)\right]}\left(T_{s,1}-T_{s,2}\right)$$

$$q_r = 4\pi r^2 q_r'' = \frac{4\pi k}{(1/r_1)-(1/r_2)}\left(T_{s,1}-T_{s,2}\right) \qquad (3.35)$$

$$R_{t,cond} = \frac{(1/r_1)-(1/r_2)}{4\pi k} \qquad (3.36)$$

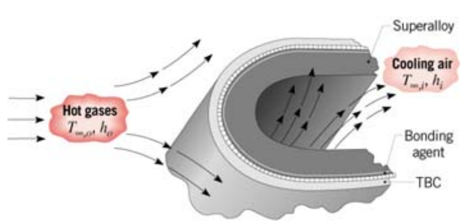
- Composite Shell:

$$q_r = \frac{\Delta T_{overall}}{R_{tot}} = UA\Delta T_{overall}$$

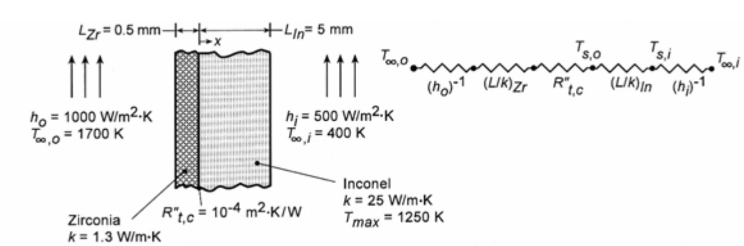
$$UA = R_{tot}^{-1} \leftrightarrow \text{Constant}$$

$$U_i = \left(A_i R_{tot}\right)^{-1} \leftrightarrow \text{ Depends on } A_i$$

Problem 3.23: Assessment of thermal barrier coating (TBC) for protection of turbine blades. Determine maximum blade temperature with and without TBC.



Schematic:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right) m^2\cdot K/W = 3.69\times10^{-3} m^2\cdot K/W$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,K}{3.69\times10^{-3}\,m^2\cdot K/W} = 3.52\times10^5\,W/m^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + (q''_w/h_i) = 400\,K + \left(\frac{3.52\times10^5\,W/m^2}{500\,W/m^2\cdot K}\right) = 1104\ K$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_w = 400\,K + \left(2\times10^{-3} + 2\times10^{-4}\right) m^2\times K/W \left(3.52\times10^5\,W/m^2\right) = 1174\,K$$

Without the TBC,

$$R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3} m^2\cdot K/W$$

$$q''_{wo} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{tot,wo} = 4.06\times10^5\ W/m^2.$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(wo)} = T_{\infty,i} + (q''_{wo}/h_i) = 1212\,K$$

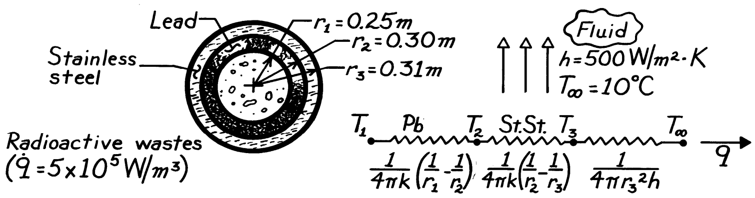
$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right] q''_{wo} = 1293\,K$$

Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to its thickness are associated with reliability considerations.

Problem 3.62: Suitability of a composite spherical shell for storing radioactive wastes in oceanic waters.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, MP = 601K; St.St.: *k* = 15.1 W/m·K

ANALYSIS: From the thermal circuit, it follows that

$$q=\frac{T_1 - T_\infty}{R_{tot}} = \dot{q}\left[\frac{4}{3}\pi\, r_1^3\right]$$

The thermal resistances are:

$$R_{Pb} = \left[1/\left(4\pi \times 35.3\ \text{W/m}\cdot\text{K}\right)\right]\left[\frac{1}{0.25\text{m}} - \frac{1}{0.30\text{m}}\right] = 0.00150\ \text{K/W}$$

$$R_{St.St.} = \left[1/\left(4\pi \times 15.1\ \text{W/m}\cdot\text{K}\right)\right]\left[\frac{1}{0.30\text{m}} - \frac{1}{0.31\text{m}}\right] = 0.000567\ \text{K/W}$$

$$R_{conv} = \left[1/\left(4\pi \times 0.31^2\,\text{m}^2 \times 500\ \text{W/m}^2\cdot\text{K}\right)\right] = 0.00166\ \text{K/W}$$

$$R_{tot} = 0.00372\ \text{K/W}.$$

The heat rate is then

$$q=5\times10^5\ \text{W/m}^3\left(4\pi/3\right)\left(0.25\text{m}\right)^3 = 32,725\ \text{W}$$

and the inner surface temperature is

$$T_1 = T_\infty + R_{tot}\ q=283\text{K}+0.00372\text{K/W}\left(32,725\ \text{W}\right) = 405\ \text{K} < \text{MP} = 601\text{K}.$$

Hence, from the thermal standpoint, the proposal is adequate.

COMMENTS: In fabrication, attention should be given to maintaining a good thermal contact. A protective outer coating should be applied to prevent long term corrosion of the stainless steel.

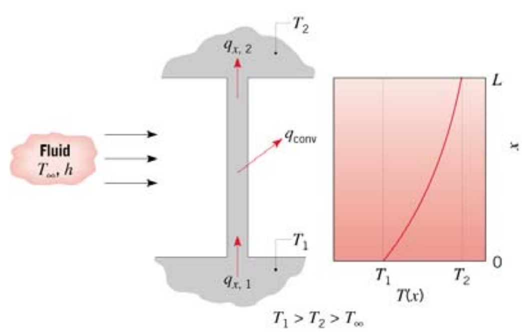
Extended Surfaces

Chapter Three

Section 3.6

Nature and Rationale of Extended Surfaces

- An extended surface (also know as a combined conduction-convection system or a fin) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.



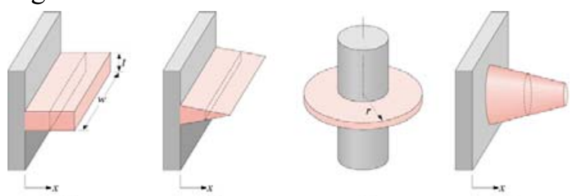
  - Why is heat transfer by conduction in the ***x***-direction not, in fact, one-dimensional?

  - If heat is transferred from the surface to the fluid by convection, what surface condition is dictated by the conservation of energy requirement?

- What is the actual functional dependence of the temperature distribution in the solid?
- If the temperature distribution is assumed to be one-dimensional, that is, $T=T(x)$, how should the value of T be interpreted for any $\boldsymbol{x}$ location?
- How does $q_{cond,x}$ vary with $\boldsymbol{x}$?
- When may the assumption of one-dimensional conduction be viewed as an excellent approximation? The thin-fin approximation.

• Extended surfaces may exist in many situations but are commonly used as fins to enhance heat transfer by increasing the surface area available for convection (and/or radiation). They are particularly beneficial when h is small, as for a gas and natural convection.

• Some typical fin configurations:



Straight fins of (a) uniform and (b) non-uniform cross sections; (c) annular fin, and (d) pin fin of non-uniform cross section.

The Fin Equation

• Assuming one-dimensional, steady-state conduction in an extended surface surface of constant conductivity (k) and uniform cross-sectional area (A_c), with negligible generation $\left(\dot{q}=0\right)$ and radiation $(q''_{rad}=0)$, the *fin equation* is of the form:

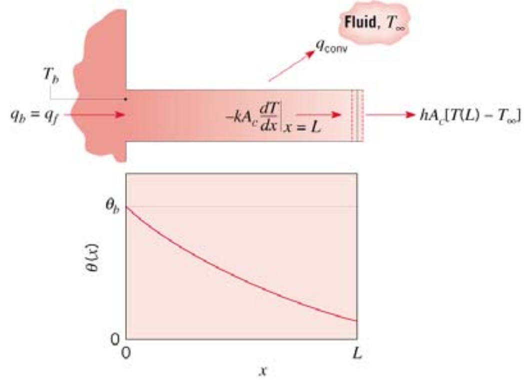
$$\frac{d^2T}{dx^2}-\frac{hP}{kA_c}(T-T_\infty)=0 \qquad (3.62)$$

or, with $m^2 \equiv (hP/kA_c)$ and the reduced temperature $\theta \equiv T-T_\infty$,

$$\frac{d^2\theta}{dx^2}-m^2\theta=0 \qquad (3.64)$$

How is the fin equation derived?

- Solutions (Table 3.4):



Base ($x = 0$) condition

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

Tip ($x = L$) conditions

A. Convection: $-kd\theta/dx|_{x=L} = h\theta(L)$
B. Adiabatic: $d\theta/dx|_{x=L} = 0$
C. Fixed temperature: $\theta(L) = \theta_L$
D. Infinite fin (mL>2.65): $\theta(L) = 0$

- Fin Heat Rate:

$$q_f = -kA_c \frac{d\theta}{dx}|_{x=0} = \int_{A_f} h\theta(x)\, dA_s$$

Fin Performance Parameters

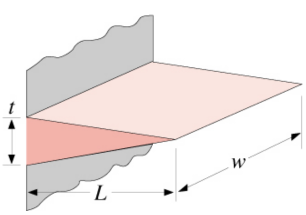
- Fin Efficiency:

$$\eta_f \equiv \frac{q_f}{q_{f,\max}} = \frac{q_f}{hA_f\theta_b} \quad (3.86)$$

How is the efficiency affected by the thermal conductivity of the fin?

Expressions for η_f are provided in Table 3.5 for common geometries.

Consider a triangular fin:



$$A_f = 2w\left[L^2 + (t/2)^2\right]^{1/2}$$

$$A_p = (t/2)L$$

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

- Fin Effectiveness:

$$\varepsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b}$$

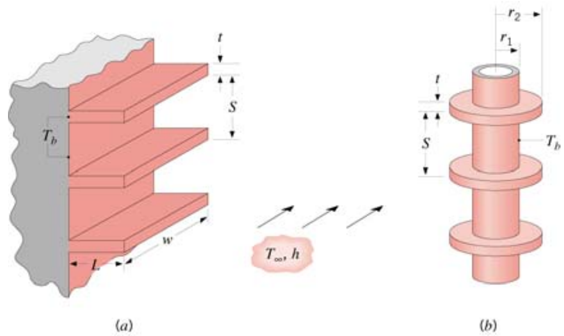
$\varepsilon_f \uparrow$ with $\downarrow h, \uparrow k$ and $\downarrow A_c/P$ (3.81)

- Fin Resistance:

$$R_{t,f} \equiv \frac{\theta_b}{q_f} = \frac{1}{hA_f\eta_f} \quad (3.92)$$

Fin Arrays

- Representative arrays of (a) rectangular and (b) annular fins.



  – Total surface area:

$$A_t = NA_f + A_b \qquad (3.99)$$

Number of fins (N) — Area of exposed base (*prime* surface) (A_b)

  – Total heat rate:

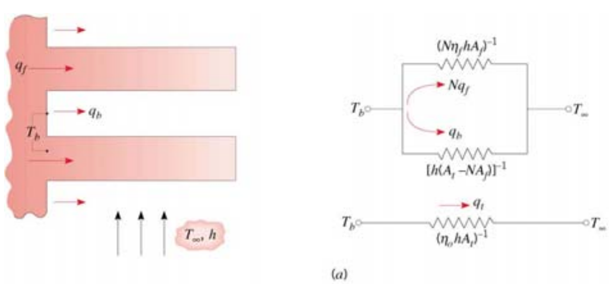
$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \equiv \eta_o hA_t\theta_b = \frac{\theta_b}{R_{t,o}} \qquad (3.100)$$

  – Overall surface efficiency and resistance:

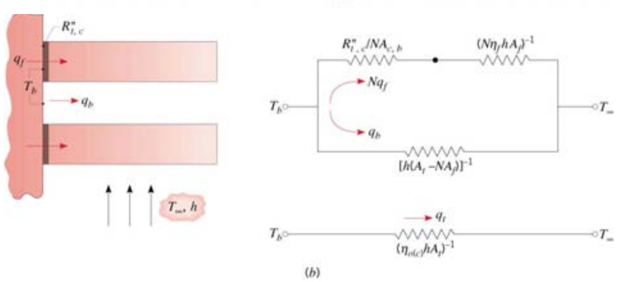
$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1-\eta_f\right) \qquad (3.102)$$

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o hA_t} \qquad (3.103)$$

- Equivalent Thermal Circuit :



- Effect of Surface Contact Resistance:



$$q_t = \eta_{o(c)} hA_t\theta_b = \frac{\theta_b}{R_{t,o(c)}}$$

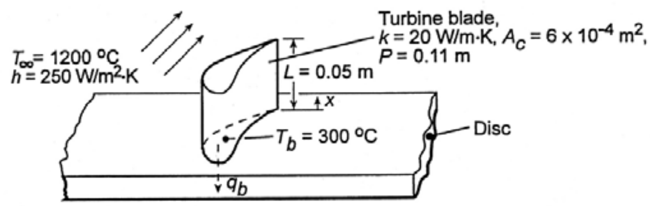
$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right) \qquad (3.105a)$$

$$C_1 = 1 + \eta_f hA_f\left(R''_{t,c} / A_{c,b}\right) \qquad (3.105b)$$

$$R_{t,o(c)} = \frac{1}{\eta_{o(c)} hA_t} \qquad (3.104)$$

Problem 3.116: Assessment of cooling scheme for gas turbine blade. Determination of whether blade temperatures are less than the maximum allowable value (1050 °C) for prescribed operating conditions and evaluation of blade cooling rate.

Schematic:



Assumptions: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

Analysis: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x=L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\text{W/m}^2\cdot\text{K}\times 0.11\text{m}/20\text{W/m}\cdot\text{K}\times 6\times 10^{-4}\text{m}^2\right)^{1/2} = 47.87\text{ m}^{-1}$$

mL = 47.87 m^{-1} × 0.05 m = 2.39

From Table B.1, $\cosh mL = 5.51$. Hence,

$$T(L)=1200^\circ\text{C}+(300-1200)^\circ\text{C}/5.51=1037^\circ\text{C}$$

and, *subject to the assumption of an adiabatic tip*, the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\theta_b=\left(250\text{W/m}^2\cdot\text{K}\times 0.11\text{m}\times 20\text{W/m}\cdot\text{K}\times 6\times 10^{-4}\text{m}^2\right)^{1/2}\left(-900^\circ C\right)=\text{-517W}$,

Eq. 3.76 and Table B.1 yield

$$q_f = \text{M}\tanh\text{mL} = -517\text{W}\left(0.983\right) = -508\text{W}$$

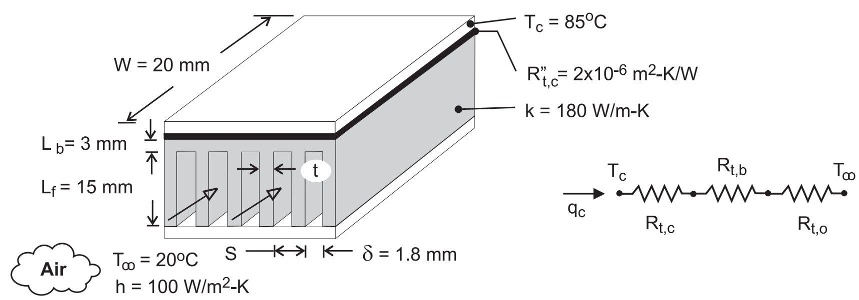
Hence,

$$q_b = -q_f = 508\text{W}$$

Comments: Radiation losses from the blade surface contribute to reducing the blade temperatures, but what is the effect of assuming an adiabatic tip condition? Calculate the tip temperature allowing for convection from the gas.

Problem 3.132: Determination of maximum allowable power q_c for a 20mm x 20mm electronic chip whose temperature is not to exceed $T_c = 85^\circ$C, when the chip is attached to an air-cooled heat sink with N=11 fins of prescribed dimensions.

Schematic:



Assumptions: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surface of heat sink, (7) Negligible radiation.

Analysis: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

$$R_{t,c} = R''_{t,c} / W^2 = 2\times10^{-6} m^2 \cdot K/W / \left(0.02m\right)^2 = 0.005 \ K/W$$

$$R_{t,b} = L_b / k\left(W^2\right) = 0.003m/180 \ W/m\cdot K\left(0.02m\right)^2 = 0.042 \ K/W$$

From Eqs. (3.103), (3.102), and (3.99)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}\left(1-\eta_f\right), \qquad A_t = N A_f + A_b$$

$A_f = 2WL_f = 2 \times 0.02m \times 0.015m = 6 \times 10^{-4}$ m^2

$A_b = W^2 - N(tW) = (0.02m)^2 - 11(0.182 \times 10^{-3}$ m $\times$ 0.02m) = 3.6 $\times 10^{-4}$ m^2

$A_t = 6.96 \times 10^{-3}$ m^2

With $mL_f = (2h/kt)^{1/2} L_f$ = (200 W/m^2·K/180 W/m·K × 0.182 × 10^{-3}m)$^{1/2}$ (0.015m) = 1.17, tanh mL_f = 0.824 and Eq. (3.87) yields

$$\eta_f = \frac{\tanh mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

$\eta_o = 0.719$,

$R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{\left(85-20\right)^\circ C}{\left(0.005+0.042+2.00\right)K/W} = 31.8 \ W$$

Comments: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with $h = 100\ \mathrm{W/m^2 \cdot K}, R_{tot} = 2.05\ \mathrm{K/W}$ from Part (a) would be replaced by $R_{cnv} = 1/hW^2 =$ 25 K/W, yielding $q_c =$ 2.60 W.

Two-Dimensional Conduction: Finite-Difference Equations and Solutions

Chapter 4

Sections 4.4

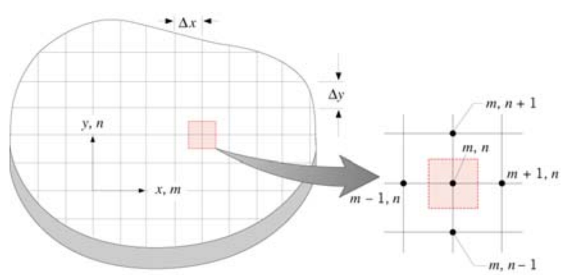
The Finite-Difference Method

- An approximate method for determining temperatures at discrete (*nodal*) points of the physical system.

- Procedure:
  - Represent the physical system by a *nodal network*.
  - Use the energy balance method to obtain a finite-difference equation for each node of unknown temperature.
  - Solve the resulting set of algebraic equations for the unknown nodal temperatures.

The Nodal Network and Finite-Difference Approximation

- The nodal network identifies discrete points at which the temperature is to be determined and uses an *m,n* notation to designate their location.



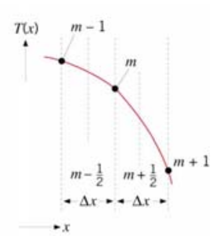
(a)

What is represented by the temperature determined at a nodal point, as for example, $T_{m,n}$?

- A finite-difference approximation is used to represent temperature gradients in the domain.

$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,\,n} = \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,\,n} = \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$



(b)

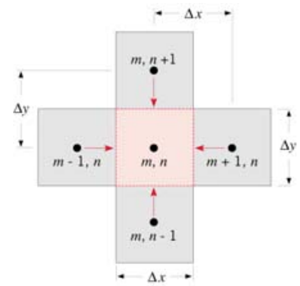
How is the accuracy of the solution affected by construction of the nodal network? What are the trade-offs between selection of a *fine* or a *coarse mesh*?

Derivation of the Finite-Difference Equations
- The Energy Balance Method -

- As a convenience that obviates the need to predetermine the direction of heat flow, assume all heat flows are into the nodal region of interest, and express all heat rates accordingly. Hence, the energy balance becomes:

$$\dot{E}_{in} + \dot{E}_g = 0 \qquad (4.30)$$

- Consider application to an *interior nodal point* (one that exchanges heat by conduction with four, equidistant nodal points):



$$\sum_{i=1}^{4} q_{(i)\rightarrow(m,n)} + \dot{q}\left(\Delta x \cdot \Delta y \cdot \ell\right) = 0$$

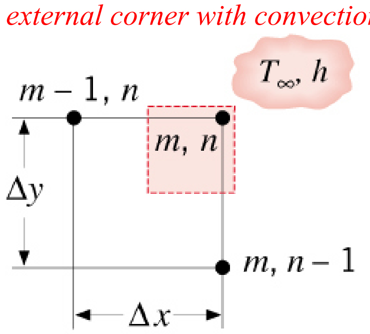
where, for example,

$$q_{(m-1,n)\rightarrow(m,n)} = k\left(\Delta y \cdot \ell\right)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} \qquad (4.31)$$

Is it possible for all heat flows to be into the m,n nodal region?

What feature of the analysis insures a correct form of the energy balance equation despite the assumption of conditions that are not realizable?

- A summary of finite-difference equations for common nodal regions is provided in Table 4.2. Consider an *external corner with convection heat transfer*.



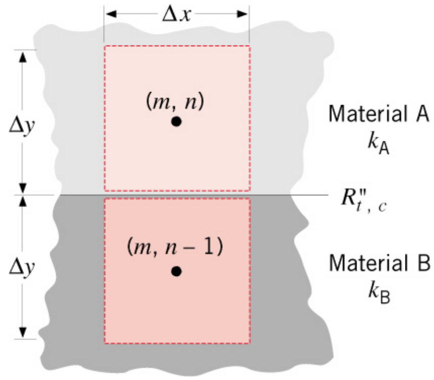
$$q_{(m-1,n)\rightarrow(m,n)} + q_{(m,n-1)\rightarrow(m,n)} + q_{(\infty)\rightarrow(m,n)} = 0$$

$$k\left(\frac{\Delta y}{2}\cdot\ell\right)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k\left(\frac{\Delta x}{2}\cdot\ell\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$
$$+\mathrm{h}\left(\frac{\Delta \mathrm{x}}{2}\cdot\ell\right)\left(T_\infty - T_{m,n}\right) + \mathrm{h}\left(\frac{\Delta \mathrm{y}}{2}\cdot\ell\right)\left(T_\infty - T_{m,n}\right) = 0$$

or, with $\Delta x = \Delta y$,

$$T_{m-1,n} + T_{m,n-1} + 2\frac{\mathrm{h}\Delta x}{k}T_\infty - 2\left(\frac{\mathrm{h}\Delta x}{k} + 1\right)T_{m,n} = 0 \qquad (4.43)$$

- Note potential utility of using thermal resistance concepts to express rate equations. E.g., conduction between adjoining dissimilar materials with an interfacial contact resistance.



$$q_{(m,n-1)\to(m,n)} = \frac{T_{m,n-1} - T_{m,n}}{R_{tot}}$$

$$R_{tot} = \frac{\Delta y/2}{k_A\left(\Delta x \cdot \ell\right)} + \frac{R''_{t,c}}{\Delta x \cdot \ell} + \frac{\Delta y/2}{k_B\left(\Delta x \cdot \ell\right)} \tag{4.46}$$

Transient Conduction: The Lumped Capacitance Method

Chapter Five

Sections 5.1 through 5.3

Transient Conduction

- A heat transfer process for which the temperature varies with time, as well as location within a solid.

- It is initiated whenever a system experiences a change in operating conditions.

- It can be induced by changes in:
  - surface convection conditions (h, T_∞),
  - surface radiation conditions (h_r, T_{sur}),
  - a surface temperature or heat flux, and/or
  - internal energy generation.

- Solution Techniques
  - The Lumped Capacitance Method
  - Exact Solutions
  - The Finite-Difference Method

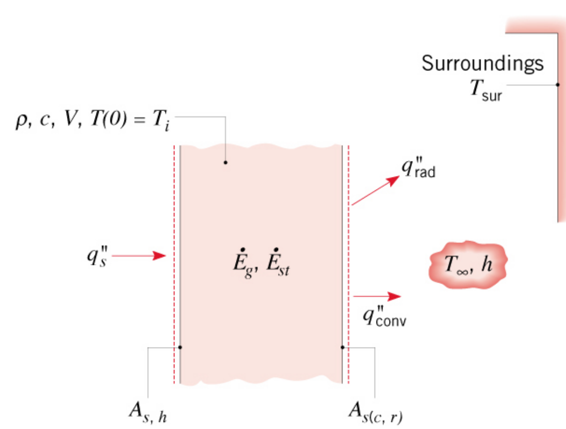
The Lumped Capacitance Method

- Based on the assumption of a spatially uniform temperature distribution throughout the transient process. Hence $T(\vec{r},t) \approx T(t)$.

- Why is the assumption never fully realized in practice?

- General Lumped Capacitance Analysis:

  - Consider a general case, which includes convection, radiation and/or an applied heat flux at specified surfaces $(A_{s,c}, A_{s,r}, A_{s,h})$, as well as internal energy generation



- First Law:

$$\frac{dE_{st}}{dt} = \rho \forall c \frac{dT}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

- Assuming energy outflow due to convection and radiation and with inflow due to an applied heat flux q_s'',

$$\rho \forall c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c} \left(T - T_\infty\right) - h_r A_{s,r} \left(T - T_{sur}\right) + \dot{E}_g$$

- Is this expression applicable in situations for which convection and/or radiation provide for energy inflow?

- May h and h_r be assumed to be constant throughout the transient process?

- How must such an equation be solved?

- Special Cases (Exact Solutions, $T(0) \equiv T_i$)
  - Negligible Radiation $\left(\theta \equiv T - T_\infty,\ \theta' \equiv \theta - b/a\right)$:

$$a \equiv h A_{s,c} / \rho \forall c \qquad\qquad b \equiv \left(q_s'' A_{s,h} + \dot{E}_g\right) / \rho \forall c$$

The non-homogeneous differential equation is transformed into a homogeneous equation of the form:

$$\frac{d\theta'}{dt} = -a\theta'$$

Integrating from *t=0 to any* t and rearranging,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}\left[1 - \exp(-at)\right] \qquad (5.25)$$

To what does the foregoing equation reduce as steady state is approached?

How else may the steady-state solution be obtained?

Special Case (Convection)

➢ Negligible Radiation and Source Terms $\left(h >> h_r,\ \dot{E}_g = 0,\ q_s'' = 0\right)$:

$$\rho \forall c \frac{dT}{dt} = -hA_{s,c}\left(T - T_\infty\right) \tag{5.2}$$

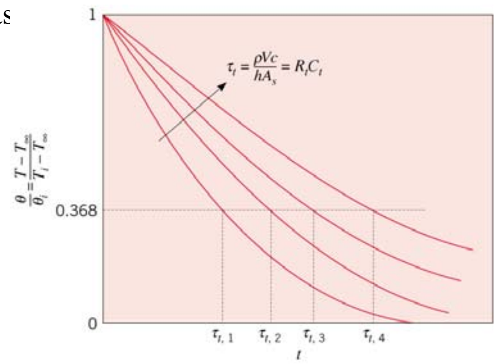
$$\frac{\rho \forall c}{hA_{s,c}} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_o^t dt$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = exp\left[-\left(\frac{hA_{s,c}}{\rho \forall c}\right)t\right] = \exp\left[-\frac{t}{\tau_t}\right]$$

The thermal time constant is defined as

$$\tau_t \equiv \left(\frac{1}{hA_{s,c}}\right)\left(\rho \forall c\right) \tag{5.7}$$

Thermal Resistance, R_t — Lumped Thermal Capacitance, C_t



The change in thermal energy storage due to the transient process is

$$\Delta E_{st} \equiv -Q = -\int_o^t \dot{E}_{out}dt = -hA_{s,c}\int_o^t \theta dt = -\left(\rho \forall c\right)\theta_i\left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right] \tag{5.8}$$

Special Case (Radiation)

➢ Negligible Convection and Source Terms $\left(h_r >> h,\ \dot{E}_g = 0,\ q_s'' = 0\right)$:

Assuming radiation exchange with large surroundings,

$$\rho \forall c \frac{dT}{dt} = -\varepsilon A_{s,r}\sigma\left(T^4 - T_{sur}^4\right)$$

$$\frac{\varepsilon A_{s,r}\sigma}{\rho \forall c}\int_o^t dt = \int_{T_i}^{T} \frac{dT}{T_{sur}^4 - T^4}$$

$$t = \frac{\rho \forall c}{4\varepsilon A_{s,r}\sigma T_{sur}^3}\left\{ \ln\left|\frac{T_{sur} + T}{T_{sur} - T}\right| - \ln\left|\frac{T_{sur} + T_i}{T_{sur} - T_i}\right| + 2\left[\tan^{-1}\left(\frac{T}{T_{sur}}\right) - \tan^{-1}\left(\frac{T_i}{T_{sur}}\right)\right]\right\} \tag{5.18}$$

Result necessitates implicit evaluation of $T(t)$.

The Biot Number and Validity of The Lumped Capacitance Method

- The Biot Number: The first of many dimensionless parameters to be considered.
  - Definition:

$$Bi \equiv \frac{hL_c}{k}$$

$h \rightarrow$ convection or radiation coefficient

$k \rightarrow$ thermal conductivity of the solid

$L_c \rightarrow$ characteristic length of the solid ($\forall / A_s$ or coordinate associated with maximum spatial temperature difference)

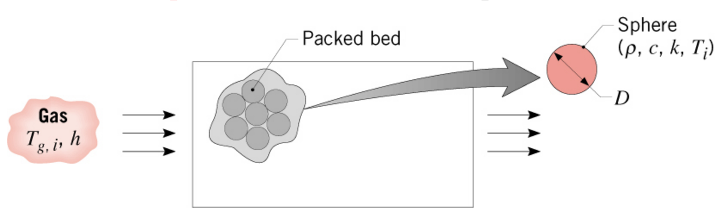
  - Physical Interpretation:

$$Bi = \frac{L_c / kA_s}{1/\mathrm{h}A_s} \approx \frac{R_{cond}}{R_{conv}} \approx \frac{\Delta T_{solid}}{\Delta T_{solid/fluid}}$$

  - Criterion for Applicability of Lumped Capacitance Method:

$$Bi << 1$$

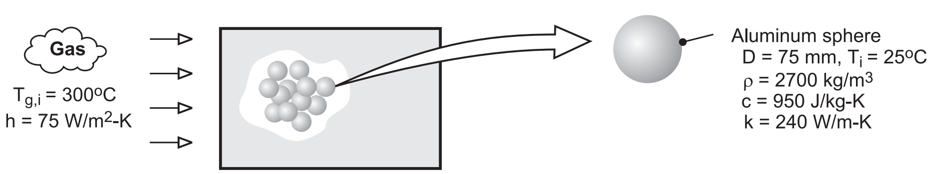
Problem 5.12: Charging a thermal energy storage system consisting of a packed bed of aluminum spheres.



KNOWN: Diameter, density, specific heat and thermal conductivity of aluminum spheres used in packed bed thermal energy storage system. Convection coefficient and inlet gas temperature.

FIND: Time required for sphere at inlet to acquire 90% of maximum possible thermal energy and the corresponding center temperature.

Schematic:



Problem: Thermal Energy Storage (cont.)

ASSUMPTIONS: (1) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres, (2) Constant properties.

ANALYSIS: To determine whether a lumped capacitance analysis can be used, first compute $Bi = h(r_o/3)/k = 75\ W/m^2\cdot K\ (0.0125m)/150\ W/m\cdot K = 0.006 << 1$.

Hence, the lumped capacitance approximation may be made, and a uniform temperature may be assumed to exist in the sphere at any time.

From Eq. 5.8a, achievement of 90% of the maximum possible thermal energy storage corresponds to

$$-\frac{\Delta E_{st}}{\rho c V \theta_i} = 0.90 = 1 - \exp(-t/\tau_t)$$

$$\tau_t = \rho Vc/hA_s = \rho Dc/6h = \frac{2700\ kg/m^3 \times 0.075m \times 950\ J/kg\cdot K}{6 \times 75\ W/m^2\cdot K} = 427s.$$

$$t = -\tau_t \ln(0.1) = 427s \times 2.30 = 984s$$

From Eq. (5.6), the corresponding temperature at any location in the sphere is

$$T(984s) = T_{g,i} + (T_i - T_{g,i})\exp(-6ht/\rho Dc)$$

$$T(984s) = 300°C - 275°C\ \exp\left(-6 \times 75\ W/m^2\cdot K \times 984s / \left(2700\ kg/m^3 \times 0.075m \times 950\ J/kg\cdot K\right)\right)$$

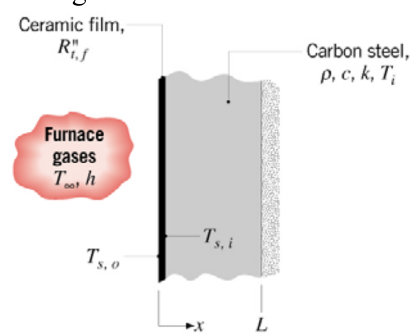
$$T(984s) = 272.5°C$$

If the product of the density and specific heat of copper is $(\rho c)_{Cu} \approx 8900\ kg/m^3 \times 400\ J/kg\cdot K = 3.56 \times 10^6\ J/m^3\cdot K$, is there any advantage to using copper spheres of equivalent diameter in lieu of aluminum spheres?

Does the time required for a sphere to reach a prescribed state of thermal energy storage change with increasing distance from the bed inlet? If so, how and why?

Problem: Furnace Start-up

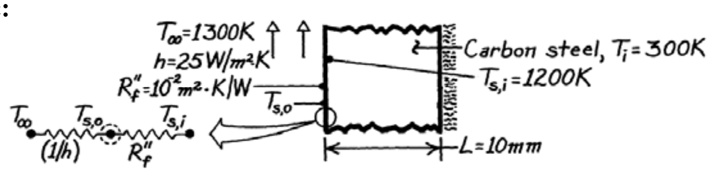
Problem 5.16: Heating of coated furnace wall during start-up.



KNOWN: Thickness and properties of furnace wall. Thermal resistance of ceramic coating on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature, (b) Corresponding value of coating surface temperature.

Schematic:



ASSUMPTIONS: (1) Constant properties, (2) Negligible coating thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel: ρ = 7850 kg/m^3, c = 430 J/kg·K, k = 60 W/m·K.

ANALYSIS: Heat transfer to the wall is determined by the total resistance to heat transfer from the gas to the surface of the steel, and not simply by the convection resistance.

Hence, with

$$U = \left(R''_{tot}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1} = \left(\frac{1}{25\ \text{W/m}^2\cdot\text{K}} + 10^{-2}\,\text{m}^2\cdot\text{K/W}\right)^{-1} = 20\ \text{W/m}^2\cdot\text{K}.$$

$$Bi = \frac{UL}{k} = \frac{20\ \text{W/m}^2\cdot\text{K}\times 0.01\ \text{m}}{60\ \text{W/m}\cdot\text{K}} = 0.0033 << 1$$

and the lumped capacitance method can be used.

(a) From Eqs. (5.6) and (5.7),

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/\tau_t) = \exp(-t/R_t C_t) = \exp(-Ut/\rho Lc)$$

$$t = -\frac{\rho Lc}{U}\ln\frac{T - T_\infty}{T_i - T_\infty} = -\frac{7850\ \text{kg/m}^3\,(0.01\ \text{m})\,430\ \text{J/kg}\cdot\text{K}}{20\ \text{W/m}^2\cdot\text{K}}\ln\frac{1200-1300}{300-1300}$$

$$t = 3886\text{s} = 1.08\text{h}.$$

(b) Performing an energy balance at the outer surface (s,o),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R''_f$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R''_f}{h + \left(1/R''_f\right)} = \frac{25\ \text{W/m}^2\cdot\text{K}\times 1300\ \text{K} + 1200\ \text{K}/10^{-2}\text{m}^2\cdot\text{K/W}}{(25+100)\,\text{W/m}^2\cdot\text{K}}$$

$$T_{s,o} = 1220\ \text{K}.$$

How does the coating affect the thermal time constant?

Transient Conduction: Spatial Effects and the Role of Analytical Solutions

Chapter 5

Sections 5.4 through 5.8

Solution to the Heat Equation for a Plane Wall with Symmetrical Convection Conditions

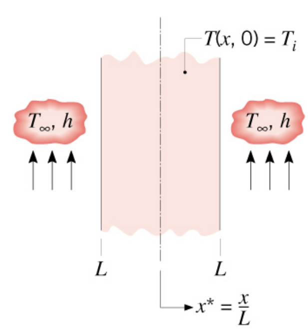
- If the lumped capacitance approximation can not be made, consideration must be given to spatial, as well as temporal, variations in temperature during the transient process.
- For a plane wall with symmetrical convection conditions and constant properties, the heat equation and initial/boundary conditions are:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (5.26)$$

$$T(x,0) = T_i \qquad (5.27)$$

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \qquad (5.28)$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h\left[T(L,t) - T_\infty\right] \qquad (5.29)$$



- Existence of seven independent variables:

$$T = T(x,t,T_i,T_\infty,k,\alpha,h) \qquad (5.30)$$

How may the functional dependence be simplified?

- Non-dimensionalization of Heat Equation and Initial/Boundary Conditions:

  Dimensionless temperature difference: $\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}$

  Dimensionless coordinate: $x^* \equiv \frac{x}{L}$

  Dimensionless time: $t^* \equiv \frac{\alpha t}{L^2} \equiv Fo$

  $Fo \rightarrow$ the Fourier Number

  The Biot Number: $Bi \equiv \frac{hL}{k_{solid}}$

  $\theta^* = f\left(x^*, Fo, Bi\right)$

- Exact Solution:

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp\left(-\zeta_n^2 Fo\right)\cos\left(\zeta_n x^*\right) \quad (5.39a)$$

$$C_n = \frac{4\sin\zeta_n}{2\zeta_n + \sin\left(2\zeta_n\right)} \qquad \zeta_n \tan\zeta_n = Bi \quad (5.39b,c)$$

See Appendix B.3 for first four roots (eigenvalues $\zeta_1,\ldots,\zeta_4$) of Eq. (5.39c)

- The One-Term Approximation $\left(\text{Valid for } Fo > 0.2\right)$
  - Variation of midplane temperature ($x^* = 0$) with time$\left(Fo\right)$:

$$\theta_o^* \equiv \frac{\left(T_o - T_\infty\right)}{\left(T_i - T_\infty\right)} \approx C_1 \exp\left(-\zeta_1^2 Fo\right) \quad (5.41)$$

  Table 5.1 $\rightarrow$ C_1 and ζ_1 as a function of Bi

  - Variation of temperature with location (x^*) and time$\left(Fo\right)$:

$$\theta^* = \theta_o^* \cos\left(\zeta_1 x^*\right) \quad (5.40b)$$

  - Change in thermal energy storage with time:

$$\Delta E_{st} = -Q \quad (5.43a)$$

$$Q = Q_o\left(1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*\right) \quad (5.46)$$

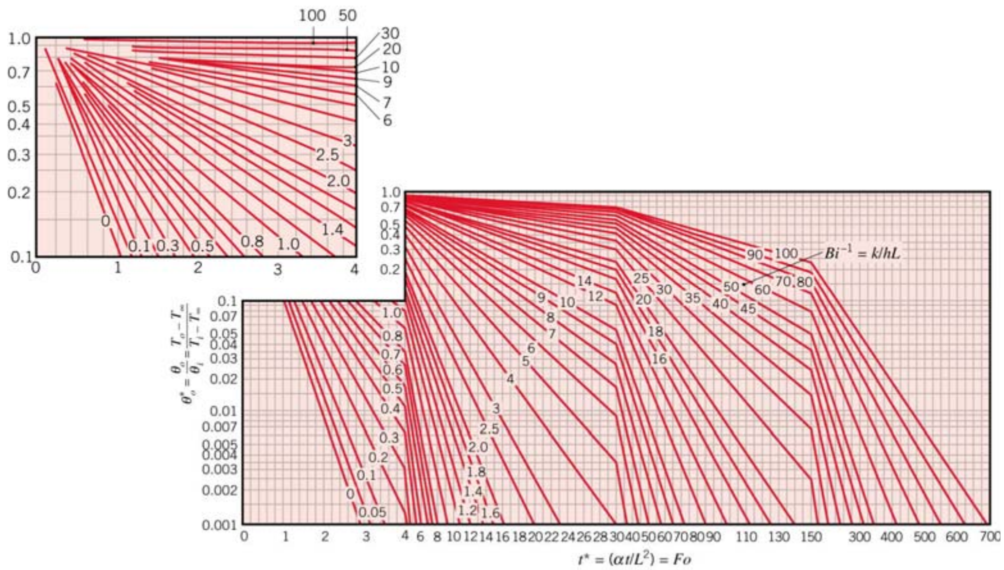
$$Q_o = \rho c V\left(T_i - T_\infty\right) \quad (5.44)$$

Can the foregoing results be used for a plane wall that is well insulated on one side and convectively heated or cooled on the other?

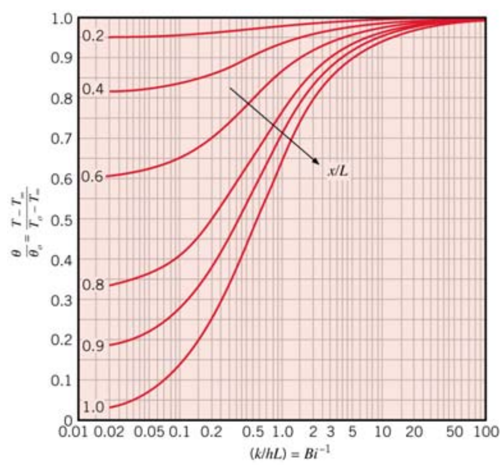
Can the foregoing results be used if an isothermal condition $\left(T_s \neq T_i\right)$ is instantaneously imposed on both surfaces of a plane wall or on one surface of a wall whose other surface is well insulated?

Graphical Representation of the One-Term Approximation
The Heisler Charts, Section 5 S.1

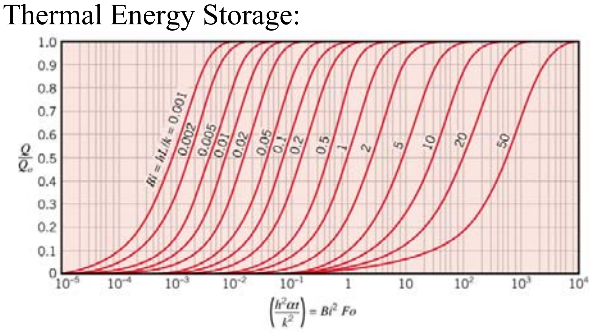
- Midplane Temperature:



- Temperature Distribution:



- Change in Thermal Energy Storage:

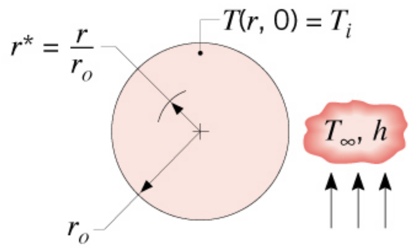


Radial Systems

- Long Rods or Spheres Heated or Cooled by Convection.

$Bi = hr_o / k$

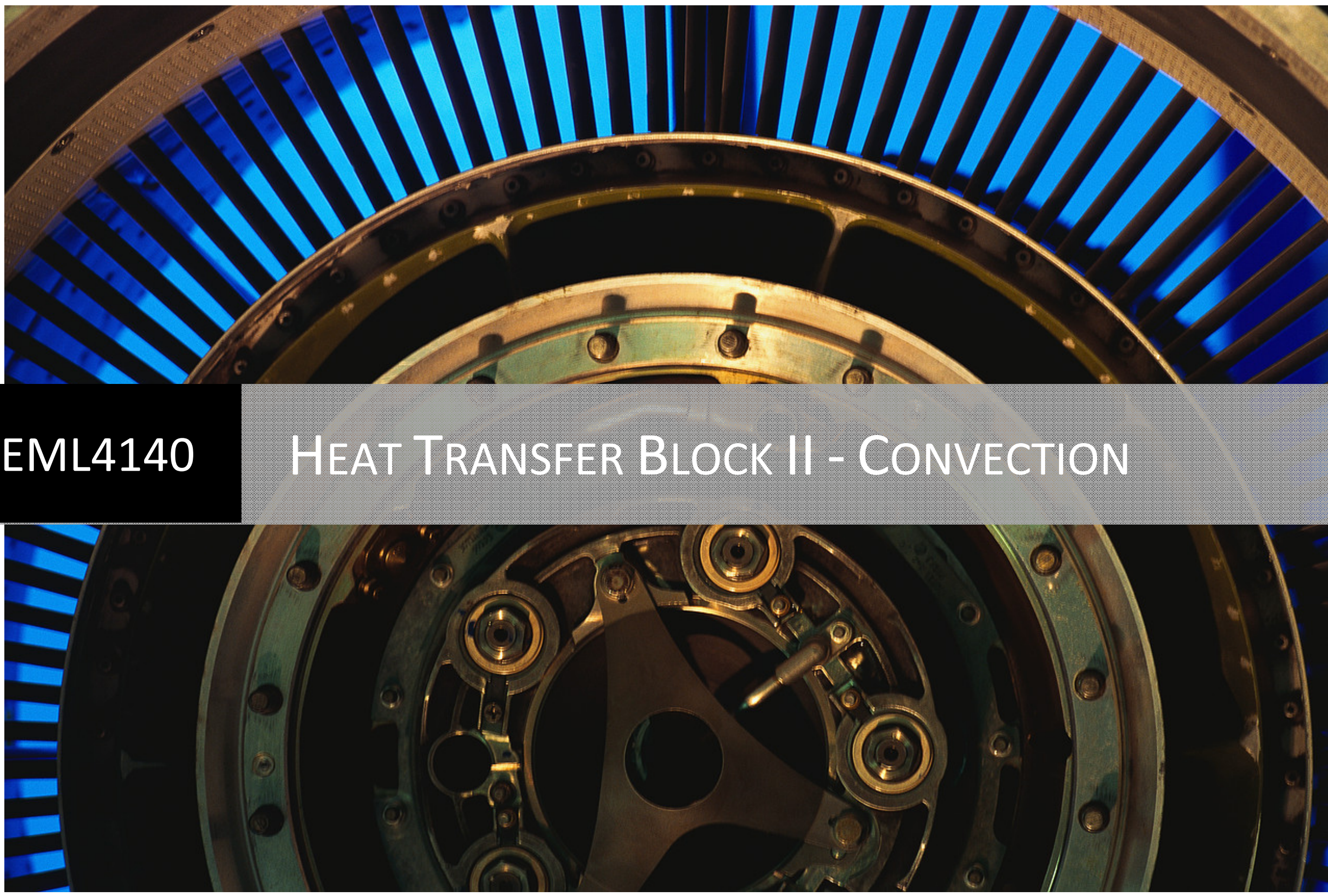
$Fo = \alpha t / r_o^2$



- One-Term Approximations:
  Long Rod: Eqs. (5.49) and (5.51)
  Sphere: Eqs. (5.50) and (5.52)

  $C_1, \zeta_1 \rightarrow$ Table 5.1

- Graphical Representations:
  Long Rod: Figs. 5 S.4 – 5 S.6
  Sphere: Figs. 5 S.7 – 5 S.9



EML4140 HEAT TRANSFER BLOCK II - CONVECTION

http://www.mae.ufl.edu/courses/spring2008/eml4140

Introduction to Convection: Flow and Thermal Considerations

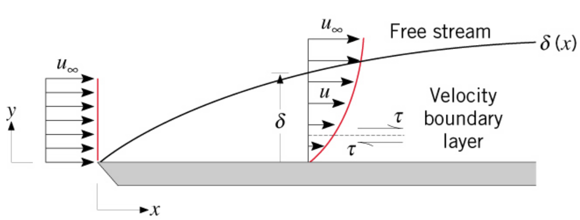
Chapter Six and Appendix D

Sections 6.1 through 6.8

and D.1 through D.3

Boundary Layers: Physical Features

- Velocity Boundary Layer
  - A consequence of viscous effects associated with relative motion between a fluid and a surface.
  - A region of the flow characterized by shear stresses and velocity gradients.
  - A region between the surface and the free stream whose thickness δ increases in the flow direction.
  - Why does δ increase in the flow direction?
  - Manifested by a surface shear stress τ_s that provides a drag force, F_D .
  - How does τ_s vary in the flow direction? Why?

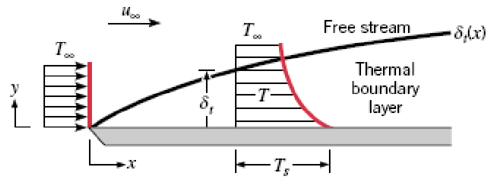


$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$F_D = \int_{A_s} \tau_s dA_s$$

- Thermal Boundary Layer



- A consequence of heat transfer between the surface and fluid.
- A region of the flow characterized by temperature gradients and heat fluxes.
- A region between the surface and the free stream whose thickness δ_t increases in the flow direction.

$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$

- Why does δ_t increase in the flow direction?

$$q_s'' = -k_f \frac{\partial T}{\partial y}\Big|_{y=0}$$

- Manifested by a surface heat flux q_s'' and a convection heat transfer coefficient h .

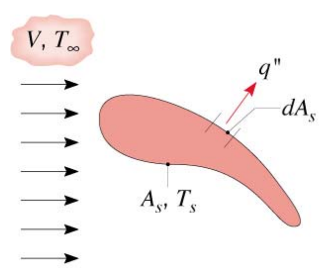
$$h \equiv \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty}$$

- If $(T_s - T_\infty)$ is constant, how do q_s'' and h vary in the flow direction?

Distinction between Local and Average Heat Transfer Coefficients

- Local Heat Flux and Coefficient:

$$q_s'' = h(T_s - T_\infty)$$



- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

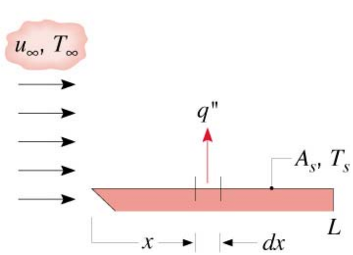
$$q = \bar{h} A_s (T_s - T_\infty)$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty) \int_{A_s} h dA_s$$

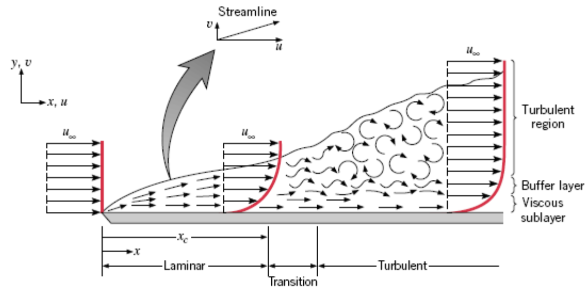
$$\bar{h} = \frac{1}{A_s} \int_{A_s} h dA_s$$

- For a flat plate in parallel flow:

$$\bar{h} = \frac{1}{L} \int_0^L h dx$$



Boundary Layer Transition



- How would you characterize conditions in the laminar region of boundary layer development? In the turbulent region?
- What conditions are associated with transition from laminar to turbulent flow?
- Why is the Reynolds number an appropriate parameter for quantifying transition from laminar to turbulent flow?
- Transition criterion for a flat plate in parallel flow:

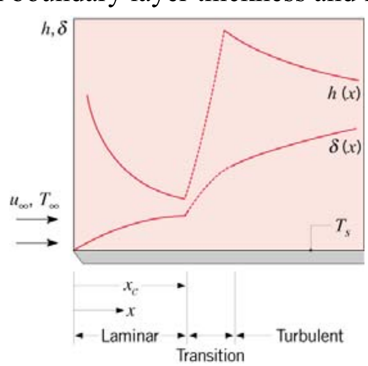
$$\text{Re}_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{critical Reynolds number}$$

$x_c \rightarrow$ location at which transition to turbulence begins

$$10^5 \lesssim \text{Re}_{x,c} \lesssim 3 \times 10^6$$

What may be said about transition if $\text{Re}_L < \text{Re}_{x,c}$? If $\text{Re}_L > \text{Re}_{x,c}$?

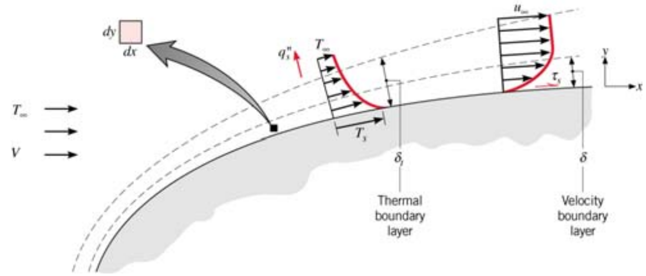
- Effect of transition on boundary layer thickness and local convection coefficient:



Why does transition provide a significant increase in the boundary layer thickness?

Why does the convection coefficient decay in the laminar region? Why does it increase significantly with transition to turbulence, despite the increase in the boundary layer thickness? Why does the convection coefficient decay in the turbulent region?

The Boundary Layer Equations



- Consider concurrent velocity and thermal boundary layer development for steady, two-dimensional, incompressible flow with constant fluid properties (μ, c_p, k) and negligible body forces.
- Apply conservation of mass, Newton's 2nd Law of Motion and conservation of energy to a differential control volume and invoke the boundary layer approximations.

Velocity Boundary Layer:

$$\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial p}{\partial x} \approx \frac{dp_\infty}{dx}$$

Thermal Boundary Layer:

$$\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$$

- Conservation of Mass:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

In the context of flow through a differential control volume, what is the physical significance of the foregoing terms, if each is multiplied by the mass density of the fluid?

- Newton's Second Law of Motion:

x-direction :

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2}$$

What is the physical significance of each term in the foregoing equation?

Why can we express the pressure gradient as dp_∞/dx instead of $\partial p / \partial x$?

- Conservation of Energy:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

What is the physical significance of each term in the foregoing equation?

What is the second term on the right-hand side called and under what conditions may it be neglected?

Boundary Layer Similarity

- As applied to the boundary layers, the principle of similarity is based on determining similarity parameters that facilitate application of results obtained for a surface experiencing one set of conditions to geometrically similar surfaces experiencing different conditions. (Recall how introduction of the similarity parameters *Bi* and *Fo* permitted generalization of results for transient, one-dimensional condition).
- Dependent boundary layer variables of interest are:

$$\tau_s \text{ and } q'' \text{ or } h$$

- For a prescribed geometry, the corresponding independent variables are:

Geometrical: Size (L), Location (x,y)
Hydrodynamic: Velocity (V)
Fluid Properties:
Hydrodynamic: ρ,μ
Thermal: c_p,k

Hence,

$$u=f\left(x,y,L,V,\rho,\mu\right)$$
$$\tau_s=f\left(x,L,V,\rho,\mu\right)$$

and

$$T = f\left(x, y, L, V, \rho, \mu, c_p, k\right)$$
$$h = f\left(x, L, V, \rho, \mu, c_p, k\right)$$

- Key similarity parameters may be inferred by non-dimensionalizing the momentum and energy equations.
- Recast the boundary layer equations by introducing dimensionless forms of the independent and dependent variables.

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$
$$u^* \equiv \frac{u}{V} \qquad v^* \equiv \frac{v}{V}$$
$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

- Neglecting viscous dissipation, the following normalized forms of the x-momentum and energy equations are obtained:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{\text{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$
$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\text{Re}_L \Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$\text{Re}_L \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu} \rightarrow \text{ the Reynolds Number}$$

$$\Pr \equiv \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \rightarrow \text{ the Prandtl Number}$$

How may the Reynolds and Prandtl numbers be interpreted physically?

- For a prescribed geometry,

$$u^* = f\left(x^*, y^*, \text{Re}_L\right)$$
$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

The dimensionless shear stress, or local friction coefficient, is then

$$C_f \equiv \frac{\tau_s}{\rho V^2 / 2} = \frac{2}{\text{Re}_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$
$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = f\left(x^*, \text{Re}_L\right)$$
$$C_f = \frac{2}{\text{Re}_L} f\left(x^*, \text{Re}_L\right)$$

What is the functional dependence of the average friction coefficient?

- For a prescribed geometry,

$$T^* = f\left(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

$$h = \frac{-k_f \partial T/\partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{\left(T_\infty - T_s\right)}{\left(T_s - T_\infty\right)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless local convection coefficient is then

$$Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, \mathrm{Re}_L, \mathrm{Pr}\right)$$

$Nu \rightarrow$ local Nusselt number

What is the functional dependence of the average Nusselt number?

How does the Nusselt number differ from the Biot number?

The Reynolds Analogy

- Equivalence of dimensionless momentum and energy equations for negligible pressure gradient (dp*/dx*~0) and Pr~1:

$$\underbrace{u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}} = \underbrace{\frac{1}{\mathrm{Re}}\frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}}^{\text{Advection terms}} = \overbrace{\frac{1}{\mathrm{Re}}\frac{\partial^2 T^*}{\partial y^{*2}}}^{\text{Diffusion}}$$

- Hence, for equivalent boundary conditions, the solutions are of the same form:

$$u^* = T^*$$

$$\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

$$C_f\frac{\mathrm{Re}}{2} = Nu$$

or, with the Stanton number defined as,

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr}$$

With Pr = 1, the Reynolds analogy, which relates important parameters of the velocity and thermal boundary layers, is

$$\frac{C_f}{2} = St$$

- Modified Reynolds (Chilton-Colburn) Analogy
  - An empirical result that extends applicability of the Reynolds analogy:

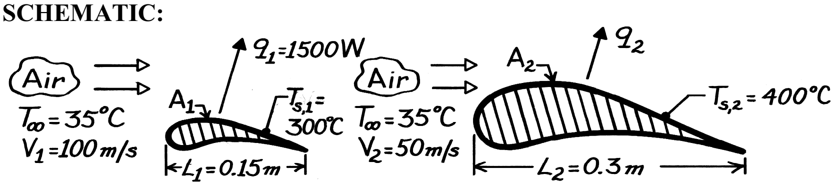
$$\frac{C_f}{2} = St\,Pr^{2/3} \equiv j_H \qquad 0.6 < Pr < 60$$

Colburn j factor for heat transfer

  - Applicable to laminar flow if dp*/dx* ~ 0.
  - Generally applicable to turbulent flow without restriction on dp*/dx*.

Problem 6.19: Determination of heat transfer rate for prescribed turbine blade operating conditions from wind tunnel data obtained for a geometrically similar but smaller blade. The blade surface area may be assumed to be directly proportional to its characteristic length $(A_s \propto L)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Surface area A is directly proportional to characteristic length L, (4) Negligible radiation, (5) Blade shapes are geometrically similar.

ANALYSIS: For a prescribed geometry,

$$\overline{Nu} = \frac{\bar{h}L}{k} = f(Re_L, Pr).$$

Problem: Turbine Blade Scaling (cont.)

The Reynolds numbers for the blades are

$$Re_{L,1} = (V_1 L_1/\nu_1) = 15\,m^2/s/\nu_1 \qquad Re_{L,2} = (V_2 L_2/\nu_2) = 15\,m^2/s/\nu_2.$$

Hence, with constant properties $(\nu_1 = \nu_2)$, $Re_{L,1} = Re_{L,2}$. Also, $Pr_1 = Pr_2$.

Therefore,

$$\overline{Nu}_2 = \overline{Nu}_1$$
$$(\bar{h}_2 L_2/k_2) = (\bar{h}_1 L_1/k_1)$$
$$\bar{h}_2 = \frac{L_1}{L_2}\bar{h}_1 = \frac{L_1}{L_2}\frac{q_1}{A_1(T_{s,1} - T_\infty)}$$

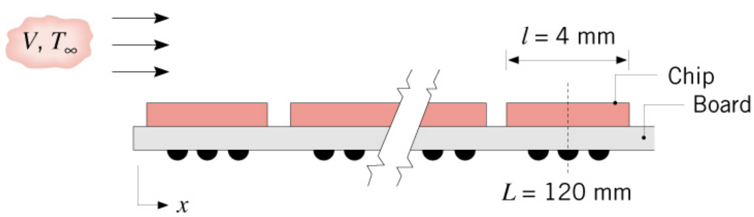
The heat rate for the *second blade* is then

$$q_2 = \bar{h}_2 A_2 (T_{s,2} - T_\infty) = \frac{L_1}{L_2}\frac{A_2}{A_1}\frac{(T_{s,2} - T_\infty)}{(T_{s,1} - T_\infty)} q_1$$
$$q_2 = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = \frac{(400 - 35)}{(300 - 35)}(1500\ W)$$
$$q_2 = 2066\ W.$$

COMMENTS: (i) The variation in ν from Case 1 to Case 2 would cause $Re_{L,2}$ to differ from $Re_{L,1}$. However, for air and the prescribed temperatures, this non-constant property effect is small. (ii) If the Reynolds numbers were not equal $(Re_{L,1} \neq Re_{L2})$, knowledge of the specific form of $f(Re_L, Pr)$ would be needed to determine h_2.

Problem: Nusselt Number

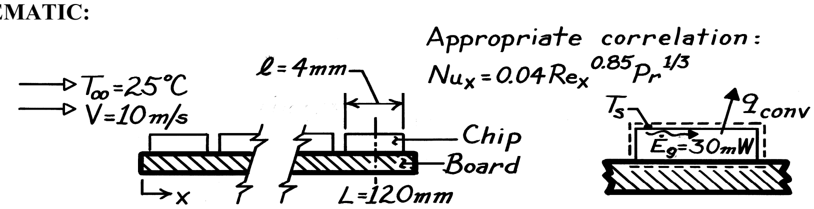
Problem 6.26: Use of a *local* Nusselt number correlation to estimate the surface temperature of a chip on a circuit board.



KNOWN: Expression for the local heat transfer coefficient of air at prescribed velocity and temperature flowing over electronic elements on a circuit board and heat dissipation rate for a 4 × 4 mm chip located 120mm from the leading edge.

FIND: Surface temperature of the chip surface, T_s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Power dissipated within chip is lost by convection across the upper surface only, (3) Chip surface is isothermal, (4) The average heat transfer coefficient for the chip surface is equivalent to the local value at x = L, (5) Negligible radiation.

PROPERTIES: *Table A-4*, Air (Evaluate properties at the *average temperature* of air in the boundary layer. Assuming $T_s = 45°C$, $T_{ave} = (45 + 25)/2 = 35°C = 308K$. Also, p = 1atm): $\nu = 16.69 \times 10^{-6} m^2/s$, $k = 26.9 \times 10^{-3}$ W/m·K, Pr = 0.703.

ANALYSIS: From an energy balance on the chip,

$$q_{conv} = \dot{E}_g = 30mW.$$

Newton's law of cooling for the upper chip surface can be written as

$$T_s = T_\infty + q_{conv} / \overline{h} A_{chip} \qquad (2)$$

where $A_{chip} = \ell^2$.

Assuming that the *average* heat transfer coefficient $(\overline{h})$ over the chip surface is equivalent to the *local* coefficient evaluated at x = L, that is, $\overline{h}_{chip} \approx h_x(L)$, the local coefficient can be evaluated by applying the prescribed correlation at x = L.

$$Nu_x = \frac{h_x x}{k} = 0.04\left[\frac{Vx}{\nu}\right]^{0.85} Pr^{1/3}$$

$$h_L = 0.04\frac{k}{L}\left[\frac{VL}{\nu}\right]^{0.85} Pr^{1/3}$$

Problem: Nusselt Number (cont.)

$$h_L = 0.04\left[\frac{0.0269 \text{ W/m}\cdot\text{K}}{0.120 \text{ m}}\right]\left[\frac{10 \text{ m/s} \times 0.120 \text{ m}}{16.69\times10^{-6} \text{ m}^2/\text{s}}\right]^{0.85}(0.703)^{1/3} = 107 \text{ W/m}^2\cdot\text{K}.$$

From Eq. (2), the surface temperature of the chip is

$$T_s = 25°C + 30\times10^{-3} \text{ W}/107 \text{ W/m}^2\cdot\text{K}\times(0.004\text{m})^2 = 42.5°C.$$

COMMENTS: (1) The estimated value of T_{ave} used to evaluate the air properties is reasonable.

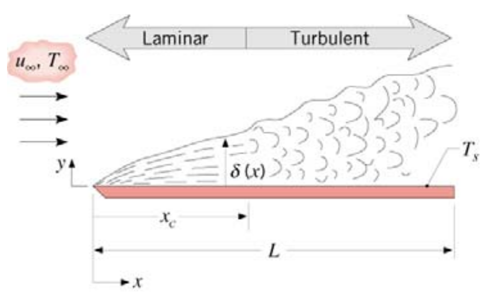
(2) How else could $\overline{h}_{chip}$ have been evaluated? Is the assumption of $\overline{h} = h_L$ reasonable?

External Flow: The Flat Plate in Parallel Flow

Chapter 7

Section 7.1 through 7.3

Physical Features



- As with all external flows, the boundary layers develop freely without constraint.
- Boundary layer conditions may be entirely laminar, laminar and turbulent, or entirely turbulent.
- To determine the conditions, compute

$$\mathrm{Re}_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu}$$

and compare with the critical Reynolds number for transition to turbulence, $\mathrm{Re}_{x,c}$.

$\mathrm{Re}_L < \mathrm{Re}_{x,c} \rightarrow$ laminar flow throughout

$\mathrm{Re}_L > \mathrm{Re}_{x,c} \rightarrow$ transition to turbulent flow at $x_c / L = \mathrm{Re}_{x,c} / \mathrm{Re}_L$

- Value of $\mathrm{Re}_{x,c}$ depends on free stream turbulence and surface roughness. Nominally,

$$\mathrm{Re}_{x,c} \approx 5 \times 10^5.$$

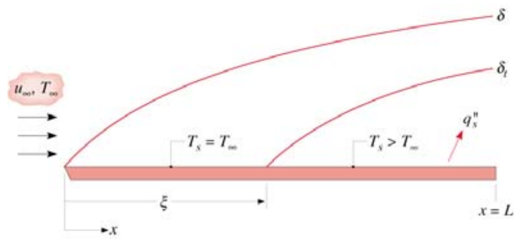
- If boundary layer is tripped at the leading edge

$$\mathrm{Re}_{x,c} = 0$$

and the flow is turbulent throughout.

- Surface thermal conditions are commonly idealized as being of uniform temperature T_s or uniform heat flux q_s'' . Is it possible for a surface to be concurrently characterized by uniform temperature and uniform heat flux?

- Thermal boundary layer development may be delayed by an unheated starting length.



Equivalent surface and free stream temperatures for $x < \xi$ and uniform T_s (or q_s'') for $x > \xi$.

Similarity Solution for Laminar, Constant-Property Flow over an Isothermal Plate

- Based on premise that the dimensionless x-velocity component, u/u_∞ , and temperature, $T^* \equiv \left[(T - T_s)/(T_\infty - T_s)\right]$, can be represented exclusively in terms of a dimensionless similarity parameter

$$\eta \equiv y\left(u_\infty / \nu x\right)^{1/2}$$

- Similarity permits transformation of the partial differential equations associated with the transfer of x-momentum and thermal energy to ordinary differential equations of the form

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

where $\left(u/u_\infty\right) \equiv df/d\eta$, and

$$\frac{d^2 T^*}{d\eta^2} + \frac{\mathrm{Pr}}{2} f \frac{dT^*}{d\eta} = 0$$

- Subject to prescribed boundary conditions, numerical solutions to the momentum and energy equations yield the following results for important local boundary layer parameters:
  - with $u/u_\infty = 0.99$ at $\eta = 5.0$,

$$\delta = \frac{5.0}{\left(u_\infty / \nu x\right)^{1/2}} = \frac{5x}{\left(\text{Re}_x\right)^{1/2}}$$

  - with $\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{u_\infty / \nu x} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0}$

and $\left. d^2 f / d\eta^2 \right|_{\eta=0} = 0.332$,

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2 / 2} = 0.664\, \text{Re}_x^{-1/2}$$

  - with $h_x = q_s'' / \left(T_s - T_\infty\right) = k \partial T^* / \partial y \big|_{y=0} = k\left(u_\infty / \nu x\right)^{1/2} dT^* / d\eta \big|_{\eta=0}$

and $\left. dT^* / d\eta \right|_{\eta=0} = 0.332\ \text{Pr}^{1/3}$ for $\text{Pr} > 0.6$,

$$Nu_x = \frac{h_x x}{k} = 0.332\ \text{Re}_x^{1/2}\, \text{Pr}^{1/3}$$

and

$$\frac{\delta}{\delta_t} = \text{Pr}^{1/3}$$

- How would you characterize relative laminar velocity and thermal boundary layer growth for a gas? An oil? A liquid metal?
- How do the local shear stress and convection coefficient vary with distance from the leading edge?
- Average Boundary Layer Parameters:

$$\overline{\tau}_{s,x} \equiv \frac{1}{x}\int_0^x \tau_s dx$$

$$\overline{C}_{f,x} = 1.328\ \text{Re}_x^{-1/2}$$

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x dx$$

$$\overline{Nu}_x = 0.664\ \text{Re}_x^{1/2}\, \text{Pr}^{1/3}$$

- The effect of variable properties may be considered by evaluating all properties at the film temperature.

$$T_f = \frac{T_s + T_\infty}{2}$$

Turbulent Flow

- Local Parameters:

Empirical Correlations
$$C_{f,x} = 0.0592\ \mathrm{Re}_x^{-1/5}$$
$$Nu_x = 0.0296\ \mathrm{Re}_x^{4/5}\ \mathrm{Pr}^{1/3}$$

How do variations of the local shear stress and convection coefficient with distance from the leading edge for turbulent flow differ from those for laminar flow?

- Average Parameters:

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam}dx + \int_{x_c}^{L} h_{turb}dx\right)$$

Substituting expressions for the local coefficients and assuming $\mathrm{Re}_{x,c} = 5\times 10^5$,

$$\bar{C}_{f,L} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}$$

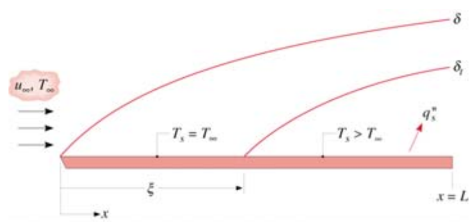
$$\overline{Nu}_L = \left(0.037\ \mathrm{Re}_L^{4/5} - 871\right)\mathrm{Pr}^{1/3}$$

For $\mathrm{Re}_{x,c} = 0$ or $L >> x_c$ $\left(\mathrm{Re}_L >> \mathrm{Re}_{x,c}\right)$,

$$\bar{C}_{f,L} = 0.074\ \mathrm{Re}_L^{-1/5}$$

$$\overline{Nu}_L = 0.037\ \mathrm{Re}_L^{4/5}\ \mathrm{Pr}^{1/3}$$

Special Cases: Unheated Starting Length (USL) and/or Uniform Heat Flux



For both uniform surface temperature (UST) and uniform surface heat flux (USF), the effect of the USL on the local Nusselt number may be represented as follows:

$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{\left[1-\left(\xi/x\right)^a\right]^b}$$

$$Nu_x\big|_{\xi=0} = C\ \mathrm{Re}_x^m\ \mathrm{Pr}^{1/3}$$

	Laminar		Turbulent	
	UST	USF	UST	USF
a	3/4	3/4	9/10	9/10
b	1/3	1/3	1/9	1/9
C	0.332	0.453	0.0296	0.0308
m	1/2	1/2	4/5	4/5

Sketch the variation of h_x versus $\left(x-\xi\right)$ for two conditions: $\xi > 0$ and $\xi = 0$.
What effect does an USL have on the local convection coefficient?

Special Cases (cont.)

- UST:

$$q_s'' = h_x (T_s - T_\infty) \quad q = \bar{h}_L A_s (T_s - T_\infty)$$

$$\overline{Nu}_L = \overline{Nu}_L\Big|_{\xi=0} \frac{L}{(L-\xi)}\left[1-(\xi/L)^{(p+1)/(p+2)}\right]^{p/(p+1)}$$

$p = 2$ for laminar flow throughout

$p = 8$ for turbulent flow throughout

$\bar{h}_L \rightarrow$ numerical integration for laminar/turbulent flow

$$\bar{h}_L = \frac{1}{L}\left[\int_{\xi}^{x_c} h_{lam}dx + \int_{x_c}^{L} h_{turb}dx\right]$$

- USF:

$$T_s = T_\infty + \frac{q_s''}{h_x} \qquad q = q_s'' A_s$$

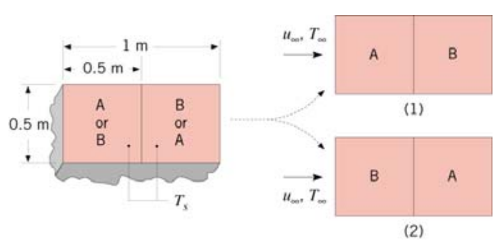
- Treatment of Non-Constant Property Effects:

Evaluate properties at the film temperature.

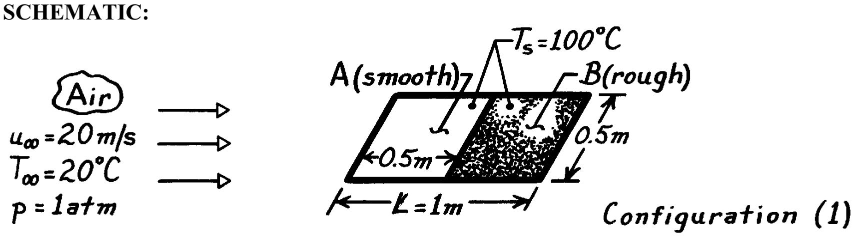
$$T_f = \frac{T_s + T_\infty}{2}$$

Problem: Orientation of Heated Surface

Problem 7.21: Preferred orientation (corresponding to lower heat loss) and the corresponding heat rate for a surface with adjoining smooth and roughened sections.



SCHEMATIC:



ASSUMPTIONS: (1) Surface B is sufficiently rough to trip the boundary layer when in the upstream position (Configuration 2); (2) $Re_{x,c} = 5 \times 10^5$ for flow over A in Configuration 1.

PROPERTIES: *Table A-4*, Air (T_f = 333K, 1 atm): $\nu = 19.2 \times 10^{-6}$ m^2/s, $k = 28.7 \times 10^{-3}$ W/m·K, Pr = 0.7.

ANALYSIS: With

$$Re_L = \frac{u_\infty L}{\nu} = \frac{20\ m/s \times 1m}{19.2 \times 10^{-6} m^2/s} = 1.04 \times 10^6.$$

transition will occur just before the rough surface (x_c = 0.48m) for Configuration 1. Hence,

$$\overline{Nu}_{L,1} = \left[0.037\left(1.04 \times 10^6\right)^{4/5} - 871\right] 0.7^{1/3} = 1366$$

Since $\overline{Nu}_{L,2} = 0.037\left(1.04 \times 10^6\right)^{4/5} (0.7)^{1/3} = 2139 > \overline{Nu}_{L,1}$, it follows that the lowest heat transfer is associated with Configuration 1.

For Configuration 1: $\frac{\bar{h}_{L,1} L}{k} = \overline{Nu}_{L,1} = 1366.$

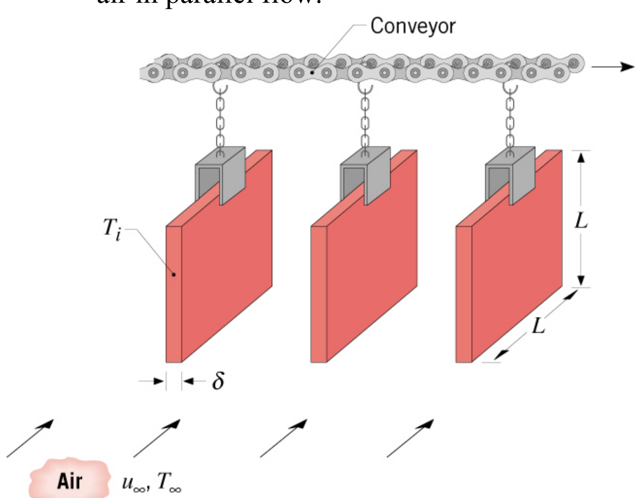
Hence

$$\bar{h}_{L,1} = 1366\left(28.7 \times 10^{-3} W/m \cdot K\right)/1m = 39.2\ W/m^2 \cdot K$$

$$q_1 = \bar{h}_{L,1} A\left(T_s - T_\infty\right) = 39.2\ W/m^2 \cdot K\left(0.5m \times 1m\right)\left(100 - 20\right)K = 1568\ W \qquad <$$

Comment: For a very short plate, a lower heat loss may be associated with Configuration 2. In fact, parametric calculations reveal that for L< 30 mm, this configuration provides the preferred orientation.

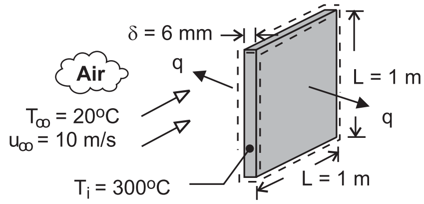
Problem 7.24: Convection cooling of steel plates on a conveyor by air in parallel flow.



KNOWN: Plate dimensions and initial temperature. Velocity and temperature of air in parallel flow over plates.

FIND: Initial rate of heat transfer from plate. Rate of change of plate temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) Negligible effect of conveyor velocity on boundary layer development, (3) Plates are isothermal, (4) Negligible heat transfer from edges of plate, (5) $Re_{x,c} = 5\times10^5$, (6) Constant properties.

PROPERTIES: *Table A-1*, AISI 1010 steel (573K): k_p = 49.2 W/m·K, c = 549 J/kg·K, ρ = 7832 kg/m^3. *Table A-4*, Air (p = 1 atm, T_f = 433K): ν = 30.4 × 10^{-6} m^2/s, k = 0.0361 W/m·K, Pr = 0.688.

ANALYSIS: The initial rate of heat transfer from a plate is

$$q = 2\bar{h}A_s\left(T_i - T_\infty\right) = 2\bar{h}L^2\left(T_i - T_\infty\right)$$

With $Re_L = u_\infty L/\nu = 10\,m/s\times1m/30.4\times10^{-6}m^2/s = 3.29\times10^5$, flow is laminar over the entire surface.

Hence,

$$\overline{Nu}_L = 0.664\,Re_L^{1/2}\,Pr^{1/3} = 0.664\left(3.29\times10^5\right)^{1/2}\left(0.688\right)^{1/3} = 336$$

$$\bar{h} = \left(k/L\right)\overline{Nu}_L = \left(0.0361\,W/m\cdot K/1m\right)336 = 12.1\,W/m^2\cdot K$$

$$q = 2\times12.1\,W/m^2\cdot K\left(1m\right)^2\left(300-20\right)°C = 6780\,W$$

Performing an energy balance at an instant of time for a control surface about the plate, $-\dot{E}_{out} = \dot{E}_{st}$,

$$\rho\delta L^2 c\left.\frac{dT}{dt}\right|_i = -\bar{h}\,2L^2\left(T_i - T_\infty\right)$$

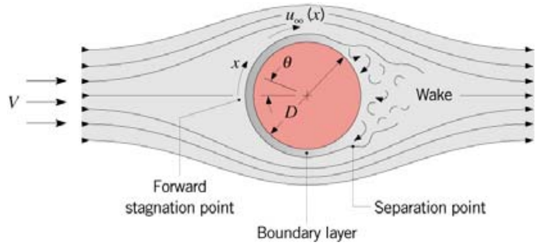
$$\left.\frac{dT}{dt}\right|_i = -\frac{2\left(12.1\,W/m^2\cdot K\right)\left(300-20\right)°C}{7832\,kg/m^3\times0.006m\times549\,J/kg\cdot K} = -0.26°C/s$$

COMMENTS: (1) With $Bi = \bar{h}\left(\delta/2\right)/k_p = 7.4\times10^{-4}$, use of the lumped capacitance method is appropriate.

(2) Despite the large plate temperature and the small convection coefficient, if adjoining plates are in close proximity, radiation exchange with the surroundings will be small and the assumption of negligible radiation is justifiable.

The Cylinder in Cross Flow

- Conditions depend on special features of boundary layer development, including onset at a stagnation point and separation, as well as transition to turbulence.

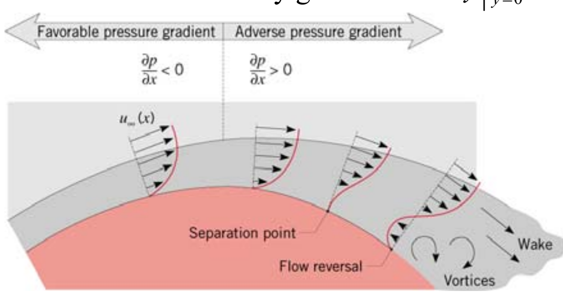


– Stagnation point: Location of zero velocity $(u_\infty = 0)$ and maximum pressure.

– Followed by boundary layer development under a favorable pressure gradient $(dp/dx < 0)$ and hence acceleration of the free stream flow $(du_\infty/dx > 0)$.

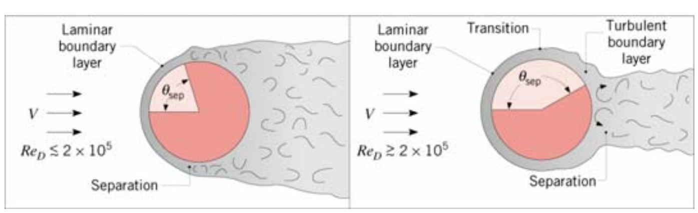
– As the rear of the cylinder is approached, the pressure must begin to increase. Hence, there is a minimum in the pressure distribution, *p(x),* after which boundary layer development occurs under the influence of an adverse pressure gradient $(dp/dx > 0,\ du_\infty/dx < 0)$.

– Separation occurs when the velocity gradient $du/dy\big|_{y=0}$ reduces to zero



and is accompanied by flow reversal and a downstream wake.

– Location of separation depends on boundary layer transition.



$$\mathrm{Re}_D \equiv \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

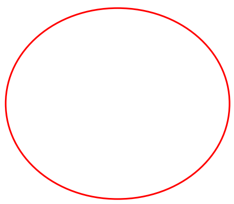
Cylinder in Cross Flow (cont.)

- What features differentiate boundary development for the flat plate in parallel flow from that for flow over a cylinder?

- Force imposed by the flow is due to the combination of *friction* and *form drag*.

  The dimensionless form of the drag force is

$$C_D = \frac{F_D}{A_f\left(\rho V^2/2\right)} \rightarrow \text{Figure 7.8}$$



Cylinder in Cross Flow (cont.)

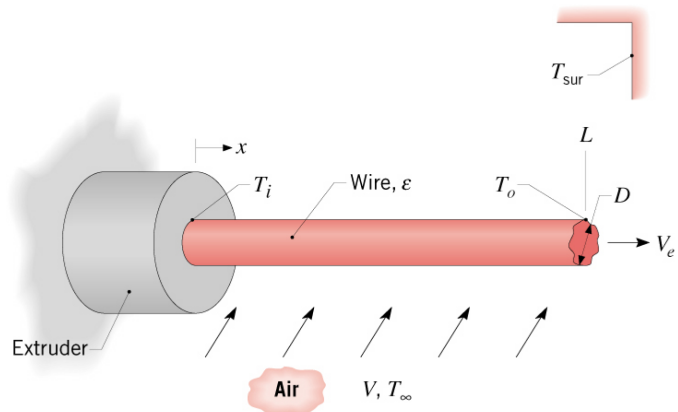
- Heat Transfer Considerations
  - The Local Nusselt Number:
    - How does the local Nusselt number vary with θ for $Re_D < 2\times10^5$? What conditions are associated with maxima and minima in the variation?
    - How does the local Nusselt number vary with θ for $Re_D > 2\times10^5$? What conditions are associated with maxima and minima in the variation?
  - The Average Nusselt Number $\left(\overline{Nu}_D \equiv \bar{h}D/k\right)$:
    - Churchill and Bernstein Correlation:

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+\left(0.4/Pr\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

    - Cylinders of Noncircular Cross Section:

$$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$$
$$C, m \rightarrow \text{Table 7.3}$$

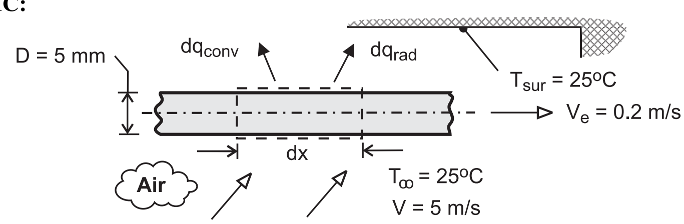
Problem 7.63: Cooling of extruded copper wire by convection and radiation.



KNOWN: Velocity, diameter, initial temperature and properties of extruded wire. Temperature and velocity of air. Temperature of surroundings.

FIND: (a) Differential equation for temperature distribution T(x) along the wire, (b) Exact solution for negligible radiation and corresponding value of temperature at prescribed length (x = L = 5m) of wire, (c) Effect of radiation on temperature of wire at prescribed length. Effect of wire velocity and emissivity on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible variation of wire temperature in radial direction, (2) Negligible effect of axial conduction along the wire, (3) Constant properties, (4) Radiation exchange between small surface and large enclosure, (5) Motion of wire has a negligible effect on the convection coefficient ($V_e << V$).

PROPERTIES: Copper: $\rho = 8900\,kg/m^3$, $c_p = 400\,J/kg \cdot K$, $\varepsilon = 0.55$. Air: $k = 0.037\,W/m \cdot K$, $\nu = 3 \times 10^{-5}\,m^2/s$, $Pr = 0.69$.

ANALYSIS: (a) Applying conservation of energy to a stationary control surface, through which the wire moves, steady-state conditions exist and $\dot{E}_{in} - \dot{E}_{out} = 0$.

Hence, with *inflow* due to *advection* and *outflow* due to *advection*, *convection* and *radiation*,

$$\rho V_e A_c c_p T - \rho V_e A_c c_p (T + dT) - dq_{conv} - dq_{rad} = 0$$

$$-\rho V_e\left(\pi D^2/4\right)c_p dT - \pi D dx\left[\bar{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = 0$$

$$\frac{dT}{dx} = -\frac{4}{\rho V_e D c_p}\left[\bar{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] \qquad (1) \quad <$$

Alternatively, if the control surface is fixed to the wire, conditions are transient and the energy balance is of the form, $-\dot{E}_{out} = \dot{E}_{st}$, or

$$-\pi D\,dx\left[\bar{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right] = \rho\left(\frac{\pi D^2}{4}dx\right)c_p\frac{dT}{dt}$$

$$\frac{dT}{dt} = -\frac{4}{\rho D c_p}\left[\bar{h}\left(T - T_\infty\right) + \varepsilon\sigma\left(T^4 - T_{sur}^4\right)\right]$$

Dividing the left- and right-hand sides of the equation by dx/dt and $V_e = dx/dt$, respectively, Eq. (1) is obtained.

(b) Neglecting radiation, separating variables and integrating, Eq. (1) becomes

$$\int_{T_i}^{T}\frac{dT}{T - T_\infty} = -\frac{4\bar{h}}{\rho V_e D c_p}\int_0^x dx$$

$$\ln\left(\frac{T - T_\infty}{T_i - T_\infty}\right) = -\frac{4\bar{h}\,x}{\rho V_e D c_p}$$

$$T = T_\infty + \left(T_i - T_\infty\right)\exp\left(-\frac{4\bar{h}x}{\rho V_e D c_p}\right) \qquad (2) \quad <$$

Problem: Extrusion Process (cont.)

With $Re_D = VD/\nu = 5\,m/s \times 0.005\,m/3\times10^{-5}\,m^2/s = 833$, the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(833)^{1/2}(0.69)^{1/3}}{\left[1 + (0.4/0.69)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{833}{282,000}\right)^{5/8}\right]^{4/5} = 14.4$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.037\,W/m\cdot K}{0.005m}14.4 = 107\,W/m^2\cdot K$$

Hence, applying Eq. (2) at x = L,

$$T_o = 25°C + (575°C)\exp\left(-\frac{4\times107\,W/m^2\cdot K\times5m}{8900\,kg/m^3\times0.2\,m/s\times0.005m\times400\,J/kg\cdot K}\right)$$

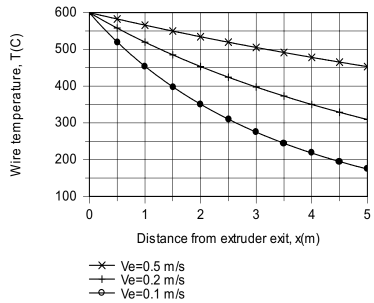
$$T_o = 340°C \quad <$$

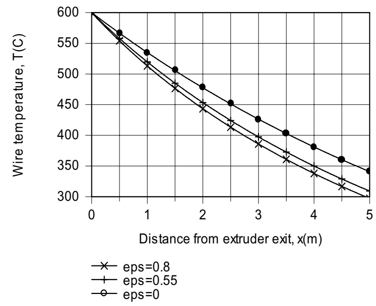
(c) Numerically integrating from x = 0 to x = L = 5.0m, we obtain

$$T_o = 309°C \quad <$$

Hence, radiation makes a discernable contribution to cooling of the wire.

Parametric calculations reveal the following distributions.





The speed with which the wire is drawn from the extruder has a significant influence on the temperature distribution. The temperature decay decreases with increasing V_e due to the increasing effect of advection on energy transfer in the x direction.

The effect of the surface emissivity is less pronounced, although, as expected, the temperature decay becomes more pronounced with increasing ε.

COMMENTS: (1) A critical parameter in wire extrusion processes is the *coiling temperature*, that is, the temperature at which the wire may be safely coiled for subsequent storage or shipment. The larger the production rate (V_e), the longer the cooling distance needed to achieve a desired coiling temperature.

(2) Cooling may be enhanced by increasing the cross-flow velocity, and the specific effect of V may also be explored.

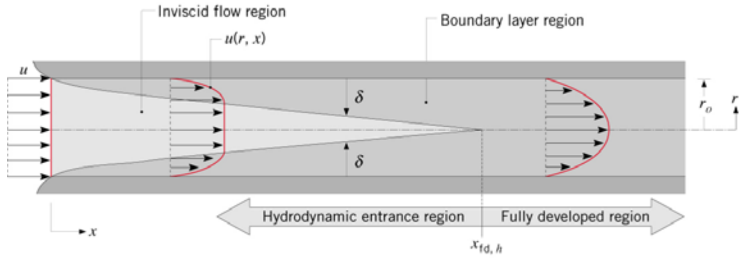
Internal Flow: General Considerations

Chapter 8

Sections 8.1 through 8.3

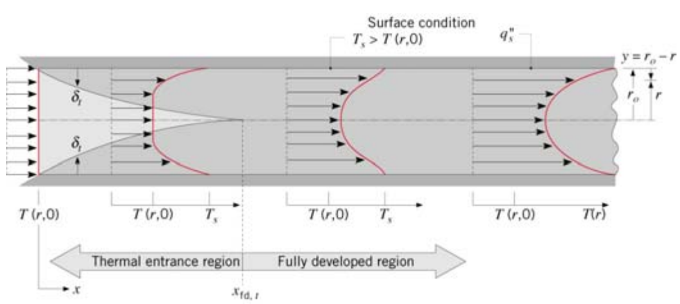
Entrance Conditions

- Must distinguish between entrance and fully developed regions.
- Hydrodynamic Effects: Assume laminar flow with uniform velocity profile at inlet of a circular tube.



  - Velocity boundary layer develops on surface of tube and thickens with increasing x.
  - Inviscid region of uniform velocity shrinks as boundary layer grows.
    - Does the centerline velocity change with increasing x? If so, how does it change?
  - Subsequent to boundary layer merger at the centerline, the velocity profile becomes parabolic and invariant with x. The flow is then said to be hydrodynamically fully developed.
    - How would the fully developed velocity profile differ for turbulent flow?

- Thermal Effects: Assume laminar flow with uniform temperature, $T(r,0) = T_i$, at inlet of circular tube with uniform surface temperature, $T_s \neq T_i$, or heat flux, q''_s.



  - Thermal boundary layer develops on surface of tube and thickens with increasing x.
  - Isothermal core shrinks as boundary layer grows.
  - Subsequent to boundary layer merger, dimensionless forms of the temperature profile (for T_s and q''_s) become independent of x. Conditions are then said to be thermally fully developed.
    - Is the temperature profile invariant with x in the fully developed region?

- For uniform surface temperature, what may be said about the change in the temperature profile with increasing *x*?
- For uniform surface heat flux, what may be said about the change in the temperature profile with increasing *x*?
- How do temperature profiles differ for laminar and turbulent flow?

The Mean Velocity and Temperature

- Absence of well-defined free stream conditions, as in external flow, and hence a reference velocity (u_∞) or temperature (T_∞), dictates the use of a cross-sectional mean velocity (u_m) and temperature (T_m) for internal flow.
- Linkage of mean velocity to mass flow rate:

$$\dot{m} = \rho u_m A_c$$

or,

$$\dot{m} = \int_{A_c} \rho u(r,x)\, dA_c$$

Hence,

$$u_m = \frac{\int_{A_c} \rho u(r,x)\, dA_c}{\rho A_c}$$

For *incompressible flow* in a *circular tube* of radius r_o,

$$u_m = \frac{2}{r_o^2} \int_o^{r_o} u(r,x) r\, dr$$

- Linkage of mean temperature to thermal energy transport associated with flow through a cross section:

$$\dot{E}_t = \int_{A_c} \rho u c_p T \, dA_c \equiv \dot{m} c_p T_m$$

Hence,

$$T_m = \frac{\int_{A_c} \rho u c_p T \, dA_c}{\dot{m} c_p}$$

- For incompressible, constant-property flow in a circular tube,

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} u(x,r) T(x,r) r \, dr$$

- Newton's Law of Cooling for the Local Heat Flux:

$$q_s'' = h(T_s - T_m)$$

What is the essential difference between use of T_m for internal flow and T_∞ for external flow?

Hydrodynamic and Thermal Entry Lengths

- Entry lengths depend on whether the flow is laminar or turbulent, which, in turn, depend on Reynolds number.

$$\mathrm{Re}_D \equiv \frac{\rho u_m D_h}{\mu}$$

The hydraulic diameter is defined as

$$D_h \equiv \frac{4 A_c}{P}$$

in which case,

$$\mathrm{Re}_D \equiv \frac{\rho u_m D_h}{\mu} = \frac{4 \dot{m}}{P \mu}$$

For a circular tube,

$$\mathrm{Re}_D = \frac{\rho u_m D}{\mu} = \frac{4 \dot{m}}{\pi D \mu}$$

– Onset of turbulence occurs at a critical Reynolds number of

$$\mathrm{Re}_{D,c} \approx 2300$$

– Fully turbulent conditions exist for

$$\mathrm{Re}_D \approx 10{,}000$$

- Hydrodynamic Entry Length

Laminar Flow: $\left(x_{fd,h}/D\right) \approx 0.05\,\mathrm{Re}_D$

Turbulent Flow: $10 < \left(x_{fd,h}/D\right) < 60$

- Thermal Entry Length

Laminar Flow: $\left(x_{fd,t}/D\right) \approx 0.05\,\mathrm{Re}_D\,\mathrm{Pr}$

Turbulent Flow: $10 < \left(x_{fd,t}/D\right) < 60$

- For laminar flow, how do hydrodynamic and thermal entry lengths compare for a gas? An oil? A liquid metal?

Fully Developed Conditions

- Assuming steady flow and constant properties, hydrodynamic conditions, including the velocity profile, are invariant in the fully developed region.

  What may be said about the variation of the mean velocity with distance from the tube entrance for steady, constant property flow?

- The pressure drop may be determined from knowledge of the friction factor f, where,

$$f \equiv -\frac{\left(dp/dx\right)D}{\rho u_m^2/2}$$

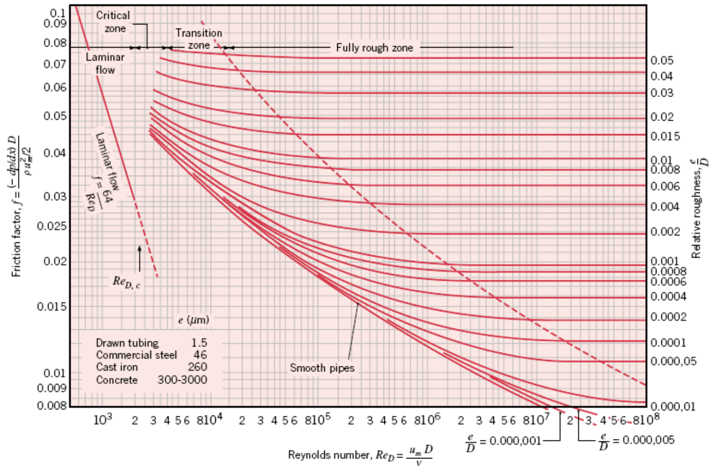
Laminar flow in a circular tube:

$$f = \frac{64}{\mathrm{Re}_D}$$

Turbulent flow in a smooth circular tube:

$$f = \left(0.790\,\ln \mathrm{Re}_D - 1.64\right)^{-2}$$

Turbulent flow in a roughened circular tube:



Pressure drop for fully developed flow from x_1 to x_2:

$$\Delta p = p_1 - p_2 = f \frac{\rho u_m^2}{2D}(x_2 - x_1)$$

and power requirement

$$P = \Delta p \dot{\forall} = \frac{\Delta p \dot{m}}{\rho}$$

- Requirement for fully developed thermal conditions:

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{fd,t} = 0$$

- Effect on the local convection coefficient:

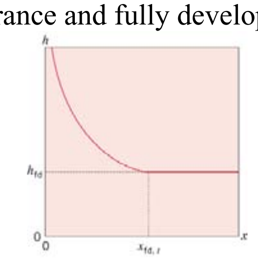
$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\Bigg|_{r=r_o} = \frac{-\partial T/\partial r|_{r=r_o}}{T_s - T_m} \neq f(x)$$

Hence, assuming constant properties,

$$\frac{q_s''/k}{T_s - T_m} = \frac{h}{k} \neq f(x)$$

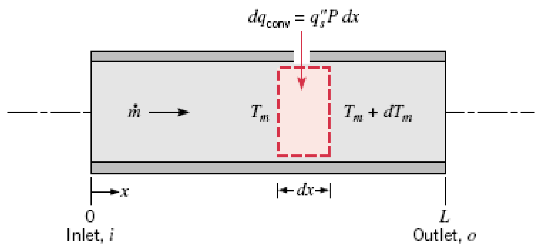
$$h \neq f(x)$$

Variation of h in entrance and fully developed regions:



Determination of the Mean Temperature

- Determination of $T_m(x)$ is an essential feature of an internal flow analysis. Determination begins with an energy balance for a differential control volume.



$$dq_{conv} = \dot{m}c_p\left[\left(T_m + dT_m\right) - T_m\right] = \dot{m}c_p dT_m$$

Integrating from the tube inlet to outlet,

$$q_{conv} = \dot{m}c_p\left(T_{m,o} - T_{m,i}\right) \qquad (1)$$

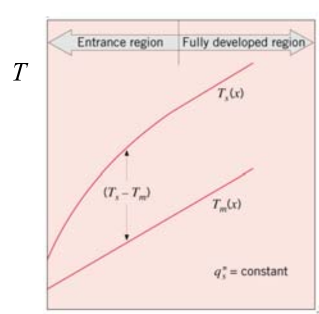
A differential equation from which $T_m(x)$ may be determined is obtained by substituting for $dq_{conv} = q_s''(P\,dx) = h(T_s - T_m)P\,dx$.

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} = \frac{P}{\dot{m}c_p}h\left(T_s - T_m\right) \qquad (2)$$

- Special Case: Uniform Surface Heat Flux

$$\frac{dT_m}{dx} = \frac{q_s''P}{\dot{m}c_p} \neq f(x)$$

$$T_m(x) = T_{m,i} + \frac{q_s''P}{\dot{m}c_p}x$$



Why does the surface temperature vary with x as shown in the figure?

In principle, what value does T_s assume at x=0?

Total heat rate:

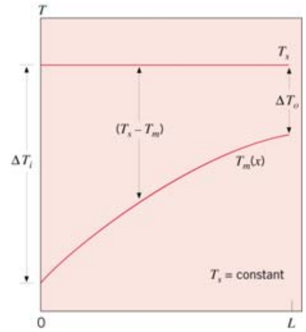
$$q_{conv} = q_s''\,PL$$

- Special Case: Uniform Surface Temperature

From Eq. (2), with $\Delta T \equiv T_s - T_m$

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{P}{\dot{m}c_p} h \Delta T$$

Integrating from x=0 to any downstream location,



$$\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}_x\right)$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x \, dx$$

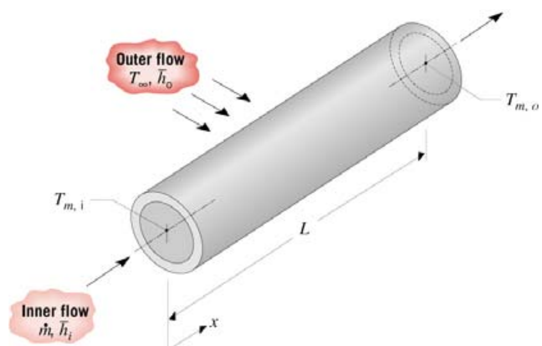
Overall Conditions:

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL}{\dot{m}c_p}\bar{h}\right) = \exp\left(-\frac{\bar{h}A_s}{\dot{m}c_p}\right)$$

$$q_{conv} = \bar{h}A_s \Delta T_{\ell m}$$

$$\Delta T_{\ell m} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)} \quad (3)$$

- Special Case: Uniform *External* Fluid Temperature



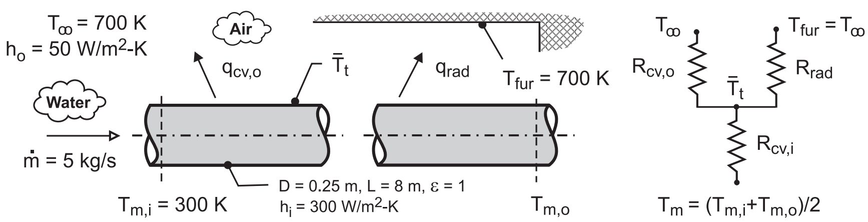
$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\bar{U}A_s}{\dot{m}c_p}\right) = \exp\left(-\frac{1}{\dot{m}c_p R_{tot}}\right)$$

$$q = \bar{U}A_s \Delta T_{\ell m} = \frac{\Delta T_{\ell m}}{R_{tot}}$$

$\Delta T_{\ell m} \rightarrow$ Eq. (3) with T_s replaced by T_∞.

Note: Replacement of T_∞ by $T_{s,o}$ if outer surface temperature is uniform.

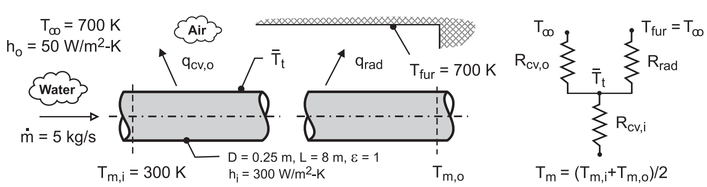
Problem 8.17: Estimate temperature of water emerging from a thin-walled tube heated by walls and air of a furnace. Inner and outer convection coefficients are known.



KNOWN: Water at prescribed temperature and flow rate enters a 0.25 m diameter, black thin-walled tube of 8-m length, which passes through a large furnace whose walls and air are at a temperature of $T_{fur} = T_\infty = 700$ K. The convection coefficients for the internal water flow and external furnace air are 300 $W/m^2 \cdot K$ and 50 $W/m^2 \cdot K$, respectively.

FIND: The outlet temperature of the water, $T_{m,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions; (2) Tube is small object with large, isothermal surroundings; (3) Furnace air and walls are at the same temperature; (4) Tube is thin-walled with black surface; and (5) Incompressible liquid with negligible viscous dissipation.

PROPERTIES: *Table A-6*, Water: $c_p \approx 4180$ J/kg·K.

ANALYSIS: The linearized radiation coefficient may be estimated from Eq. 1.9 with ε = 1,

$$\bar{h}_{rad} \approx \sigma\left(\bar{T}_t + T_{fur}\right)\left(\bar{T}_t^2 + T_{fur}^2\right)$$

where $\bar{T}_t$ represents the average tube wall surface temperature, which can be estimated from an energy balance on the tube.

As represented by the thermal circuit, the energy balance may be expressed as

$$\frac{T_m - \bar{T}_t}{R_{cv,i}} = \frac{\bar{T}_t - T_{fur}}{\left(1/R_{cv,o} + 1/R_{rad}\right)^{-1}}$$

The thermal resistances, with $A_s = PL = \pi DL$, are

$$R_{cv,i} = 1/h_i A_s \qquad R_{cv,o} = 1/h_o A_s \qquad R_{rad} = 1/\bar{h}_{rad} A_s$$

and the mean temperature of the water is approximated as

$$T_m = \left(T_{m,i} + T_{m,o}\right)/2$$

The outlet temperature can be calculated from Eq. 8.45b, with $T_{fur} = T_\infty$,

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{1}{\dot{m} c_p R_{tot}}\right)$$

where

$$R_{tot} = R_{cv,i} + \frac{1}{1/R_{cv,o} + 1/R_{rad}}$$

With

$R_{cv,i} = 5.305 \times 10^{-4}$ K/W $\qquad R_{cv,o} = 3.183 \times 10^{-3}$ K/W $\qquad R_{rad} = 2.296 \times 10^{-3}$ K/W

It follows that

$T_m = 304.1$ K $\qquad \overline{T}_t = 395.6$ K $\qquad T_{m,o} = 308.3$ K $\qquad$ <

Fully Developed Flow

- Laminar Flow in a Circular Tube:

  The local Nusselt number is a constant throughout the fully developed region, but its value depends on the surface thermal condition.

  – Uniform Surface Heat Flux (q_s''):

  $$Nu_D = \frac{hD}{k} = 4.36$$

  – Uniform Surface Temperature (T_s):

  $$Nu_D = \frac{hD}{k} = 3.66$$

- Turbulent Flow in a Circular Tube:

  – For a smooth surface and fully turbulent conditions $\left(Re_D > 10{,}000\right)$, the Dittus – Boelter equation may be used as a first approximation:

  $$Nu_D = 0.023 Re_D^{4/5} Pr^n \qquad \begin{cases} n = 0.3 & (T_s < T_m) \\ n = 0.4 & (T_s > T_m) \end{cases}$$

  – The effects of wall roughness and transitional flow conditions $\left(Re_D > 3000\right)$ may be considered by using the Gnielinski correlation:

  $$Nu_D = \frac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}\left(Pr^{2/3} - 1\right)}$$

Smooth surface:

$$f = (0.790 \ln Re_D - 1.64)^{-2}$$

Surface of roughness $e > 0$:

$$f \rightarrow \text{Figure 8.3}$$

- Noncircular Tubes:
  - Use of hydraulic diameter as characteristic length:

$$D_h \equiv \frac{4A_c}{P}$$

  - Since the local convection coefficient varies around the periphery of a tube, approaching zero at its corners, correlations for the fully developed region are associated with convection coefficients averaged over the periphery of the tube.
  - Laminar Flow:

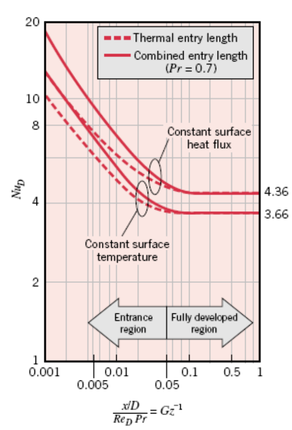
    The local Nusselt number is a constant whose value (Table 8.1) depends on the surface thermal condition $(T_s \text{ or } q_s'')$ and the duct aspect ratio.
  - Turbulent Flow:

    As a first approximation, the Dittus-Boelter or Gnielinski correlation may be used with the hydraulic diameter, irrespective of the surface thermal condition.

Effect of the Entry Region

- The manner in which the Nusselt decays from inlet to fully developed conditions for laminar flow depends on the nature of thermal and velocity boundary layer development in the entry region, as well as the surface thermal condition.

Laminar flow in a circular tube.



  - Combined Entry Length:

    Thermal and velocity boundary layers develop concurrently from uniform profiles at the inlet.

– Thermal Entry Length:

Velocity profile is fully developed at the inlet, and boundary layer development in the entry region is restricted to thermal effects. Such a condition may also be assumed to be a good approximation for a uniform inlet velocity profile if $\Pr >> 1$. Why?

- Average Nusselt Number for Laminar Flow in a Circular Tube with Uniform Surface Temperature:
  - Combined Entry Length:

$\left[\mathrm{Re}_D \Pr/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} > 2$:

$$\overline{Nu}_D = 1.86\left(\frac{\mathrm{Re}_D \Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$\left[\mathrm{Re}_D \Pr/(L/D)\right]^{1/3}(\mu/\mu_s)^{0.14} < 2$:

$$\overline{Nu}_D = 3.66$$

  - Thermal Entry Length:

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)\mathrm{Re}_D \Pr}{1+0.04\left[(D/L)\mathrm{Re}_D \Pr\right]^{2/3}}$$

Entry Region (cont)

- Average Nusselt Number for Turbulent Flow in a Circular Tube :
  - Effects of entry and surface thermal conditions are less pronounced for turbulent flow and can often be neglected.
  - For long tubes $(L/D > 60)$:

$$\overline{Nu}_D \approx Nu_{D,fd}$$

  - For short tubes $(L/D < 60)$:

$$\frac{\overline{Nu}_D}{Nu_{D,fd}} \approx 1 + \frac{C}{(L/D)^m}$$

$$C \approx 1$$
$$m \approx 2/3$$

- Noncircular Tubes:
  - Laminar Flow:

$\overline{Nu}_{D_h}$ depends strongly on aspect ratio, as well as entry region and surface thermal conditions. See references 11 and 12.

– Turbulent Flow:
As a first approximation, correlations for a circular tube may be used with D replaced by D_h.

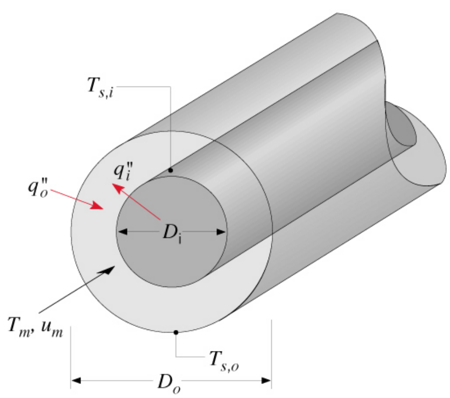
- When determining $\overline{Nu}_D$ for any tube geometry or flow condition, all properties are to be evaluated at

$$\overline{T}_m \equiv \left(T_{m,i} + T_{m,o}\right)/2$$

Why do solutions to internal flow problems often require iteration?

The Concentric Tube Annulus

- Fluid flow through region formed by concentric tubes.



- Convection heat transfer may be from or to inner surface of outer tube and outer surface of inner tube.

- Surface thermal conditions may be characterized by uniform temperature $\left(T_{s,i}, T_{s,o}\right)$ or uniform heat flux $\left(q''_i, q''_o\right)$.

- Convection coefficients are associated with each surface, where

$$q''_i = h_i\left(T_{s,i} - T_m\right)$$

$$q''_o = h_o\left(T_{s,o} - T_m\right)$$

$$Nu_i \equiv \frac{h_i D_h}{k} \qquad Nu_o \equiv \frac{h_o D_h}{k}$$

$$D_h = D_o - D_i$$

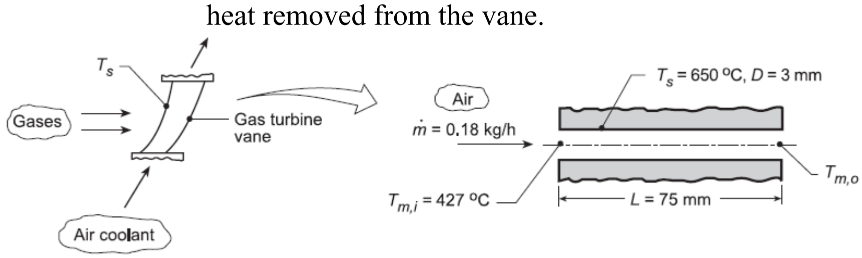
- Fully Developed Laminar Flow

  Nusselt numbers depend on D_i / D_o and surface thermal conditions (Tables 8.2, 8.3)

- Fully Developed Turbulent Flow

  Correlations for a circular tube may be used with D replaced by D_h .

Problem 8.43: For an air passage used to cool a gas turbine vane, calculate the air outlet temperature and heat removed from the vane.



KNOWN: Gas turbine vane approximated as a tube of prescribed diameter and length maintained at a Known surface temperature. Air inlet temperature and flow rate.

FIND: (a) Outlet temperature of the air coolant for the prescribed conditions and (b) Compute and plot the air outlet temperature $T_{m,o}$ as a function of flow rate, $0.1 \le \dot{m} \le 0.6$ kg/h. Compare this result with those for vanes having passage diameters of 2 and 4 mm.

ASSUMPTIONS: (1) Steady-state conditions, (2) Ideal gas with negligible viscous dissipation and pressure variation.

PROPERTIES: *Table A.4*, Air (assume $\overline{T}_m$ = 780 K, 1 atm): c_p = 1094 J/kg·K, k = 0.0563 W/m·K, $\mu = 363.7 \times 10^{-7}$ N·s/m^2, Pr = 0.706; (T_s = 650°C = 923 K, 1 atm): $\mu = 404.2 \times 10^{-7}$ N·s/m^2.

ANALYSIS: (a) For constant wall temperature heating, from Eq. 8.41b,

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m}c_p}\right) \tag{1}$$

where P = πD. For flow in a circular passage,

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\times 0.18\,kg/h\,(1/3600\,s/h)}{\pi(0.003\,m)363.7\times 10^{-7}\,N\cdot s/m^2} = 584\,. \tag{2}$$

The flow is laminar. Let us use the Sieder-Tate correlation including the combined entry length which yields

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = 1.86\left(\frac{Re_D Pr}{L/D}\right)^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} = 1.86\left(\frac{584\times 0.706}{25}\right)^{1/3}\left(\frac{363.7}{404.2}\right)^{0.14} = 4.66 > 3.66$$

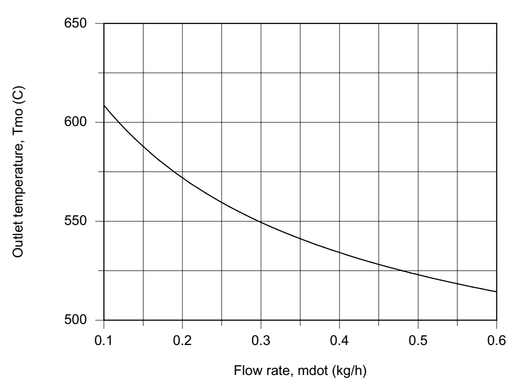
$$\bar{h} = 4.66\times\frac{0.0563\ W/m\cdot K}{0.003\ m} = 87.5\ W/m^2\cdot K$$

Hence, the air outlet temperature is

$$\frac{650 - T_{m,o}}{(650-427)^\circ C} = \exp\left(-\frac{\pi(0.003\,m)\times 0.075\,m\times 87.5\,W/m^2\cdot K}{(0.18/3600)\,kg/s\times 1094\,J/kg\cdot K}\right)$$

$$T_{m,o} = 578^\circ C$$

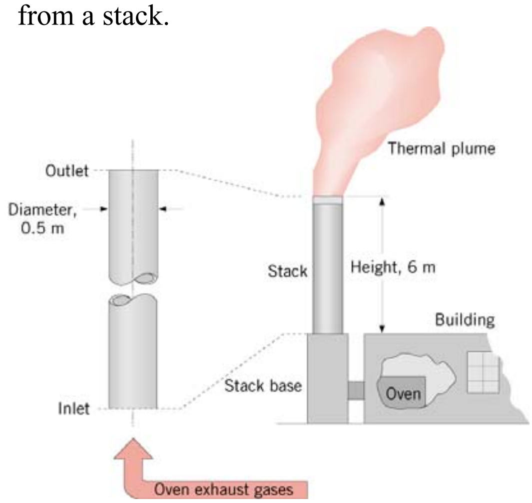
(b) Using the *IHT Correlations Tool*, *Internal Flow*, for *Laminar Flow* with *combined entry length*, along with the energy balance and rate equations above, the outlet temperature $T_{m,o}$ was calculated as a function of flow rate for diameters of D = 2, 3 and 4 mm. The plot below shows that $T_{m,o}$ decreases nearly linearly with increasing flow rate, but is independent of passage diameter.



Based upon the calculation for $T_{m,o}$ = 578°C, $\bar{T}_m$ = 775 K which is in good agreement with our assumption to evaluate the thermophysical properties.

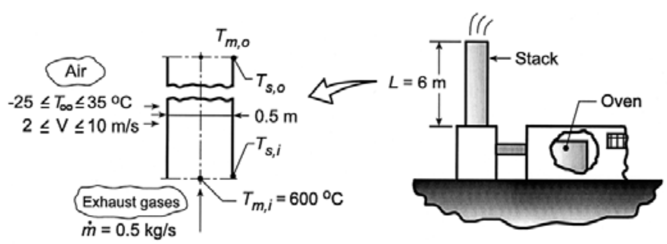
Why is $T_{m,o}$ independent of D? From Eq. (3), note that $\bar{h}$ is inversely proportional to D, $\bar{h} \sim D^{-1}$. From Eq. (1), note that on the right-hand side the product $P\cdot\bar{h}$ will be independent of D. Hence, $T_{m,o}$ will depend only on $\dot{m}$. This is, of course, a consequence of the laminar flow condition and will not be the same for turbulent flow.

Problem 8.52: Determine effect of ambient air temperature and wind velocity on temperature at which oven gases are discharged from a stack.



KNOWN: Thin-walled, tall stack discharging exhaust gases from an oven into the environment.

FIND: (a) Outlet gas and stack surface temperatures, $T_{m,o}$ and $T_{s,o}$, for V=5 m/s and $T_\infty = 4^\circ C$;
(b) Effect of wind temperature and velocity on $T_{m,o}$.



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible wall thermal resistance, (3) Exhaust gas properties approximately those of atmospheric air, (4) Negligible radiative exchange with surroundings, (5) Ideal gas with negligible viscous dissipation and pressure variation, (6) Fully developed flow, (7) Constant properties.

PROPERTIES: *Table A.4*, air (assume $T_{m,o}$ = 773 K, $\bar{T}_m$ = 823 K, 1 atm): c_p = 1104 J/kg·K, $\mu = 376.4 \times 10^{-7}$ N·s/m^2, k = 0.0584 W/m·K, Pr = 0.712; *Table A.4*, air (assume T_s = 523 K, T_∞ = 4°C = 277 K, T_f = 400 K, 1 atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, k = 0.0338 W/m·K, Pr = 0.690.

ANALYSIS: (a) From Eq. 8.45a,

$$T_{m,o} = T_\infty - \left(T_\infty - T_{m,i}\right)\exp\left[-\frac{PL}{\dot{m}c_p}\bar{U}\right] \qquad U = 1\Big/\left(\frac{1}{\bar{h}_i} + \frac{1}{\bar{h}_o}\right)$$

where $\bar{h}_i$ and $\bar{h}_o$ are average coefficients for internal and external flow, respectively.

Internal flow: With a Reynolds number of

$$Re_{D_i} = \frac{4\dot{m}}{\pi D \mu} = \frac{4 \times 0.5\,\text{kg/s}}{\pi \times 0.5\,\text{m} \times 376.4 \times 10^{-7}\,\text{N}\cdot\text{s/m}^2} = 33{,}827$$

The flow is turbulent, and assuming fully developed conditions throughout the stack, the Dittus-Boelter correlation may be used to determine $\bar{h}_i$.

$$\overline{Nu}_D = \frac{\bar{h}_i D}{k} = 0.023\,Re_{D_i}^{4/5}\,Pr^{0.3}$$

$$\bar{h}_i = \frac{58.4 \times 10^{-3}\,\text{W/m}\cdot\text{K}}{0.5\,\text{m}} \times 0.023(33{,}827)^{4/5}(0.712)^{0.3} = 10.2\,\text{W/m}^2\cdot\text{K}$$

External flow: Working with the Churchill/Bernstein correlation and

$$Re_{D_o} = \frac{VD}{\nu} = \frac{5\,\text{m/s} \times 0.5\,\text{m}}{26.41 \times 10^{-6}\,\text{m}^2\text{/s}} = 94{,}660$$

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} = 205$$

Hence,

$$\bar{h}_o = \left(0.0338\,\text{W/m}\cdot\text{K}/0.5\,\text{m}\right) \times 205 = 13.9\,\text{W/m}^2\cdot\text{K}$$

The outlet gas temperature is then

$$T_{m,o} = 4^\circ\text{C} - (4 - 600)^\circ\,\text{C}\exp\left[-\frac{\pi \times 0.5\,\text{m} \times 6\,\text{m}}{0.5\,\text{kg/s} \times 1104\,\text{J/kg}\cdot\text{K}}\left(\frac{1}{1/10.2 + 1/13.9}\,\text{W/m}^2\cdot\text{K}\right)\right] = 543^\circ\text{C}$$

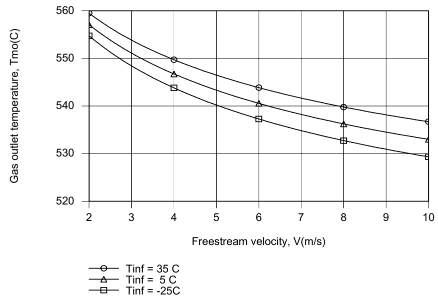
The outlet stack surface temperature can be determined from a local surface energy balance of the form,

$$h_i(T_{m,o} - T_{s,o}) = h_o(T_{s,o} - T_\infty),$$

which yields

$$T_{s,o} = \frac{h_i T_{m,o} + h_o T_\infty}{h_i + h_o} = \frac{(10.2 \times 543 + 13.9 \times 4)\,\text{W/m}^2}{(10.2 + 13.9)\,\text{W/m}^2\cdot\text{K}} = 232^\circ\text{C}$$

b) The effects of the air temperature and velocity are as follows



Due to the elevated temperatures of the gas, the variation in ambient temperature has only a small effect on the gas exit temperature. However, the effect of the freestream velocity is more pronounced. Discharge temperatures of approximately 530 and 560°C would be representative of cold/windy and warm/still atmospheric conditions, respectively.

COMMENTS: If there are constituents in the gas discharge that condense or precipitate out at temperatures below $T_{s,o}$, related operating conditions should be avoided.

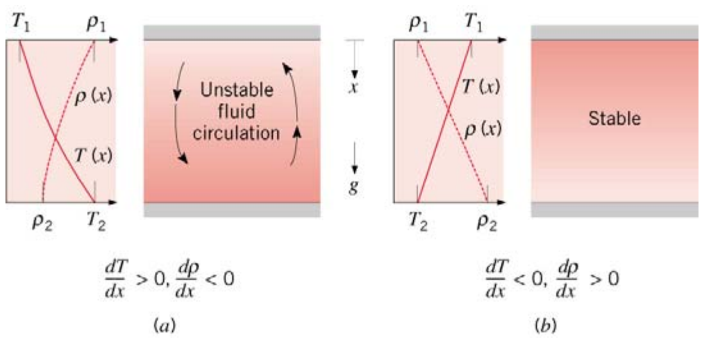
Free Convection: General Considerations and Results for Vertical and Horizontal Plates

Chapter 9

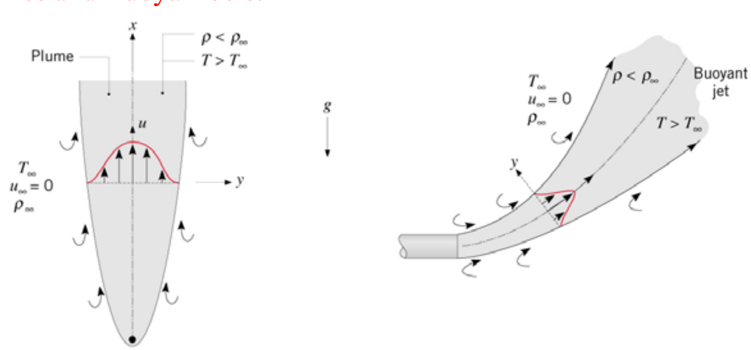
Sections 9.1 through 9.6.2, 9.9

General Considerations

- Free convection refers to fluid motion induced by buoyancy forces.
- Buoyancy forces may arise in a fluid for which there are density gradients and a body force that is proportional to density.
- In heat transfer, density gradients are due to temperature gradients and the body force is gravitational.
- Stable and Unstable Temperature Gradients



- Free Boundary Flows
  - Occur in an extensive (in principle, infinite), quiescent (motionless at locations far from the source of buoyancy) fluid.
  - Plumes and Buoyant Jets:



- Free Convection Boundary Layers
  - Boundary layer flow on a heated or cooled surface $(T_s \neq T_\infty)$ induced by buoyancy forces.

- Pertinent Dimensionless Parameters
  - Grashof Number:

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \approx \frac{\text{Buoyancy Force}}{\text{Viscous Force}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ thermal expansion coefficient (a thermodynamic property of the fluid)

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Liquids: $\beta \rightarrow$ Tables A.5, A.6

Perfect Gas: $\beta = 1/T\,(\text{K})$

  - Rayleigh Number:

$$Ra_L = Gr_L \, \text{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}$$

- Mixed Convection
  - A condition for which forced and free convection effects are comparable.
  - Exists if

$$\left(Gr_L / \text{Re}_L^2\right) \approx (1)$$

- Free convection $\rightarrow \left(Gr_L / \text{Re}_L^2\right) >> 1$
- Forced convection $\rightarrow \left(Gr_L / \text{Re}_L^2\right) << 1$

  - Heat Transfer Correlations for Mixed Convection:

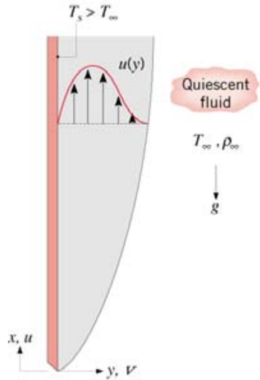
$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

$+ \rightarrow$ assisting and transverse flows
$- \rightarrow$ opposing flows

$n \approx 3$

Vertical Plates

- Free Convection Boundary Layer Development on a Heated Plate:



  - Ascending flow with the maximum velocity occurring in the boundary layer and zero velocity at both the surface and outer edge.
  - How do conditions differ from those associated with forced convection?
  - How do conditions differ for a cooled plate $(T_s < T_\infty)$?

- Form of the *x*-Momentum Equation for Laminar Flow

$$\underbrace{u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y}}_{\text{Net Momentum Fluxes (Inertia Forces)}} = \underbrace{g\beta(T - T_\infty)}_{\text{Buoyancy Force}} + \underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}}$$

  - Temperature dependence requires that solution for $u\,(x,y)$ be obtained concurrently with solution of the boundary layer energy equation for $T\,(x,y)$.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

    – The solutions are said to be coupled.

Vertical Plates (cont)

- Similarity Solution
  - Based on existence of a similarity variable, η, through which the x-momentum equation may be transformed from a partial differential equation with two-independent variables (x and y) to an ordinary differential equation expressed exclusively in terms of η.

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

  - Transformed momentum and energy equations:

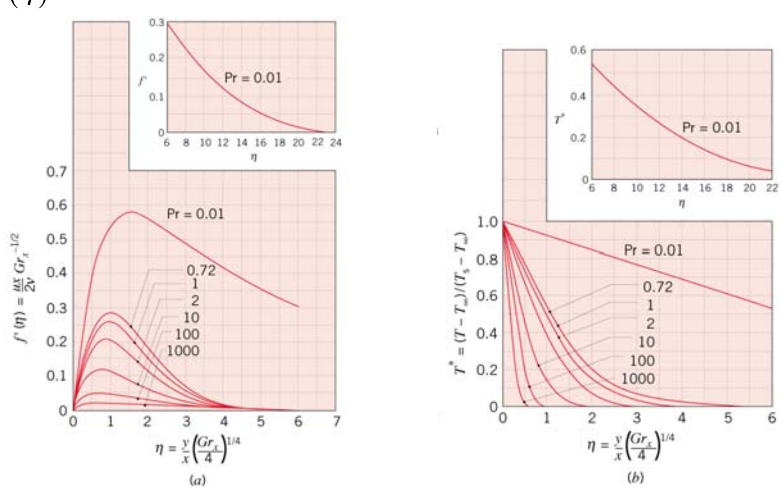
$$f''' + 3ff'' - 2(f')^2 + T^* = 0$$

$$T^{*''} + 3\Pr fT^{*'} = 0$$

$$f'(\eta) \equiv \frac{df}{d\eta} = \frac{x}{2\nu}\left(Gr_x^{-1/2}\right)u \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

Vertical Plates (cont)

- Numerical integration of the equations yields the following results for $f'(\eta)$ and T^* :



- Velocity boundary layer thickness $(\delta) \rightarrow \eta \approx 5$ for $\Pr > 0.6$

- $\Pr > 0.6: \quad \delta = 5x\left(\frac{Gr_x}{4}\right)^{-1/4} = 7.07\frac{x}{(Gr_x)^{1/4}} \propto x^{1/4}$

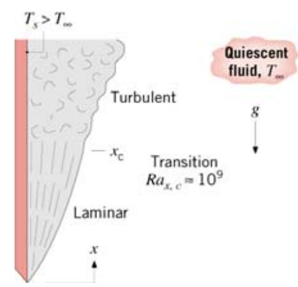
- Nusselt Numbers $\left(Nu_x \text{ and } \overline{Nu}_L\right)$:

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \left.\frac{dT^*}{d\eta}\right|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g(\Pr)$$

$$g(\Pr) = \frac{0.75 \Pr^{1/2}}{\left(0.609 + 1.221 \Pr^{1/2} + 1.238 \Pr\right)^{1/4}} \qquad \left(0 < \Pr < \infty\right)$$

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx \rightarrow \overline{Nu}_L = \frac{4}{3} Nu_L$$

- Transition to Turbulence
  - Amplification of disturbances depends on relative magnitudes of buoyancy and viscous forces.
  - Transition occurs at a critical Rayleigh Number.



$$Ra_{x,c} = Gr_{x,c} \Pr = \frac{g\beta\left(T_s - T_\infty\right)x^3}{\nu\alpha} \approx 10^9$$

- Empirical Heat Transfer Correlations
  - Laminar Flow $\left(Ra_L < 10^9\right)$:

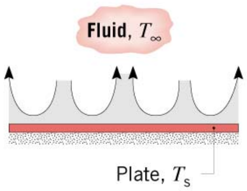
$$\overline{Nu}_L = 0.68 + \frac{0.670\, Ra_L^{1/4}}{\left[1 + \left(0.492/\Pr\right)^{9/16}\right]^{4/9}}$$

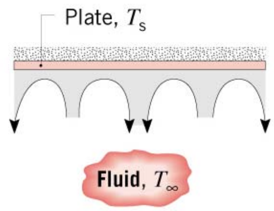
  - All Conditions:

$$\overline{Nu}_L = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(0.492/\Pr\right)^{9/16}\right]^{8/27}}\right\}^2$$

Horizontal Plates

- Buoyancy force is normal, instead of parallel, to the plate.
- Flow and heat transfer depend on whether the plate is heated or cooled and whether it is facing upward or downward.
- Hot Surface Facing Upward or Cold Surface Facing Downward





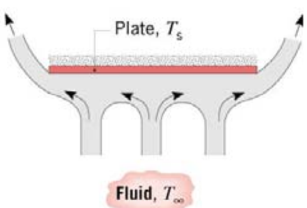
$T_s > T_\infty$ $\qquad$ $T_s < T_\infty$

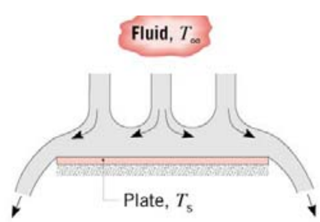
$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \left(10^4 < Ra_L < 10^7\right)$$

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \left(10^7 < Ra_L < 10^{11}\right)$$

How does $\bar{h}$ depend on L when $\overline{Nu}_L \propto Ra_L^{1/3}$?

- Hot Surface Facing Downward or Cold Surface Facing Upward



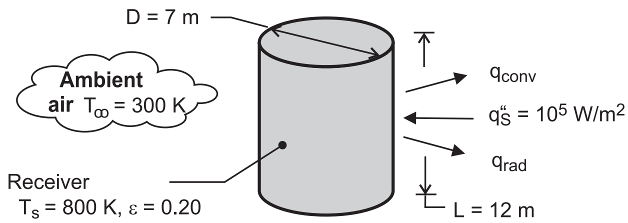


$T_s > T_\infty$ $\qquad$ $T_s < T_\infty$

$$\overline{Nu}_L = 0.27 Ra_L^{1/4} \qquad \left(10^5 < Ra_L < 10^{10}\right)$$

➢ Why do these flow conditions yield smaller heat transfer rates than those for a heated upper surface or cooled lower surface?

Problem 9.31: Convection and radiation losses from the surface of a central solar receiver.



KNOWN: Dimensions and emissivity of cylindrical solar receiver. Incident solar flux. Temperature of ambient air.

FIND: (a) Heat loss and collection efficiency for a prescribed receiver temperature, (b) Effect of receiver temperature on heat losses and collector efficiency.

ASSUMPTIONS: (1) Steady-state, (2) Ambient air is quiescent, (3) Incident solar flux is uniformly distributed over receiver surface, (4) All of the incident solar flux is absorbed by the receiver, (5) Negligible irradiation from the surroundings, (6) Uniform receiver surface temperature, (7) Curvature of cylinder has a negligible effect on boundary layer development, (8) Constant properties, (9) Negligible effect of top and bottom surfaces.

PROPERTIES: *Table A-4*, air (T_f = 550 K): k = 0.0439 W/m·K, $\nu = 45.6 \times 10^{-6}$ m^2/s, $\alpha = 66.7 \times 10^{-6}$ m^2/s, Pr = 0.683, $\beta = 1.82 \times 10^{-3}$ K^{-1}.

ANALYSIS: (a) The total heat loss is

$$q = q_{rad} + q_{conv} = A_s \varepsilon\sigma T_s^4 + \bar{h} A_s (T_s - T_\infty)$$

With $Ra_L = g\beta(T_s - T_\infty)L^3/\nu\alpha = 9.8$ m/s^2 $(1.82 \times 10^{-3}$ K$^{-1})$ 500K (12m)^3/(45.6 × 66.7 × 10^{-12} m^4/s^2) = 5.07 × 10^{12}, the Churchill and Chu correlation yields

$$\bar{h} = \frac{k}{L}\left\{0.825 + \frac{0.387\,Ra_L^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2 = \frac{0.0439\text{ W/m}\cdot\text{K}}{12\text{m}}\{0.825 + 42.4\}^2 = 6.83\text{ W/m}^2\cdot\text{K}$$

Hence, with $A_s = \pi DL = 264$ m^2

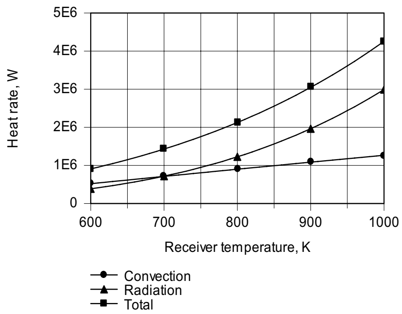
$$q = 264\text{ m}^2 \times 0.2 \times 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4(800\text{ K})^4 + 264\text{ m}^2 \times 6.83\text{ W/m}^2\cdot\text{K}(500\text{ K})$$

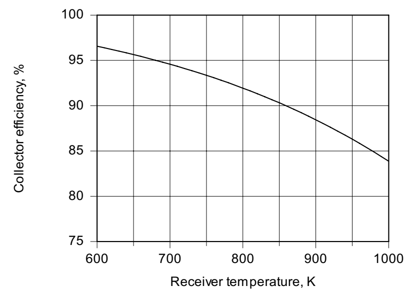
$$q = q_{rad} + q_{conv} = 1.23\times10^6\text{ W} + 9.01\times10^5\text{ W} = 2.13\times10^6\text{ W}$$

With $A_s q''_s = 2.64 \times 10^7$ W, the collector efficiency is

$$\eta = \left(\frac{A_s q''_s - q}{A_s q''_s} \right) 100 = \frac{\left(2.64 \times 10^7 - 2.13 \times 10^6\right) W}{2.64 \times 10^7 \ W} (100) = 91.9\%$$

(b) As shown below, because of its dependence on temperature to the fourth power, q_{rad} increases more significantly with increasing T_s than does q_{conv}, and the effect on the efficiency is pronounced





COMMENTS: The collector efficiency is also reduced by the inability to have a perfectly absorbing receiver. Partial reflection of the incident solar flux will reduce the efficiency by at least several percent.

Boiling and Condensation

Chapter 10

Sections 10.1 through 10.6

eneral Considerations

- Boiling is associated with transformation of liquid to vapor at a solid/liquid interface due to convection heat transfer from the solid.

- Agitation of fluid by vapor bubbles provides for large convection coefficients and hence large heat fluxes at low-to-moderate surface-to-fluid temperature differences.

- Special form of Newton's law of cooling:

$$q_s'' = h\left(T_s - T_{sat}\right) = h\,\Delta T_e$$

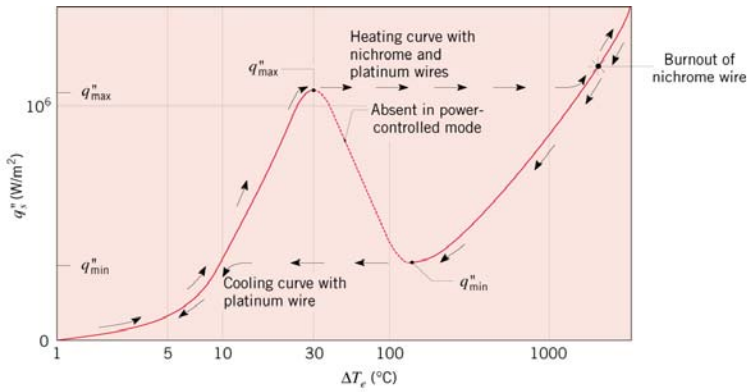
  - $T_{sat} \rightarrow$ saturation temperature of liquid
  - $\Delta T_e \equiv \left(T_s - T_{sat}\right) \rightarrow$ excess temperature

- Special Cases
  - Pool Boiling:
    Liquid motion is due to natural convection and bubble-induced mixing.
  - Forced Convection Boiling:
    Fluid motion is induced by external means, as well as by bubble-induced mixing.
  - Saturated Boiling:
    Liquid temperature is slightly larger than saturation temperature.
  - Subcooled Boiling:
    Liquid temperature is less than saturation temperature.

The Boiling Curve

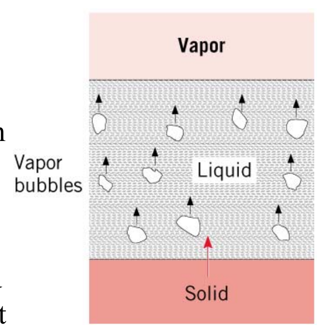
Reveals range of conditions associated with saturated pool boiling on a $q''_s - \Delta T_e$ plot.

Water at Atmospheric Pressure

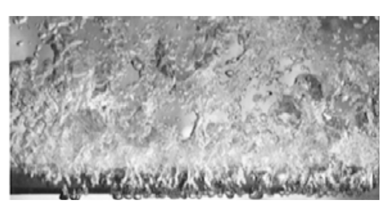


- Free Convection Boiling $(\Delta T_e < 5^\circ C)$
  - Little vapor formation.
  - Liquid motion is due principally to single-phase natural convection.
- nset of Nucleate Boiling - NB$(\Delta T_e \approx 5^\circ C)$

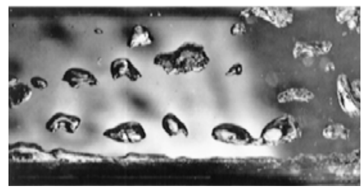
- Nucleate Boiling $(5 < \Delta T_e < 30^\circ C)$
  - solated Vapor Bubbles $(5 < \Delta T_e < 10^\circ C)$
    - Liquid motion is strongly influenced by nucleation of bubbles at the surface.
    - h and q''_s increase sharply with increasing ΔT_e.
    - Heat transfer is principally due to contact of liquid with the surface (single-phase convection) and not to vaporization.



  - Jets and Columns $(10 < \Delta T_e < 30^\circ C)$
    - ncreasing number of nucleation sites causes bubble interactions and coalescence into jets and slugs.
    - Liquid/surface contact is impaired.
    - q''_s continues to increase with ΔT_e while h begins to decrease.



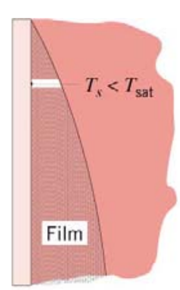
- *Critical Heat Flux - CHF,* $q''_{max}\left(\Delta T_e \approx 30^\circ\mathrm{C}\right)$
  - Maximum attainable heat flux in nucleate boiling.
  - $q''_{max} \approx 1\ \mathrm{MW/m^2}$ for water at atmospheric pressure.
- Potential Burnout for Power-Controlled Heating
  - An increase in q''_s beyond q''_{max} causes the surface to be blanketed by vapor, and the surface temperature can spontaneously achieve a value that potentially exceeds its melting point $\left(\Delta T_s > 1000^\circ\mathrm{C}\right)$.
  - f the surface survives the temperature shock, conditions are characterized by *film boiling*.
- Film Boiling
  - Heat transfer is by conduction and radiation across the vapor blanket.
  - A reduction in q''_s follows the cooling curve continuously to the Leidenfrost point corresponding to the minimum heat flux q''_{min} for film boiling.



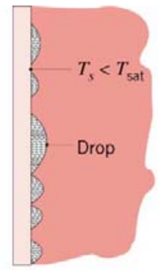
- A reduction in q''_s below q''_{min} causes an abrupt reduction in surface temperature to the nucleate boiling regime.
- Transition Boiling for Temperature-Controlled Heating
  - Characterized by a continuous decay of q''_s (from q''_{max} to q''_{min}) with increasing ΔT_e.
  - Surface conditions oscillate between nucleate and film boiling, but portion of surface experiencing film boiling increases with ΔT_e.
  - Also termed unstable or partial film boiling.

eneral Considerations - Condensation

- Heat transfer to a surface occurs by condensation when the surface temperature is less than the saturation temperature of an adjoining vapor.
- Film Condensation
  - Entire surface is covered by the condensate, which flows continuously from the surface and provides a resistance to heat transfer between the vapor and the surface.



- Thermal resistance is reduced through use of short vertical surfaces and horizontal cylinders.
- Characteristic of clean, uncontaminated surfaces.

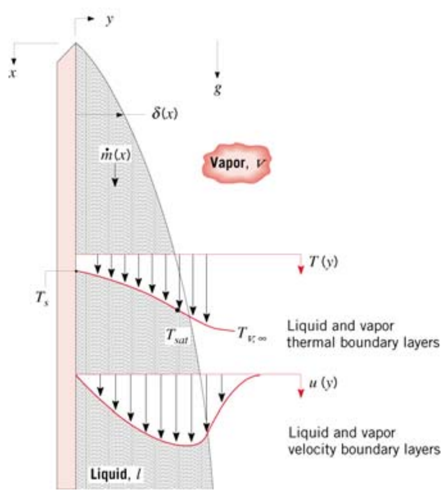


- Dropwise Condensation
  - Surface is covered by drops ranging from a few micrometers to agglomerations visible to the naked eye.

- Thermal resistance is greatly reduced due to absence of a continuous film.
- Surface coatings may be applied to inhibit *wetting* and stimulate dropwise condensation.

Film Condensation on a Vertical Plate

- Distinguishing Features
  - Thickness (δ) and flow rate $(\dot{m})$ of condensate increase with increasing x
  - Generally, the vapor is superheated $(T_{v,\infty} > T_{sat})$ and may be part of a mixture that includes noncondensibles.
  - A shear stress at the liquid/vapor interface induces a velocity gradient in the vapor, as well as the liquid.



- Nusselt Analysis for Laminar Flow

  Assumptions:
  - A pure vapor at T_{sat}.
  - Negligible shear stress at liquid/vapor interface.

$$\rightarrow \left.\frac{\partial u}{\partial y}\right|_{y=\delta} = 0$$

- Negligible advection in the film. Hence, the steady-state x-momentum and energy equations for the film are

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu_l}\frac{dp_\infty}{dx} - \frac{\rho_l g}{\mu_l}$$

$$\frac{\partial^2 T}{\partial y^2} = 0$$

- The boundary layer approximation, $\partial p / \partial y = 0$, may be applied to the film. Hence,

$$\frac{dp_\infty}{dx} = \rho_v g$$

- Solutions to momentum and energy equations $\longrightarrow$

Film thickness:

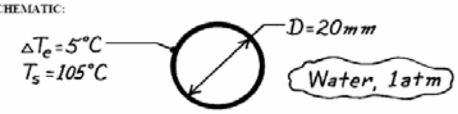
$$\delta(x) = \left[\frac{4k_l \mu_l (T_{sat} - T_s) x}{g \rho_l (\rho_l - \rho_v) h_{fg}}\right]^{1/4}$$

PROBLEM 10.2

KNOWN: Horizontal 20 mm diameter cylinder with $\Delta T_e = T_s - T_{sat} = 5°C$ in saturated water, 1 atm.

FIND: Heat flux based upon free convection correlation; compare with boiling curve. Estimate maximum value of the heat transfer coefficient from the boiling curve.

SCHEMATIC:



ASSUMPTIONS: (1) Horizontal cylinder, (2) Free convection, no bubble nucleation.

PROPERTIES: *Table A-6*, Water (Saturated liquid, $T_f = (T_{sat} + T_s)/2 = 102.5\ °C \approx 375K$): $\rho_\ell = 956.9\ kg/m^3$, $c_{p,\ell} = 4220\ J/kg \cdot K$, $\mu_\ell = 274 \times 10^{-6}\ N \cdot s/m^2$, $k_\ell = 0.681\ W/m \cdot K$, $Pr = 1.70$, $\beta = 761 \times 10^{-6}\ K^{-1}$.

ANALYSIS: To estimate the free convection heat transfer coefficient, use the Churchill-Chu correlation,

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = \left\{0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2.$$

Substituting numerical values, with $\Delta T = \Delta T_e = 5°C$, find

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha} = \frac{9.8 m/s^2 \times 761\times10^{-6} K^{-1} \times 5°C (0.020m)^3}{\left[274\times10^{-6} N\cdot s/m^2 / 956.9\ kg/m^3\right] \times 1.686\times10^{-7} m^2/s} = 6.178\times10^6$$

where $\alpha = k/\rho c_p = (0.681\ W/m \cdot K / 956.9\ kg/m^3 \times 4220\ J/kg \cdot K) = 1.686 \times 10^{-7}\ m^2/s$. Note that Ra_D is within the prescribed limits of the correlation. Hence,

$$\overline{Nu}_D = \left\{0.60 + \frac{0.387\left(6.178\times10^6\right)^{1/6}}{\left[1+(0.559/1.70)^{9/16}\right]^{8/27}}\right\}^2 = 27.22$$

$$\bar{h}_{fc} = Nu_D \frac{k}{D} = \frac{27.22\times0.681 W/m\cdot K}{0.020m} = 928\ W/m^2 \cdot K. \qquad <$$

Hence, $q_s'' = h_{fc}\Delta T_e = 4640\ W/m^2$

From the typical boiling curve for water at 1 atm, Fig. 10.4, find at $\Delta T_e = 5°C$ that

$$q_s'' \approx 8.5\times10^3\ W/m^2 \qquad <$$

The free convection correlation underpredicts (by 1.8) the boiling curve. The maximum value of h_{bc} can be estimated as

$$h_{max} \approx q_{max}'' / \Delta T_e = 1.2\times10^6 MW/m^2 / 30°C = 40,000\ W/m^2 \cdot K. \qquad <$$

COMMENTS: (1) Note the large increase in h with a slight change in ΔT_e.

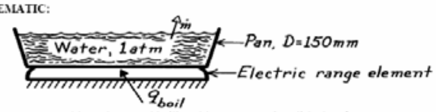
(2) The maximum value of h occurs at point P on the boiling curve.

PROBLEM 10.10

KNOWN: Copper pan, 150 mm diameter and filled with water at 1 atm, is maintained at 115°C.

FIND: Power required to boil water and the evaporation rate; ratio of heat flux to critical heat flux; pan temperature required to achieve critical heat flux.

SCHEMATIC:



ASSUMPTIONS: (1) Nucleate pool boiling, (2) Copper pan is polished surface.

PROPERTIES: *Table A-6*, Water (1 atm): $T_{sat} = 100°C$, $\rho_\ell = 957.9\ kg/m^3$, $\rho_v = 0.5955\ kg/m^3$, $c_{p,\ell} = 4217\ J/kg{\cdot}K$, $\mu_\ell = 279 \times 10^{-6}\ N{\cdot}s/m^2$, $Pr_\ell = 1.76$, $h_{fg} = 2257\ kJ/kg$, $\sigma = 58.9 \times 10^{-3}\ N/m$.

ANALYSIS: The power requirement for boiling and the evaporation rate can be expressed as follows,

$$q_{boil} = q_s'' \cdot A_s \qquad \dot{m} = q_{boil} / h_{fg}.$$

The heat flux for nucleate pool boiling can be estimated using the Rohsenow correlation.

$$q_s'' = \mu_\ell h_{fg}\left[\frac{g(\rho_\ell - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,\ell}\Delta T_e}{C_{s,f}\, h_{fg}\, Pr_\ell^n}\right)^3.$$

Selecting $C_{s,f} = 0.0128$ and $n = 1$ from Table 10.1 for the polished copper finish, find

$$q_s'' = 279\times10^{-6}\frac{N\cdot s}{m^2}\times2257\times10^3\frac{J}{kg}\left[\frac{9.8\frac{m}{s^2}(957.9-0.5955)\frac{kg}{m^3}}{58.9\times10^{-3}N/m}\right]^{1/2}\left(\frac{4217\frac{J}{kg\cdot K}\times15°C}{0.0128\times2257\times10^3\frac{J}{kg}\times1.76}\right)^3$$

$$q_s'' = 4.839\times10^5\ W/m^2.$$

The power and evaporation rate are

$$q_{boil} = 4.839\times10^5\ W/m^2 \times \frac{\pi}{4}(0.150m)^2 = 8.55kW$$ <

$$\dot{m}_{boil} = 8.55kW / 2257\times10^3 J/kg = 3.79\times10^{-3} kg/s = 14 kg/h.$$ <

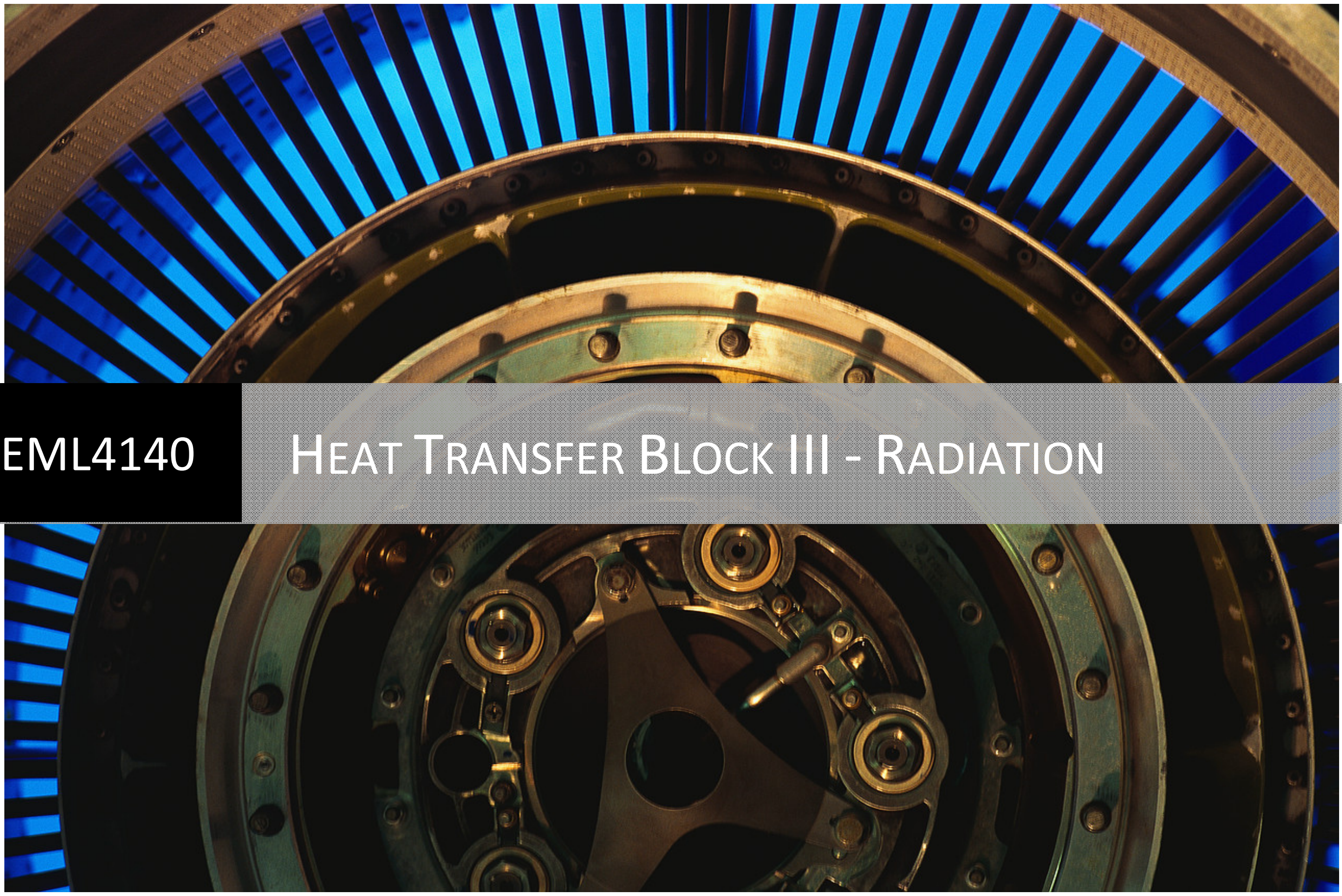
The maximum or critical heat flux was found in Example 10.1 as

$$q''_{max} = 1.26 MW/m^2.$$

Hence, the ratio of the operating to maximum heat flux is

$$\frac{q_s''}{q''_{max}} = 4.619\times10^5\ W/m^2 / 1.26MW/m^2 = 0.384.$$ <

From the boiling curve, Fig. 10.4, $\Delta T_e \approx 30°C$ will provide the maximum heat flux. <



EML4140

HEAT TRANSFER BLOCK III - RADIATION

http://www.mae.ufl.edu/courses/spring2008/eml4140

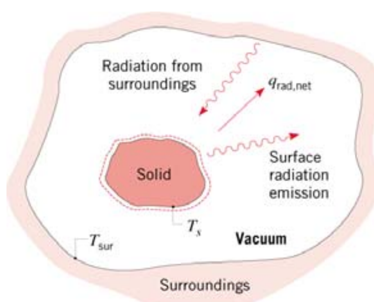
Radiation: Processes and Properties -Basic Principles and Definitions-

Chapter 12

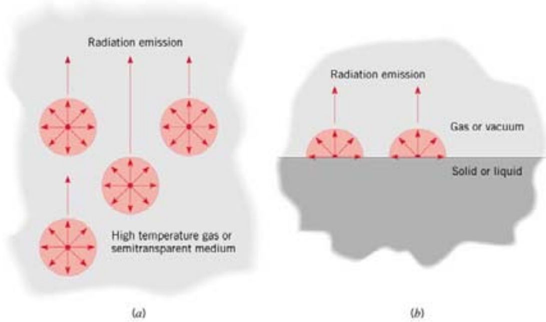
Sections 12.1 through 12.3

General Considerations

- Attention is focused on thermal radiation, whose origins are associated with emission from matter at an absolute temperature $T > 0$.
- Emission is due to oscillations and transitions of the many electrons that comprise matter, which are, in turn, sustained by the thermal energy of the matter.
- Emission corresponds to heat transfer from the matter and hence to a reduction in thermal energy stored by the matter.
- Radiation may also be intercepted and absorbed by matter.
- Absorption results in heat transfer to the matter and hence an increase in thermal energy stored by the matter.
- Consider a solid of temperature T_s in an evacuated enclosure whose walls are at a fixed temperature T_{sur} :
  - What changes occur if $T_s > T_{sur}$? Why?
  - What changes occur if $T_s < T_{sur}$? Why?



- Emission from a gas or a semitransparent solid or liquid is a volumetric phenomenon. Emission from an opaque solid or liquid is a surface phenomenon.



(a) (b)

For an opaque solid or liquid, emission originates from atoms and molecules within 1 μm of the surface.

- The dual nature of radiation:
  – In some cases, the physical manifestations of radiation may be explained by viewing it as particles (aka photons or quanta).
  – In other cases, radiation behaves as an electromagnetic wave.

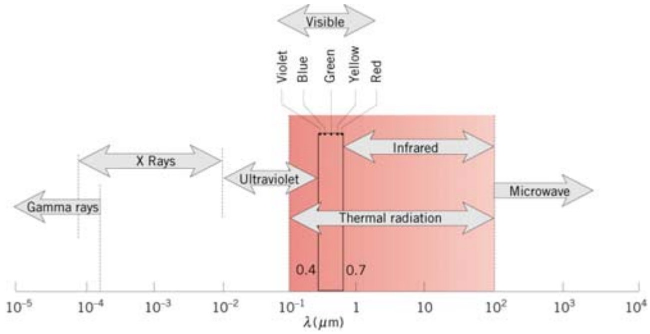
– In all cases, radiation is characterized by a wavelength λ and frequency ν, which are related through the speed at which radiation propagates in the medium of interest:

$$\lambda = \frac{c}{\nu}$$

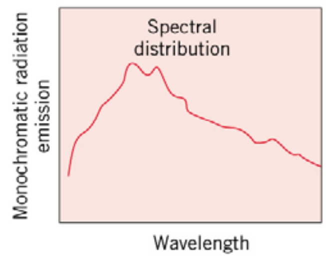
For propagation in a vacuum,

$$c = c_o = 2.998 \times 10^8 \text{ m/s}$$

The Electromagnetic Spectrum

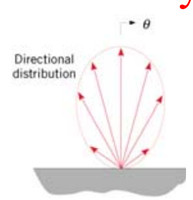


- Thermal radiation is confined to the infrared, visible and ultraviolet regions of the spectrum $(0.1 < \lambda < 100 \mu m)$.
- The amount of radiation emitted by an opaque surface varies with wavelength, and we may speak of the spectral distribution over all wavelengths or of monochromatic/spectral components associated with particular wavelengths.

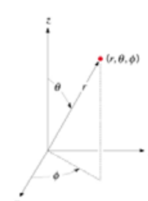


Directional Considerations and the Concept of Radiation Intensity

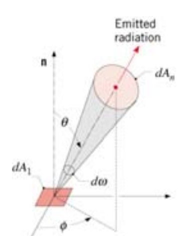
- Radiation emitted by a surface will be in all directions associated with a hypothetical hemisphere about the surface and is characterized by a directional distribution.



- Direction may be represented in a spherical coordinate system characterized by the zenith or polar angle θ and the azimuthal angle ϕ .



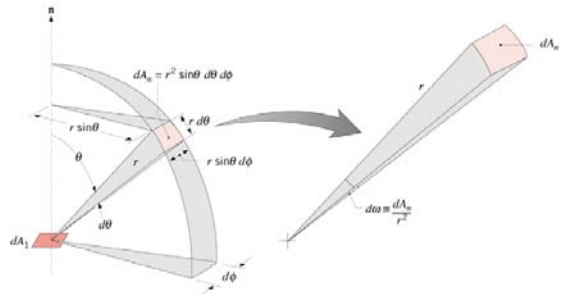
- The amount of radiation emitted from a surface, dA_1, and propagating in a particular direction, θ, ϕ, is quantified in terms of a differential solid angle associated with the direction.



$$d\omega \equiv \frac{dA_n}{r^2}$$

$dA_n \rightarrow$ unit element of surface on a hypothetical sphere and normal to the θ, ϕ direction.

Directional Considerations (cont)



$$dA_n = r^2 \sin\theta d\theta d\phi$$

$$d\omega = \frac{dA_n}{r^2} = \sin\theta d\theta d\phi$$

– The solid angle ω has units of steradians (sr).

– The solid angle associated with a complete hemisphere is

$$\omega_{hemi} = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta d\theta d\phi = 2\pi \text{sr}$$

- Spectral Intensity: A quantity used to specify the radiant heat flux $\left(\text{W/m}^2\right)$ within a unit solid angle about a prescribed direction $\left(\text{W/m}^2 \cdot \text{sr}\right)$ and within a unit wavelength interval about a prescribed wavelength $\left(\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}\right)$.

Directional Considerations (cont)

- The spectral intensity $I_{\lambda,e}$ associated with emission from a surface element dA_1 in the solid angle $d\omega$ about θ,ϕ and the wavelength interval $d\lambda$ about λ is defined as:

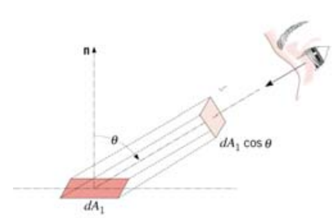
$$I_{\lambda,e}(\lambda,\theta,\phi) \equiv \frac{dq}{(dA_1 \cos\theta)\cdot d\omega \cdot d\lambda}$$

- The rationale for defining the radiation flux in terms of the projected surface area $(dA_1 \cos\theta)$ stems from the existence of surfaces for which, to a good approximation, $I_{\lambda,e}$ is independent of direction. Such surfaces are termed diffuse, and the radiation is said to be isotropic.

  - The projected area is how dA_1 would appear if observed along θ,ϕ .

    – What is the projected area for $\theta = 0$?

    – What is the projected area for $\theta = \pi / 2$?



- The spectral heat rate and heat flux associated with emission from dA_1 are, respectively,

$$dq_\lambda \equiv \frac{dq}{d\lambda} = I_{\lambda,e}(\lambda,\theta,\phi) dA_1 \cos\theta d\omega$$

$$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta d\omega = I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta \sin\theta d\theta d\phi$$

Relation of Intensity to Emissive Power, Irradiation, and Radiosity

- The spectral emissive power $\left(\text{W/m}^2 \cdot \mu\text{m}\right)$ corresponds to spectral emission over all possible directions.

$$E_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

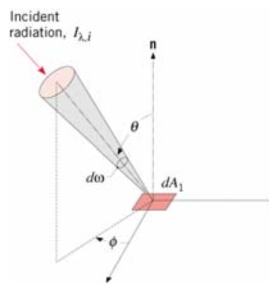
- The total emissive power $\left(\text{W/m}^2\right)$ corresponds to emission over all directions and wavelengths.

$$E = \int_0^\infty E_\lambda(\lambda)d\lambda$$

- For a diffuse surface, emission is isotropic and

$$E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda) \qquad E = \pi I_e$$

- The spectral intensity of radiation incident on a surface, $I_{\lambda,i}$, is defined in terms of the unit solid angle about the direction of incidence, the wavelength interval $d\lambda$ about λ, and the projected area of the receiving surface, $dA_1 \cos\theta$.



- The spectral irradiation $\left(\text{W/m}^2 \cdot \mu\text{m}\right)$ is then:

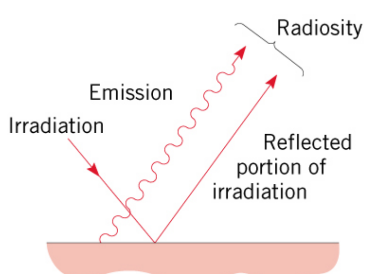
$$G_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

and the total irradiation $\left(\text{W/m}^2\right)$ is

$$G = \int_0^\infty G_\lambda(\lambda)d\lambda$$

➢ How may G_λ and G be expressed if the incident radiation is diffuse?

- The radiosity of an opaque surface accounts for all of the radiation leaving the surface in all directions and may include contributions from both reflection and emission.



- With $I_{\lambda,e+r}$ designating the spectral intensity associated with radiation emitted by the surface and the reflection of incident radiation, the spectral radiosity $\left(\mathrm{W/m^2 \cdot \mu m}\right)$ is:

$$J_\lambda(\lambda) = \int_0^{2\pi} \int_0^{\pi/2} I_{\lambda,e+r}(\lambda,\theta,\phi) \cos\theta \sin\theta d\theta d\phi$$

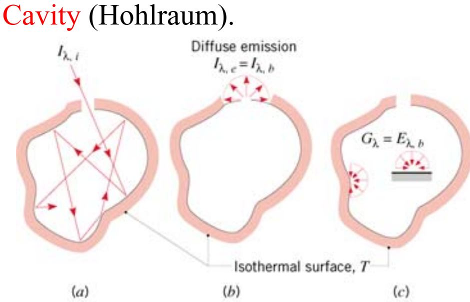
and the total radiosity $\left(\mathrm{W/m^2}\right)$ is

$$J = \int_0^\infty J_\lambda(\lambda) d\lambda$$

  - How may J_λ and J be expressed if the surface emits and reflects diffusely?

Blackbody Radiation

- The Blackbody
  - An idealization providing limits on radiation emission and absorption by matter.
    - For a prescribed temperature and wavelength, no surface can emit more radiation than a blackbody: the ideal emitter.
    - A blackbody is a diffuse emitter.
    - A blackbody absorbs all incident radiation: the ideal absorber.
- The Isothermal Cavity (Hohlraum).



(a) After multiple reflections, virtually all radiation entering the cavity is absorbed.

(b) Emission from the aperture is the maximum possible emission achievable for the temperature associated with the cavity and is diffuse.

(c) The cumulative effect of radiation emission from and reflection off the cavity wall is to provide diffuse irradiation corresponding to emission from a blackbody $\left(G_\lambda = E_{\lambda,b}\right)$ for any surface in the cavity.

- Does this condition depend on whether the cavity surface is highly reflecting or absorbing?

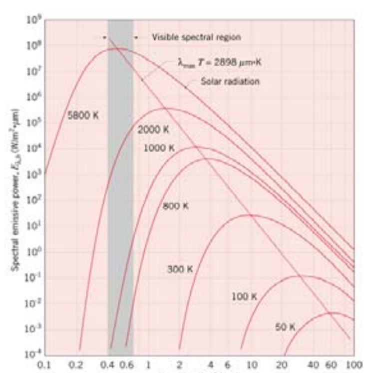
The Spectral (Planck) Distribution of Blackbody Radiation

- The spectral distribution of the blackbody emissive power (determined theoretically and confirmed experimentally) is

$$E_{\lambda,b}\left(\lambda,T\right) = \pi I_{\lambda,b}\left(\lambda,T\right) = \frac{C_1}{\lambda^5\left[\exp\left(C_2 / \lambda T\right)-1\right]}$$

First radiation constant: $C_1 = 3.742\, x 10^8\ \mathrm{W}\cdot\mu\mathrm{m}^4/\mathrm{m}^2$

Second radiation constant: $C_2 = 1.439\, x 10^4\ \mu\mathrm{m}\cdot\mathrm{K}$



- $E_{\lambda,b}$ varies continuously with λ and increases with T.
- The distribution is characterized by a maximum for which λ_{max} is given by Wien's displacement law:

$$\lambda_{max} T = C_3 = 2898 \ \mu\text{m} \cdot \text{K}$$

- The fractional amount of total blackbody emission appearing at lower wavelengths increases with increasing T.

The Stefan-Boltzmann Law and Band Emission

- The total emissive power of a blackbody is obtained by integrating the Planck distribution over all possible wavelengths.

$$E_b = \pi I_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

$\longrightarrow$ the Stefan-Boltzmann law, where

$$\sigma = 5.670 \, x \, 10^{-8} \ \text{W/m}^2 \cdot \text{K}^4 \rightarrow \text{the Stefan-Boltzmann constant}$$

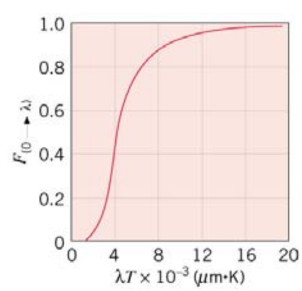
- The fraction of total blackbody emission that is in a prescribed wavelength interval or band $(\lambda_1 < \lambda < \lambda_2)$ is

$$F_{(\lambda_1 - \lambda_2)} = F_{(0-\lambda_2)} - F_{(0-\lambda_1)} = \frac{\int_0^{\lambda_2} E_{\lambda,b} d\lambda - \int_o^{\lambda_1} E_{\lambda,b} d\lambda}{\sigma T^4}$$

where, in general,

$$F_{(0-\lambda)} = \frac{\int_0^{\lambda} E_{\lambda,b} d\lambda}{\sigma T} = f(\lambda T)$$

and numerical results are given in Table 12.1



- Table 12.1

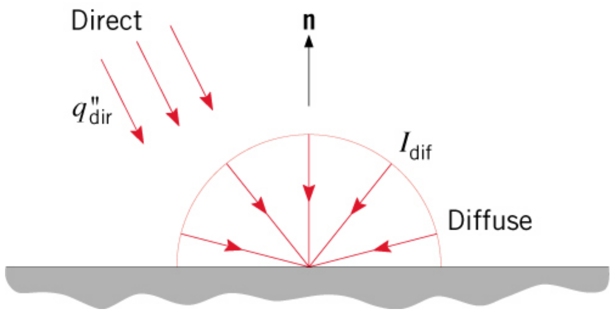
TABLE 12.1 Blackbody Radiation Functions

λT ($\mu m \cdot K$)	$F_{(0 \rightarrow \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu m \cdot K \cdot sr)^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291

Note ability to readily determine $I_{\lambda,b}$ and its relation to the maximum intensity from the 3rd and 4th columns, respectively.

- If emission from the sun may be approximated as that from a blackbody at 5800 K, at what wavelength does peak emission occur?
- Would you expect radiation emitted by a blackbody at 800 K to be discernible by the naked eye?
- As the temperature of a blackbody is increased, what color would be the first to be discerned by the naked eye?

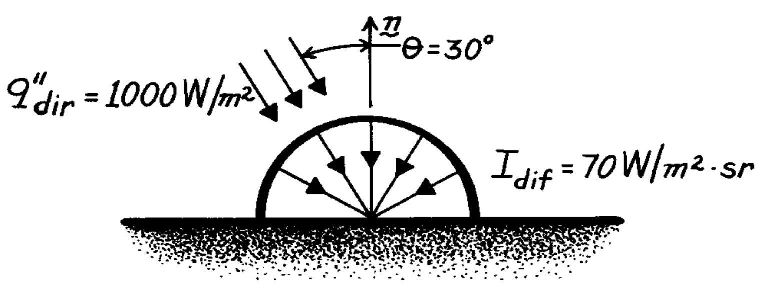
Problem 12.6: Evaluation of total solar irradiation at the earth's surface from knowledge of the direct and diffuse components of the incident radiation.



KNOWN: Flux and intensity of direct and diffuse components, respectively, of solar irradiation

FIND: Total irradiation.

SCHEMATIC:



ANALYSIS: Since the irradiation is based on the actual surface area, the contribution due to the direct solar radiation is

$$G_{dir} = q''_{dir} \cdot \cos\theta.$$

For the contribution due to the diffuse radiation

$$G_{dif} = \pi I_{dif}.$$

Hence

$$G = G_{dir} + G_{dif} = q''_{dir} \cdot \cos\theta + \pi I_{dif}$$

or

$$G = 1000\,W/m^2 \times 0.866 + \pi sr \times 70\,W/m^2 \cdot sr$$

$$G = (866 + 220)\,W/m^2$$

$$G = 1086\ W/m^2.$$

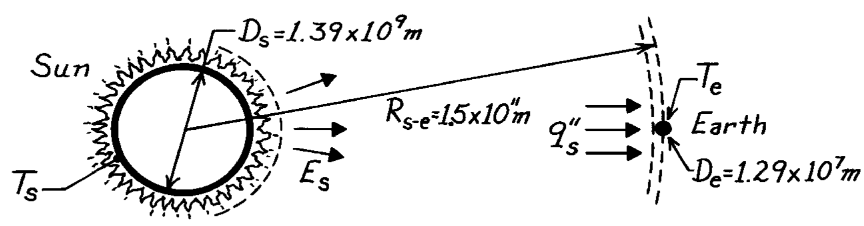
COMMENTS: Although a diffuse approximation is often made for the non-direct component of solar radiation, the actual directional distribution deviates from this condition, providing larger intensities at angles close to the direct beam.

Problem 12.20: Determination of the sun's emissive power, temperature and wavelength of maximum emission, as well as the earth's temperature, from knowledge of the sun/earth geometry and the solar flux at the outer edge of the earth's atmosphere.

KNOWN: Solar flux at outer edge of earth's atmosphere, 1353 W/m^2.

FIND: (a) Emissive power of sun, (b) Surface temperature of sun, (c) Wavelength of maximum solar emission, (d) Earth equilibrium temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Sun and earth emit as blackbodies, (2) No attenuation of solar radiation enroute to earth, (3) Earth atmosphere has no effect on earth energy balance.

ANALYSIS: (a) Applying conservation of energy to the solar energy crossing two concentric spheres, one having the radius of the sun and the other having the radial distance from the edge of the earth's atmosphere to the center of the sun, it follows that

$$E_s\left(\pi D_s^2\right) = 4\pi\left(R_{s-e} - \frac{D_e}{2}\right)^2 q_s''.$$

Hence

$$E_s = \frac{4\left(1.5\times10^{11}\,m - 0.65\times10^{7}\,m\right)^2 \times 1353\ W/m^2}{\left(1.39\times10^{9}\,m\right)^2} = 6.302\times10^{7}\ W/m^2.$$

(b) From the Stefan-Boltzmann law, the temperature of the sun is

$$T_s = \left(\frac{E_s}{\sigma}\right)^{1/4} = \left(\frac{6.302\times10^{7}\ W/m^2}{5.67\times10^{-8}\ W/m^2\cdot K^4}\right)^{1/4} = 5774\ K.$$

(c) From Wien's displacement law, the wavelength of maximum emission is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2898\,\mu m\cdot K}{5774\ K} = 0.50\ \mu m.$$

Problem: Solar/Earth Temperatures (cont)

(d) From an energy balance on the earth's surface

$$E_e\left(\pi D_e^2\right) = q_S''\left(\pi D_e^2/4\right).$$

Hence, from the Stefan-Boltzmann law,

$$T_e = \left(\frac{q_S''}{4\sigma}\right)^{1/4} = \left(\frac{1353\ W/m^2}{4\times5.67\times10^{-8}\ W/m^2\cdot K^4}\right)^{1/4} = 278\ K.$$

COMMENTS: The average earth temperature is higher than 278 K due to the shielding effect of the earth's atmosphere (transparent to solar radiation but not to longer wavelength earth emission).

Radiation: Processes and Properties
Surface Radiative Properties

Chapter 12

Sections 12.4 through 12.7

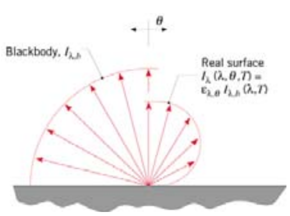
Surface Emissivity

- Radiation emitted by a surface may be determined by introducing a property (the emissivity) that contrasts its emission with the ideal behavior of a blackbody at the same temperature.

- The definition of the emissivity depends upon one's interest in resolving directional and/or spectral features of the emitted radiation, in contrast to averages over all directions (hemispherical) and/or wavelengths (total).

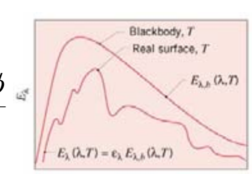
- The spectral, directional emissivity:

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) \equiv \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)}$$



- The spectral, hemispherical emissivity (a directional average):

$$\varepsilon_\lambda(\lambda,T) \equiv \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)} = \frac{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,e}(\lambda,\theta,\phi,T)\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,b}(\lambda,T)\cos\theta\sin\theta d\theta d\phi}$$



Emissivity (cont)

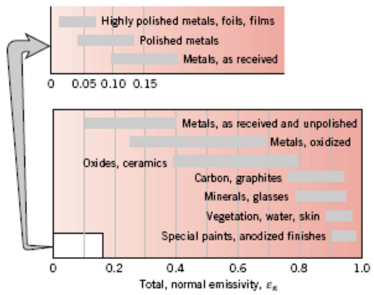
- The total, hemispherical emissivity (a directional and spectral average):

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)\,E_{\lambda,b}(\lambda,T)\,d\lambda}{E_b(T)}$$

- To a reasonable approximation, the hemispherical emissivity is equal to the normal emissivity.

$$\varepsilon \approx \varepsilon_n$$

- Representative values of the total, normal emissivity:

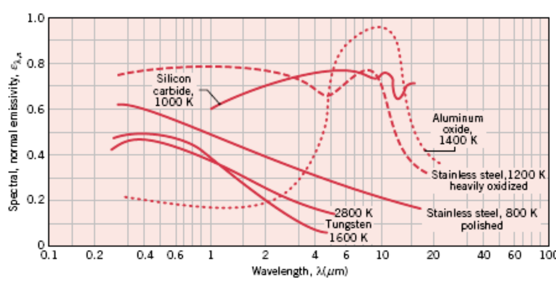


Note:

- ➢ Low emissivity of polished metals and increasing emissivity for unpolished and oxidized surfaces.
- ➢ Comparatively large emissivities of nonconductors.

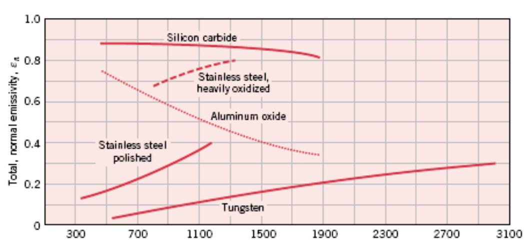
Emissivity (cont)

- Representative spectral variations:



Note decreasing $\varepsilon_{\lambda,n}$ with increasing λ for metals and different behavior for nonmetals.

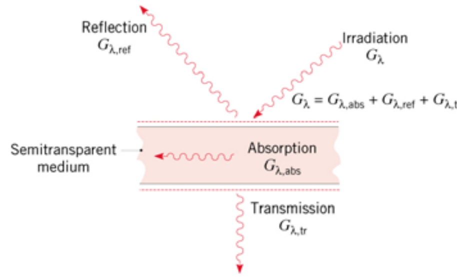
- Representative temperature variations:



Why does ε_n increase with increasing λ for tungsten and not for aluminum oxide?

Response to Surface Irradiation: Absorption, Reflection and Transmission

- There may be three responses of a semitransparent medium to irradiation:
  - Reflection from the medium $\left(G_{\lambda,ref}\right)$.
  - Absorption within the medium $\left(G_{\lambda,abs}\right)$.
  - Transmission through the medium $\left(G_{\lambda,tr}\right)$.



Radiation balance →

$$G_{\lambda} = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,tr}$$

- In contrast to the foregoing volumetric effects, the response of an opaque material to irradiation is governed by surface phenomena and $G_{\lambda,tr} = 0$.

$$G_{\lambda} = G_{\lambda,ref} + G_{\lambda,tr}$$

- The wavelength of the incident radiation, as well as the nature of the material, determine whether the material is semitransparent or opaque.
  - Are glass and water semitransparent or opaque?

- Unless an opaque material is at a sufficiently high temperature to emit visible radiation, its *color* is determined by the spectral dependence of reflection in response to visible irradiation.
  - What may be said about reflection for a white surface?   A black surface?
  - Why are leaves green?

Absorptivity of an Opaque Material

- The spectral, directional absorptivity: Assuming negligible temperature dependence,

$$\alpha_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

- The spectral, hemispherical absorptivity:

$$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\alpha_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}$$

  - ➢ To what does the foregoing result simplify, if the irradiation is diffuse? If the surface is diffuse?

- The total, hemispherical absorptivity:

$$\alpha \equiv \frac{G_{abs}}{G} = \frac{\int_o^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

  - ➢ If the irradiation corresponds to emission from a blackbody, how may the above expression be rewritten?
  - ➢ The absorptivity is approximately independent of the surface temperature, but if the irradiation corresponds to emission from a blackbody, why does α depend on the temperature of the blackbody?

Reflectivity of an Opaque Material

- The spectral, directional reflectivity: Assuming negligible temperature dependence:

$$\rho_{\lambda,\theta}(\lambda,\theta,\phi) \equiv \frac{I_{\lambda,i,ref}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

- The spectral, hemispherical reflectivity:

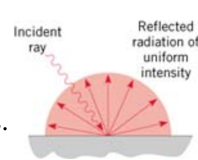
$$\rho_\lambda \equiv \frac{G_{\lambda,ref}(\lambda)}{G_\lambda(\lambda)} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\rho_{\lambda,\theta}(\lambda,\theta,\phi) I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta d\theta d\phi}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

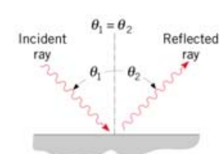
  - ➢ To what does the foregoing result simplify if the irradiation is diffuse? If the surface is diffuse?

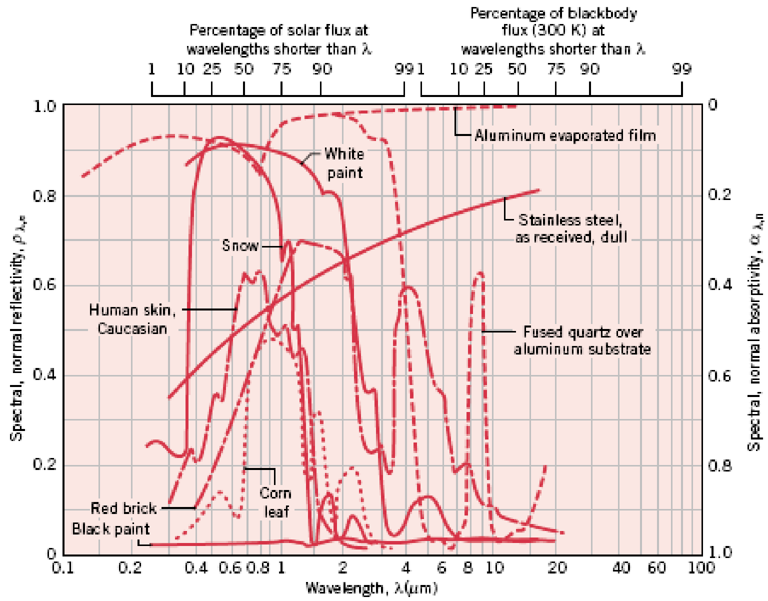
- The total, hemispherical reflectivity:

$$\rho \equiv \frac{G_{ref}}{G} = \frac{\int_0^\infty \rho_\lambda(\lambda) G_\lambda(\lambda) d\lambda}{\int_0^\infty G_\lambda(\lambda) d\lambda}$$

- Limiting conditions of diffuse and specular reflection. Polished and rough surfaces.





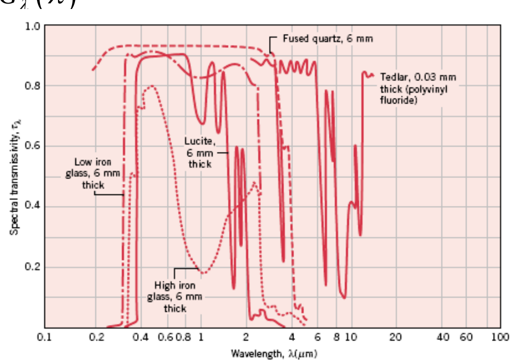


- ➢ Note strong dependence of ρ_λ (and $\alpha_\lambda = 1-\rho_\lambda$) on λ.
- ➢ Is snow a highly reflective substance? White paint?

Transmissivity

- The spectral, hemispherical transmissivity: Assuming negligible temperature dependence,

$$\tau_\lambda \equiv \frac{G_{\lambda,tr}\lambda}{G_\lambda(\lambda)}$$



Note shift from semitransparent to opaque conditions at large and small wavelengths.

- The total, hemispherical transmissivity:

$$\tau \equiv \frac{G_{tr}}{G} = \frac{\int_0^\infty G_{\lambda,tr}(\lambda)\,d\lambda}{\int_0^\infty G_\lambda(\lambda)\,d\lambda}$$

- For a semitransparent medium,

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$$
$$\rho + \alpha + \tau + 1$$

Kirchhoff's Law

- Kirchhoff's law equates the total, hemispherical emissivity of a surface to its total, hemispherical absorptivity:

$$\varepsilon = \alpha$$

However, conditions associated with its derivation are highly restrictive:

Irradiation of the surface corresponds to emission from a blackbody at the same temperature as the surface.

- However, Kirchhoff's law may be applied to the spectral, directional properties without restriction:

$$\varepsilon_{\lambda,\theta} = \alpha_{\lambda,\theta}$$

Why are there no restrictions on use of the foregoing equation?

Diffuse/Gray Surfaces

- With

$$\varepsilon_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \varepsilon_{\lambda,\theta}\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} \cos\theta\sin\theta d\theta d\phi}$$

and

$$\alpha_\lambda = \frac{\int_0^{2\pi}\int_0^{\pi/2} \alpha_{\lambda,\theta} I_{\lambda,i}\cos\theta\sin\theta d\theta d\phi}{\int_0^{2\pi}\int_0^{\pi/2} I_{\lambda,i}\cos\theta\sin\theta d\theta d\phi}$$

Under what conditions may we equate ε_λ to α_λ?

- With

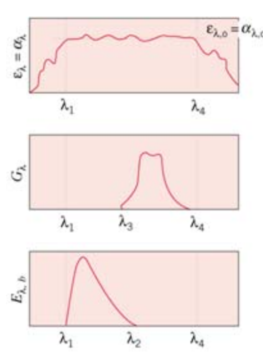
$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda)d\lambda}{E_b(T)}$$

and

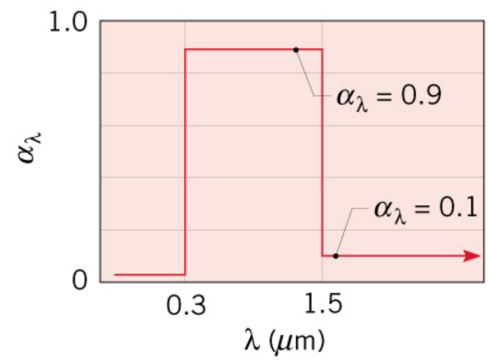
$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda)d\lambda}{G}$$

Under what conditions may we equate ε to α ?

- Conditions associated with assuming a gray surface:



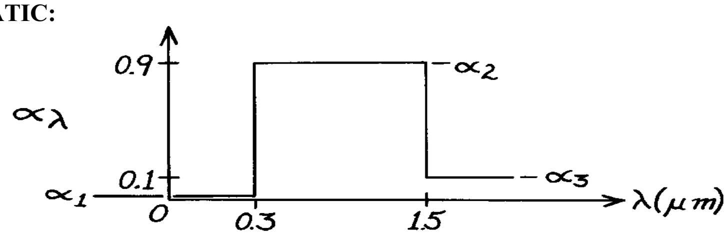
Problem 12.49: Determination of the solar absorptivity and total emissivity of a diffuse surface from knowledge of the spectral distribution of $\alpha_\lambda(\lambda)$ and the surface temperature.



KNOWN: Spectral, hemispherical absorptivity of an opaque surface.

FIND: (a) Solar absorptivity, (b) Total, hemispherical emissivity for $T_s = 340$ K.

SCHEMATIC:



ASSUMPTIONS: (1) Surface is opaque, (2) $\varepsilon_\lambda = \alpha_\lambda$, (3) Solar spectrum has $G_\lambda = G_{\lambda,S}$ proportional to $E_{\lambda,b}$ (λ, 5800 K).

ANALYSIS: (a) The solar absorptivity may be expressed as

$$\alpha_S = \int_0^\infty \alpha_\lambda(\lambda) E_{\lambda,b}(\lambda, 5800K) d\lambda \Big/ \int_0^\infty E_{\lambda,b}(\lambda, 5800K) d\lambda.$$

The integral can be written in three parts using $F_{(0\to\lambda)}$ terms.

$$\alpha_S = \alpha_1 F_{(0\to 0.3\mu m)} + \alpha_2 \left[F_{(0\to 1.5\mu m)} - F_{(0\to 0.3\mu m)}\right] + \alpha_3 \left[1 - F_{(0\to 1.5\mu m)}\right].$$

From Table 12.1,

$\lambda T = 0.3 \times 5800 = 1740$ μm·K $\quad F_{(0\to 0.3\,\mu m)} = 0.0335$

$\lambda T = 1.5 \times 5800 = 8700$ μm·K $\quad F_{(0\to 1.5\,\mu m)} = 0.8805.$

Hence,

$$\alpha_S = 0\times 0.0355 + 0.9\left[0.8805 - 0.0335\right] + 0.1\left[1 - 0.8805\right] = 0.774.$$

(b) The total, hemispherical emissivity for the surface at 340 K may be expressed as

$$\varepsilon = \int_0^\infty \varepsilon_\lambda(\lambda) E_{\lambda,b}(\lambda, 340K)\, d\lambda / E_b(340K).$$

With $\varepsilon_\lambda = \alpha_\lambda$, the integral can be written in terms of the $F_{(0\to\lambda)}$ function. However, it is readily recognized that since

$$F_{(0\to 1.5\,\mu m, 340K)} \approx 0.000 \qquad \text{at} \qquad \lambda T = 1.5\times 340 = 510\,\mu m\cdot K$$

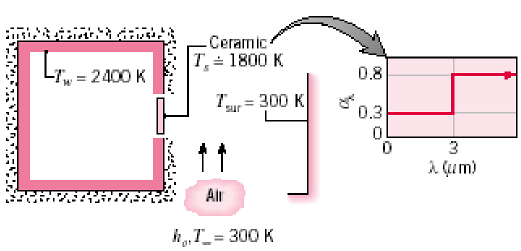
there is negligible emission below 1.5 µm.

It follows that

$$\varepsilon = \varepsilon_\lambda = \alpha_\lambda = 0.1$$

COMMENTS: The assumption $\varepsilon_\lambda = \alpha_\lambda$ is satisfied if the surface is irradiated diffusely or if the surface itself is diffuse. Note that for this surface under the specified conditions of solar irradiation and surface temperature, $\alpha_S \neq \varepsilon$. Such a surface is *spectrally selective*.

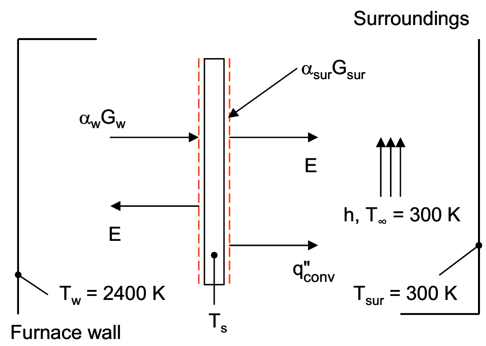
Problem 12.101: Determination of the minimum value of the outside convection coefficient on a ceramic plate to maintain prescribed plate temperature, and the sensitivity of the plate temperature to the outside convection coefficient.

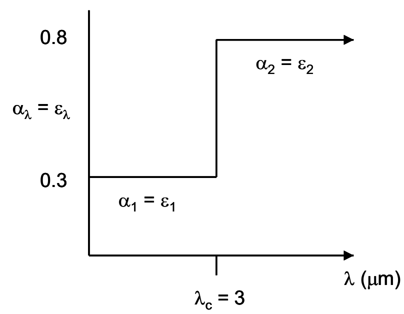


KNOWN: Temperature of interior wall of a furnace and the surroundings. Spectral distribution of plate absorptivity. Specified maximum plate temperature.

FIND: (a) Value of the outside convection coefficient, h_o, to maintain plate at T_s = 1800 K. (b) Plate temperature for various values of h_o.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) Plate is opaque and diffuse, (3) Negligible temperature gradient in the plate, (4) Negligible convection on interior surface of plate, (5) Furnace cavity and surroundings are large.

Problem: Energy Balance for an Irradiated Surface (cont)

ANALYSIS: (a) Perform an energy balance on the plate as shown in the schematic.

$$E_{in} - E_{out} = 0$$

$$\alpha_w G_w + \alpha_{sur} G_{sur} - 2E - q''_{conv} = 0$$

$$\alpha_w \sigma T_w^4 + \alpha_{sur} \sigma T_{sur}^4 - 2\varepsilon\sigma T_s^4 - h_o(T_s - T_\infty) = 0$$

or

$$h_o = \frac{\sigma\left[\alpha_w T_w^4 + \alpha_{sur} T_{sur}^4 - 2\varepsilon T_s^4\right]}{(T_s - T_\infty)}$$

where α_w, α_{sur}, and ε are determined from $\alpha_\lambda = \varepsilon_\lambda$.

Plate total emissivity: Expressing the emissivity in terms of the band emission factor, $F_{(0-\lambda T)}$,

$$\varepsilon = \varepsilon_1 F_{(0-\lambda_c T_s)} + \varepsilon_2\left[1 - F_{(0-\lambda_c T_s)}\right]$$

$$\varepsilon = 0.3 \times 0.6804 + 0.8[1 - 0.6804] = 0.460$$

where from Table 12.1, with $\lambda_c T_s = 3$ μm × 1800 K = 5400 μm·K, $F_{(0-\lambda_c T_s)} = 0.6804$.

Plate total absorptivity to irradiation from the furnace:

$$\alpha_w = \alpha_1 F_{(0-\lambda_c T_w)} + \alpha_2 \left[1 - F_{(0-\lambda_c T_w)}\right]$$

$$\alpha_w = 0.3 \times 0.8192 + 0.8[1 - 0.8912] = 0.390$$

where from Table 12.1, with $\lambda_c T_w$ = 3 μm × 2400 K = 7200 μm·K, $F_{(0-\lambda_c T_w)} = 0.8192$.

Plate total absorptivity to irradiation from the surroundings:

$$\alpha_{sur} = \alpha_1 F_{(0-\lambda_c T_s)} + \alpha_2 \left[1 - F_{(0-\lambda_c T_s)}\right]$$

$$\alpha_{sur} = 0.3 \times 0 + 0.8[1 - 0] = 0.800$$

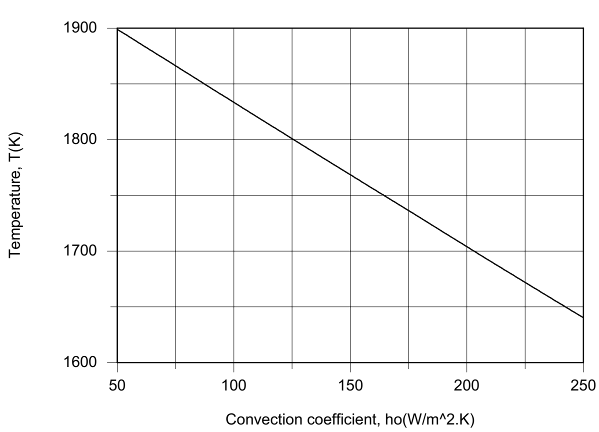
where from Table 12.1, with $\lambda_c T_{sur}$ = 3 μm × 300 K = 900 μm·K, $F_{(0-\lambda_c T_{sur})} = 0.0$.

Therefore,

$$h_o = \frac{5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\left[0.390 \times (2400\ \mathrm{K})^4 + 0.800 \times (300\ \mathrm{K})^4 - 2 \times 0.460 \times (1800\ \mathrm{K})^4\right]}{(1800 - 300)\ \mathrm{K}}$$

$$h_o = 125\ \mathrm{W/m^2 \cdot K}$$ <

(b)



With increasing values of h_o, the plate temperature is reduced. <

COMMENTS: (1) The plate is not gray. (2) The required value of the exterior heat transfer coefficient is easily obtainable with air cooling. (3) The gas inside the furnace may heat or cool the plate by convection, depending whether the gas temperature is greater or less than the plate temperature.

Radiation Exchange Between Surfaces: Enclosures with Nonparticipating Media

Chapter 13

Sections 13.1 through 13.3

Basic Concepts

- Enclosures consist of two or more surfaces that envelop a region of space (typically gas-filled) and between which there is radiation transfer. Virtual, as well as real, surfaces may be introduced to form an enclosure.

- A nonparticipating medium within the enclosure neither emits, absorbs, nor scatters radiation and hence has no effect on radiation exchange between the surfaces.

- Each surface of the enclosure is assumed to be isothermal, opaque, diffuse and gray, and to be characterized by uniform radiosity and irradiation.

The View Factor (also Configuration or Shape Factor)

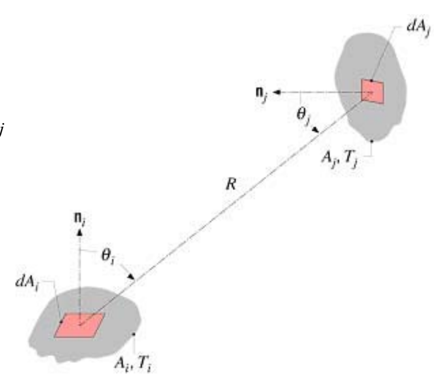
- The view factor, F_{ij}, is a geometrical quantity corresponding to the fraction of the radiation leaving surface *i* that is intercepted by surface *j*.

$$F_{ij} = \frac{q_{i\to j}}{A_i J_i}$$

- The view factor integral provides a general expression for F_{ij}. Consider exchange between differential areas dA_i and dA_j :

$$dq_{i\to j} = I_i \cos\theta_i dA_i d\omega_{j-i} = J_i \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$

$$F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_i dA_j$$



View Factor Relations

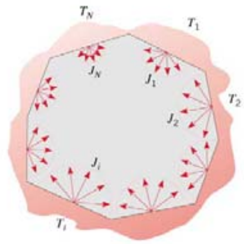
- Reciprocity Relation. With

$$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j}\frac{\cos\theta_i\cos\theta_j}{\pi R^2}dA_i dA_j$$

$$A_i F_{ij} = A_j F_{ji}$$

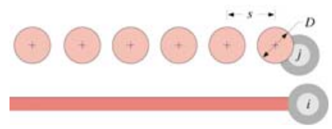
- Summation Rule for Enclosures.

$$\sum_{j=1}^{N} F_{ij} = 1$$



- Two-Dimensional Geometries (Table 13.1) For example,

An Infinite Plane and a Row of Cylinders



$$F_{ij} = 1 - \left[1-\left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2-D^2}{D^2}\right)^{1/2}\right]$$

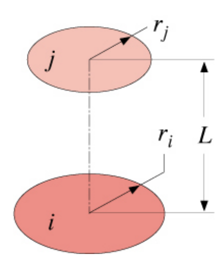
- Three-Dimensional Geometries (Table 13.2). For example,

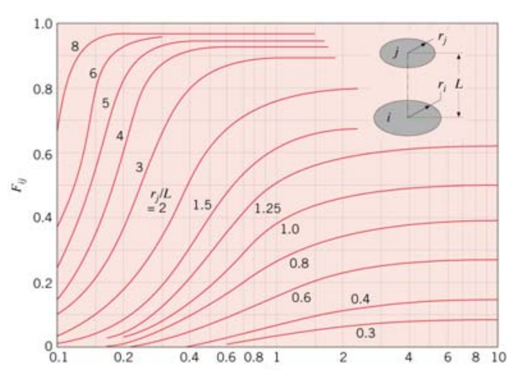
Coaxial Parallel Disks

$$F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(r_j/r_i\right)^2\right]^{1/2}\right\}$$

$$S = 1 + \frac{1+R_j^2}{R_i^2}$$

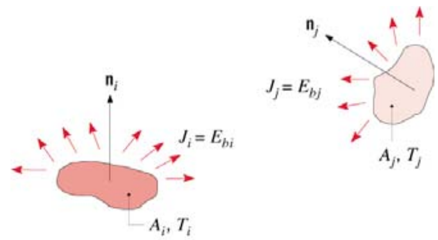
$$R_i = r_i/L \qquad R_j = r_j/L$$





Blackbody Radiation Exchange

- For a blackbody, $J_i = E_{bi}$.



- Net radiative exchange between two surfaces that can be approximated as blackbodies ⟶ *net* rate at which radiation leaves surface *i* due to its interaction with *j*

  or net rate at which surface *j* gains radiation due to its interaction with *i*

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

$$q_{ij} = A_i F_{ij} E_{bi} - A_j F_{ji} E_{bj}$$

$$q_{ij} = A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right)$$

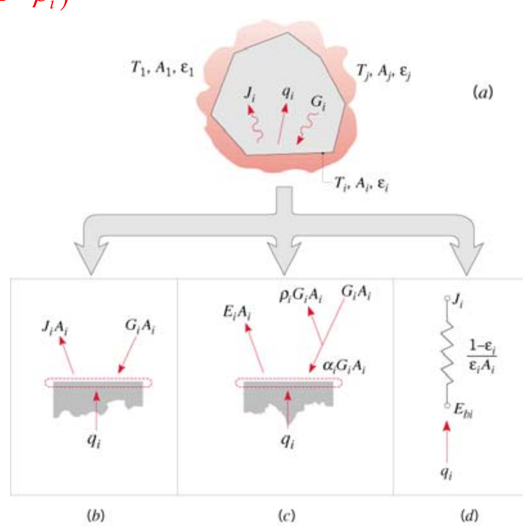
- Net radiation transfer from surface *i* due to exchange with all (*N*) surfaces of an enclosure:

$$q_i = \sum_{j=1}^{N} A_i F_{ij} \sigma \left(T_i^4 - T_j^4 \right)$$

General Radiation Analysis for Exchange between the *N* Opaque, Diffuse, Gray Surfaces of an Enclosure

$(\varepsilon_i = \alpha_i = 1 - \rho_i)$

- Alternative expressions for net radiative transfer from surface *i*:



$$q_i = A_i \left(J_i - G_i \right) \to \text{ Fig. (b)} \qquad (1)$$

$$q_i = A_i \left(E_i - \alpha_i G_i \right) \to \text{ Fig. (c)} \qquad (2)$$

$$q_i = \frac{E_{bi} - J_i}{\left(1 - \varepsilon_i\right) / \varepsilon_i A_i} \to \text{ Fig. (d)} \qquad (3)$$

↳ Suggests a surface radiative resistance of the form: $\left(1 - \varepsilon_i\right) / \varepsilon_i A_i$

General Enclosure Analysis (cont)

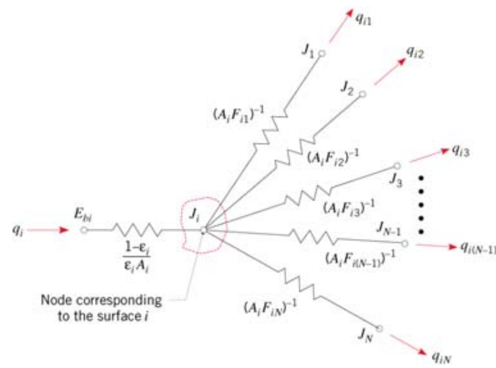
$$q_i = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (4)$$

Suggests a space or geometrical resistance of the form: $\left(A_i F_{ij}\right)^{-1}$

- Equating Eqs. (3) and (4) corresponds to a radiation balance on surface *i*:

$$\frac{E_{bi} - J_i}{\left(1-\varepsilon_i\right)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_i - J_j}{\left(A_i F_{ij}\right)^{-1}} \qquad (5)$$

which may be represented by a radiation network of the form

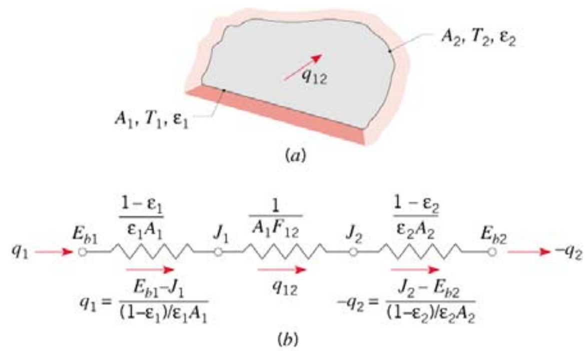


General Enclosure Analysis (cont)

- Methodology of an Enclosure Analysis
  - Apply Eq. (4) to each surface for which the net radiation heat rate q_i is known.
  - Apply Eq. (5) to each of the remaining surfaces for which the temperature T_i, and hence E_{bi}, is known.
  - Evaluate all of the view factors appearing in the resulting equations.
  - Solve the system of *N* equations for the unknown radiosities, $J_1, J_2, \ldots, J_N$.
  - Use Eq. (3) to determine q_i for each surface of known T_i and T_i for each surface of known q_i.
- Treatment of the virtual surface corresponding to an opening (aperture) of area A_i, through which the interior surfaces of an enclosure exchange radiation with large surroundings at T_{sur}:
  - Approximate the opening as blackbody of area, A_i, temperature, $T_i = T_{sur}$, and properties, $\varepsilon_i = \alpha_i = 1$.

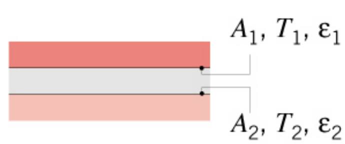
Two-Surface Enclosures

- Simplest enclosure for which radiation exchange is exclusively between two surfaces and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange.



$$q_1 = -q_2 = q_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

- Special cases are presented in Table 13.3. For example,
  - Large (Infinite) Parallel Plates



$$A_1 = A_2 \equiv A \qquad F_{12} = 1$$

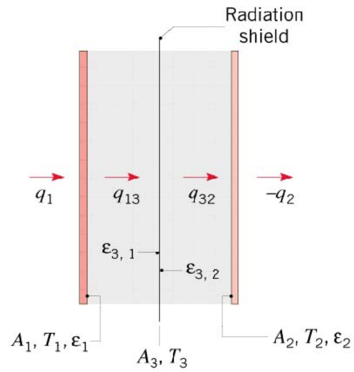
$$q_{12} = \frac{A_1 \sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

  - Note result for *Small Convex Object in a Large Cavity.*

Radiation Shields

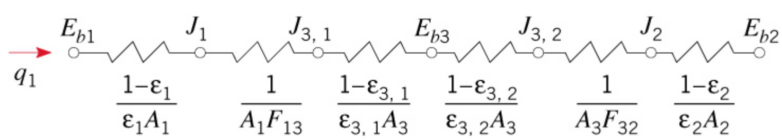
- High reflectivity (low $\alpha = \varepsilon$) surface(s) inserted between two surfaces for which a reduction in radiation exchange is desired.

- Consider use of a single shield in a two-surface enclosure, such as that associated with large parallel plates:



Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.

- Radiation Network:

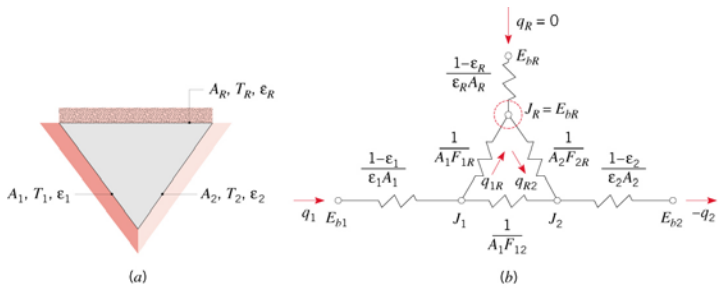


$$q_{12} = q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{13}} + \frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1} A_3} + \frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2} A_3} + \frac{1}{A_3 F_{32}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

- The foregoing result may be readily extended to account for multiple shields and may be applied to long, concentric cylinders and concentric spheres, as well as large parallel plates.

The Reradiating Surface

- An idealization for which $G_R = J_R$. Hence, $q_R = 0$ and $J_R = E_{bR}$.
- Approximated by surfaces that are well insulated on one side and for which convection is negligible on the opposite (radiating) side.
- Three-Surface Enclosure with a Reradiating Surface:



$$q_1 = -q_2 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1F_{12} + \left[\left(1/A_1F_{1R}\right) + \left(1/A_2F_{2R}\right)\right]^{-1}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

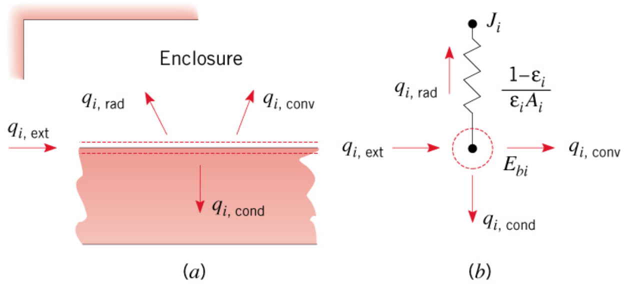
- Temperature of reradiating surface T_R may be determined from knowledge of its radiosity J_R . With $q_R = 0$, a radiation balance on the surface yields

$$\frac{J_1 - J_R}{\left(1/A_1F_{1R}\right)} = \frac{J_R - J_2}{\left(1/A_2F_{2R}\right)}$$

$$T_R = \left(\frac{J_R}{\sigma}\right)^{1/4}$$

Multimode Effects

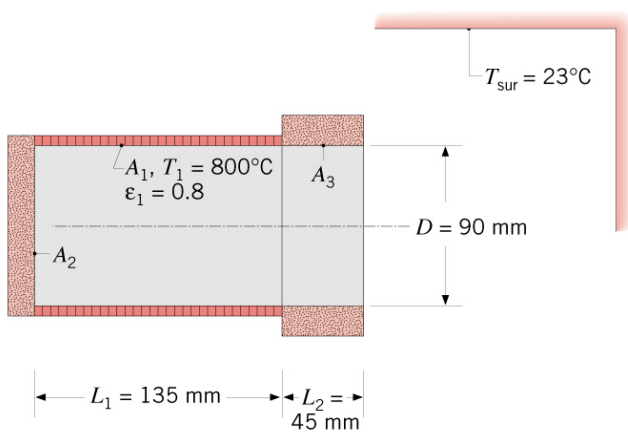
- In an enclosure with conduction and convection heat transfer to or from one or more surfaces, the foregoing treatments of radiation exchange may be combined with surface energy balances to determine thermal conditions.
- Consider a general surface condition for which there is external heat addition (e.g., electrically), as well as conduction, convection and radiation.



$$q_{i,ext} = q_{i,rad} + q_{i,conv} + q_{i,rad}$$

$q_{i,rad} \rightarrow$ Appropriate analysis for N-surface, two-surface, etc., enclosure.

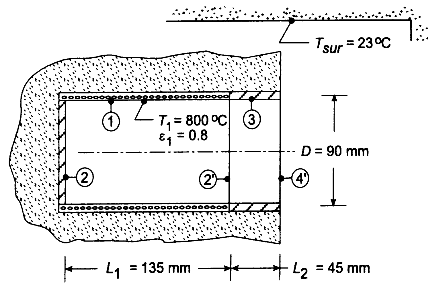
Problem 13.88: Power requirement for a cylindrical furnace with two reradiating surfaces and an opening to large surroundings.



KNOWN: Cylindrical furnace of diameter D = 90 mm and overall length L = 180 mm. Heating elements maintain the refractory lining ($\varepsilon = 0.8$) of section (1), L_1 = 135 mm, at T_1 = 800°C. The bottom (2) and upper (3) sections are refractory lined, but are insulated. Furnace operates in a spacecraft vacuum environment.

FIND: Power required to maintain the furnace operating conditions with the surroundings at 23°C.

SCHEMATIC:



ASSUMPTIONS: (1) All surfaces are diffuse gray, and (2) Uniform radiosity over the sections 1, 2, and 3.

ANALYSIS: By defining the furnace opening as the hypothetical area A_4, the furnace can be represented as a four-surface enclosure.

The power required to maintain A_1 at T_1 is q_1, the net radiation leaving A_1.

To obtain q_1, we must determine the radiosity at each surface by simultaneously solving radiation energy balance equations of the form

$$q_i = \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_j - J_j}{1/A_i F_{ij}} \qquad (1,2)$$

Problem: Furnace in Spacecraft Environment (cont)

However, since $\varepsilon_4 = 1$, $J_4 = E_{b4}$, and only three energy balances are needed for A_1, A_2, and A_3.

$$A_1: \quad \frac{E_{b1} - J_1}{(1-\varepsilon_1)/\varepsilon_1 A_1} = \frac{J_1 - J_2}{1/A_1F_{12}} + \frac{J_1 - J_3}{1/A_1F_{13}} + \frac{J_1 - J_4}{1/A_1F_{14}} \qquad (3)$$

$$A_2: \quad 0 = \frac{J_2 - J_1}{1/A_2F_{21}} + \frac{J_2 - J_3}{1/A_2F_{23}} + \frac{J_2 - J_4}{1/A_2F_{24}} \qquad (4)$$

$$A_3: \quad 0 = \frac{J_3 - J_1}{1/A_3F_{31}} + \frac{J_3 - J_2}{1/A_3F_{32}} + \frac{J_3 - J_4}{1/A_3F_{34}} \qquad (5)$$

where $q_2 = q_3 = 0$ since the surfaces are insulated (adiabatic) and hence reradiating.

From knowledge of J_1, q_1 can be determined using Eq. (1).

Of the $N^2 = 4^2 = 16$ view factors, $N(N-1)/2 = 6$ must be independently evaluated, while the remaining can be determined by the summation rule and appropriate reciprocity relations. The six independently determined F_{ij} are:

By inspection: (1) $F_{22} = 0$ (2) $F_{44} = 0$

Coaxial parallel disks: From Table 13.2,

$$(3) \qquad F_{24} = 0.5\left\{S - \left[S^2 - 4\left(r_4 / r_2\right)^2\right]^{1/2}\right\} = 0.05573$$

where

$$S = 1 + \frac{1 + R_4^2}{R_2^2} = 1 + \frac{1 + 0.250^2}{0.250^2} = 18.00 \qquad R_2 = r_2 / L = 45/180 = 0.250 \qquad R_4 = r_4 / L = 0.250$$

Enclosure 1-2-2′: From the summation rule for A_2,

(4) $\quad F_{21} = 1 - F_{22'} = 1 - 0.09167 = 0.9083$
where $F_{22'}$ can be evaluated from the coaxial parallel disk relation, Table 13.2, with $R_2 = r_2/L_1 = 45/135 = 0.333$, $R_{2'} = r_2/L_1 = 0.333$, and $S = 11.00$.

From the summation rule for A_1,
(5) $\quad F_{11} = 1 - F_{12} - F_{12'} = 1 - 0.1514 - 0.1514 = 0.6972$
From symmetry $F_{12} = F_{12'}$ and using reciprocity

$$F_{12} = A_2 F_{21} / A_1 = \left[\pi\left(0.090\text{m}\right)^2 / 4\right] \times 0.9083 / \pi \times 0.090\text{m} \times 0.135\text{m} = 0.1514$$

Enclosure 2′-3-4: From the summation rule for A_4,
(6) $\quad F_{43} = 1 - F_{42'} - F_{44} = 1 - 0.3820 - 0 = 0.6180$
where $F_{44} = 0$ and using the coaxial parallel disk relation from Table 13.2, $F_{42'} = 0.3820$ with $R_4 = r_4/L_2 = 45/45 = 1$, $R_{2'} = r_2/L_2 = 1$, and $S = 3$.

The View Factors: Using summation rules and appropriate reciprocity relations, the remaining 10 view factors can be evaluated. Written in matrix form, the F_{ij} are

0.6972*	0.1514	0.09704	0.05438
0.9083*	0*	0.03597	0.05573*
0.2911	0.01798	0.3819	0.3090
0.3262	0.05573	0.6180*	0*

The F_{ij} shown with an asterisk were independently determined.

From knowledge of the relevant view factors, the energy balances, Eqs. (3, 4, 5), can be solved simultaneously to obtain the radiosities,

$$J_1 = 73{,}084\ \text{W/m}^2 \qquad J_2 = 67{,}723\ \text{W/m}^2 \qquad J_3 = 36{,}609\ \text{W/m}^2$$

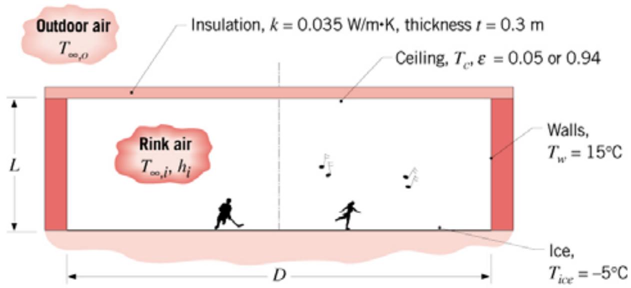
The net heat rate leaving A_1 can be evaluated using Eq. (1) written as

$$q_1 = \frac{E_{b1} - J_1}{\left(1 - \varepsilon_1\right)/\varepsilon_1 A_1} = \frac{\left(75{,}159 - 73{,}084\right)\text{W/m}^2}{\left(1 - 0.8\right)/0.8 \times 0.03817\,\text{m}^2} = 317\ \text{W} \qquad <$$

where $E_{b1} = \sigma T_1^4 = \sigma(800 + 273\text{K})^4 = 75{,}159\ \text{W/m}^2$ and $A_1 = \pi D L_1 = \pi \times 0.090\text{m} \times 0.135\text{m} = 0.03817\ \text{m}^2$.

COMMENTS: Recognize the importance of defining the furnace opening as the hypothetical area A_4 which completes the four-surface enclosure representing the furnace. The temperature of A_4 is that of the surroundings and its emissivity is unity since it absorbs all radiation incident on it.

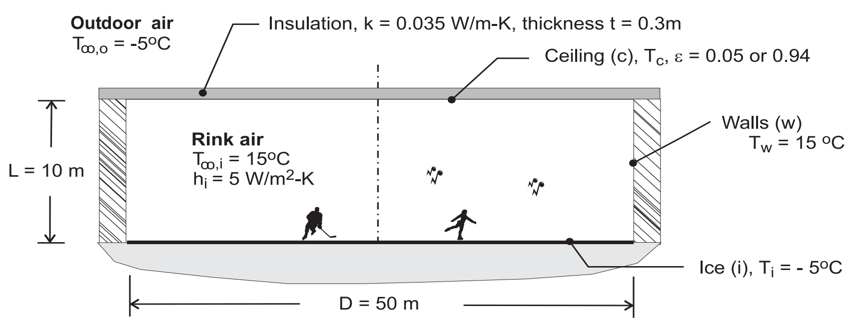
Problem 13.93: Assessment of ceiling radiative properties for an ice rink in terms of ability to maintain surface temperature above the dew point.

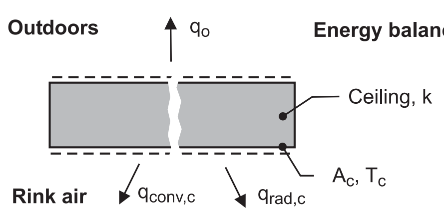


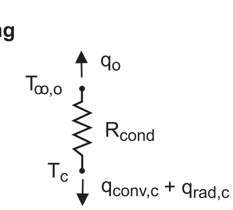
KNOWN: Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

FIND: (a) Temperature of the ceiling, T_c, for an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \le t \le 1$ m; identify conditions for which condensation will occur on the ceiling.

SCHEMATIC:







ASSUMPTIONS: (1) Rink comprised of the ice, walls and ceiling approximates a three-surface, diffuse-gray enclosure, (2) Surfaces have uniform radiosities, (3) Ice surface and walls are black, (4) Panels are diffuse-gray, and (5) Thermal resistance for convection on the outdoor side of the ceiling is negligible compared to the conduction resistance of the ceiling insulation.

PROPERTIES: *Psychrometric chart* (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{\infty,i}$ = 15°C; relative humidity, RH = 70%): Dew point temperature, T_{dp} = 9.4°C.

ANALYSIS: Applying an energy balance to the inner surface of the ceiling and treating all heat rates as energy *outflows*,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$-q_o - q_{conv,c} - q_{rad,c} = 0 \quad (1)$$

where the rate equations for each process are

$$q_o = \left(T_c - T_{\infty,o}\right)/R_{cond} \qquad R_{cond} = t/kA_c \quad (2,3)$$

$$q_{conv,c} = h_i\, A_c\, \left(T_c - T_{\infty,i}\right) \quad (4)$$

$$q_{rad,c} = \varepsilon E_b\left(T_c\right)A_c - \alpha A_w F_{wc} E_b\left(T_w\right) - \alpha A_i F_{ic} E_b\left(T_i\right) \quad (5)$$

Since the ceiling panels are diffuse-gray, $\alpha = \varepsilon$.

From Table 13.2 for parallel, coaxial disks

$$F_{ic} = 0.672$$

From the summation rule applied to the ice (i) and the reciprocity rule,

$$F_{ic} + F_{iw} = 1 \qquad F_{iw} = F_{cw} \text{ (symmetry)}$$

$$F_{cw} = 1 - F_{ic}$$

$$F_{wc} = \left(A_c / A_w\right)F_{cw} = \left(A_c / A_w\right)\left(1 - F_{ic}\right) = 0.410$$

where $A_c = \pi D^2/4$ and $A_w = \pi DL$.

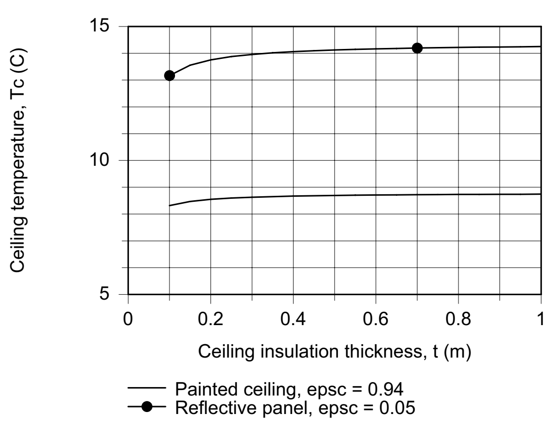
Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

Ceiling panel	ε	T_c (°C)	
Reflective	0.05	14.0	
Paint	0.94	8.6	$T_c < T_{dp}$

<

. Condensation will occur on the painted panel since $T_c < T_{dp}$.

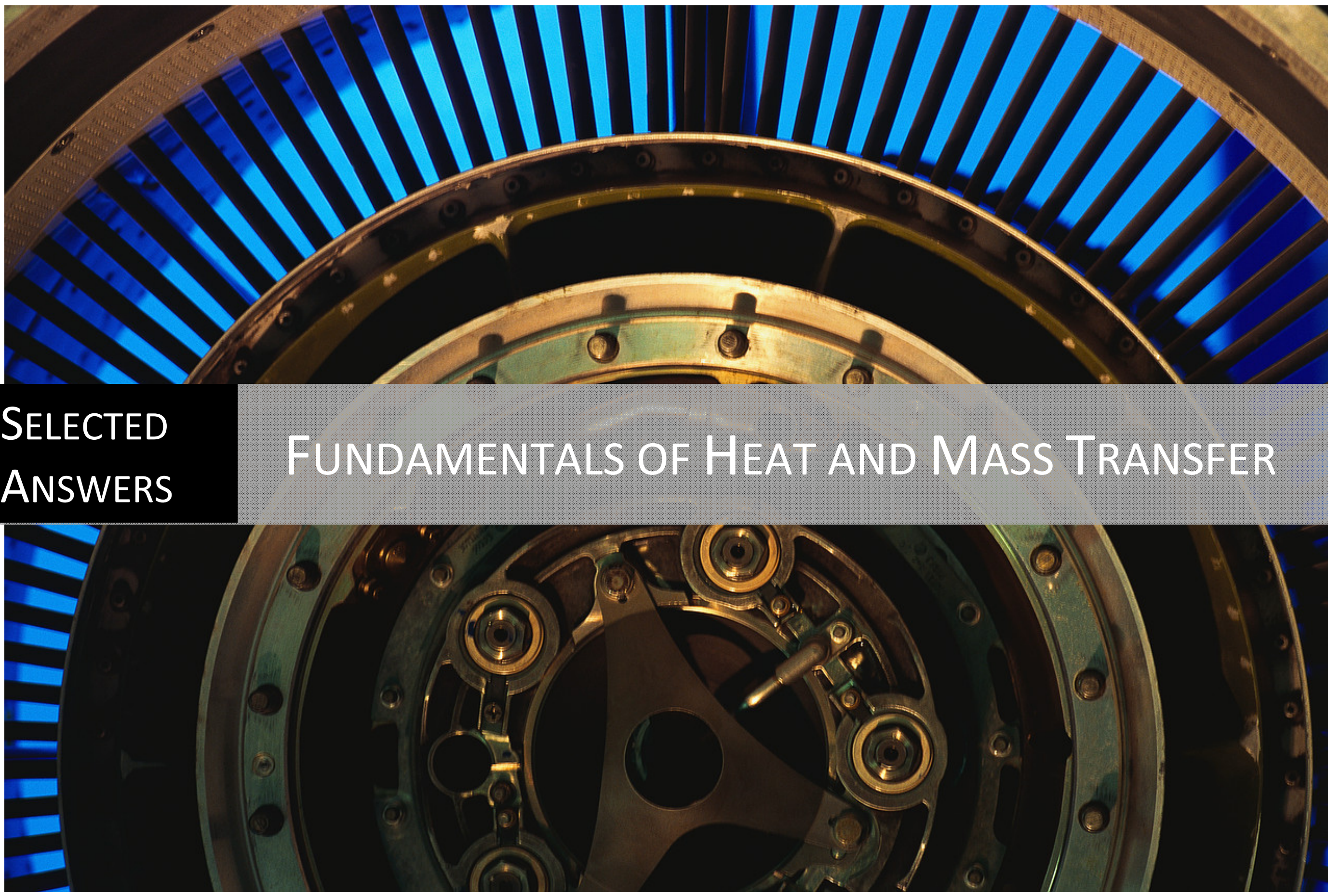
(b) Applying the foregoing model for $0.1 \le t \le 1.0$ m, the following result is obtained



For the reflective panel ($\varepsilon = 0.05$), the ceiling surface temperature is considerably above the dew point. Therefore, condensation will not occur for the range of insulation thicknesses. For the painted panel ($\varepsilon = 0.94$), the ceiling surface temperature is always below the dew point, and condensation occurs for the range of insulation thicknesses.

COMMENTS: From the analysis, recognize that radiative exchange between the ice and the ceiling has the dominant effect on the ceiling temperature. With the reflective panel, the rate is reduced nearly 20-fold relative to that for the painted panel. With the painted panel ceiling, condensation will occur for most of the conditions likely to exist in the rink.

University of Florida EML4140 §5964 Spring 2008



Selected Answers

Fundamentals of Heat and Mass Transfer

http://www.mae.ufl.edu/courses/spring2008/eml4140

ANSWERS TO END-OF-CHAPTER PROBLEMS
Fundamentals of Heat and Mass Transfer (6th Edition)
Introduction to Heat Transfer (5th Edition)

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CHAPTER 1

1.1	14.5 W/m^2, 58 W
1.2	2667 W
1.3	4312 W, \$4.14/d
1.4	0.10 W/m·K
1.5	8400 W
1.6	19,600 W, 120 W
1.7	54 mm
1.8	16.6 W/m^2, 35.9 W
1.9	375 mm
1.10	110.40°C, 110.24C
1.11	1.1°C
1.12	(a) 9800 W/m^2
1.13	(a) 1400 W/m^2; (b) 18,000 W/m^2
1.14	(a) 22.0 $W/m^2 \cdot K$; (b) 22.12, 0.6
1.15	4570 $W/m^2 \cdot K$, 65 $W/m^2 \cdot K$
1.16	51.8°C, 3203°C
1.17	6.3 m/s
1.18	0.35 W, 5.25 W
1.19	2.94 W
1.21	15 mW
1.22	6.3 $W/m^2 \cdot K$
1.23	102.5°C
1.25	254.7 K
1.27	0.42, 264 W
1.28	(a) 18,405 W; (b) \$6450
1.30	3.5%
1.31	(a) 0.223 W; (b) 3.44 W
1.32	(a) 8.1 W; (b) 0.23 kg/h
1.33	100°C
1.34	(a) 0, 144 W, 144 W, 0; 0, 144 W, 144 W, 0; (b) 2.04×10^5 W/m^3; (c) 39.0 $W/m^2 \cdot K$
1.35	(a) 0.052°C/s; (b) 48.4°C
1.36	375 W, 1.8×10^{-4} W, 0.065 W
1.37	6380 kWh, \$510 or \$170
1.38	(a) 4180 s; (b) 319 K, 359 K; (c) 830 K

1.39 (a) 0.0181 m^3/s, 4.7 m/s; (b) 5.97 W
1.40 840 kW
1.41 (a) 32.5 kW/m^2, 126 kW/m^2, 17.6 K/s, 68.6 K/s
1.43 (a) 104 K/s; (b) 1251 K
1.46 (a) 704~~1950~~ A
1.47 132 J/kg·K
1.48 (a) –0.084 K/s; (b) 439 K
1.49 (a) 1.41×10^{-3} kg/s
1.50 3.2 h
1.51 (a) 60.6×10^{-3} kg/s·m^2, 121 g/m^2; (b) 32.3×10^{-3} kg/s·m^2
1.53 49°C
1.54 (a) 7.13×10^{-3} m^3/s; (b) 70°C
1.55 (a) 86.7°C; (b) 47°C
1.56 (a) –0.044 K/s; (b) 230 W, 230 W
1.57 (a) 26°C; (b) 4.0×10^{-5} ℓ/s
1.58 (b) 80°C
1.59 12.2 W/m^2·K, 12.2 W/m^2·°C
1.60 (a) 600 K
1.61 375 W/m^2·K
1.62 (a) 5500 W/m^2; (b) 87.8°C
1.63 (a) 5268 W; (b) 41°C
1.65 345°C
1.66 (a) 84°C
1.67 214 K, 20.0 mW
1.68 (a) 190.6 W
1.69 (a) 153°C
1.70 (a) 386 W/m^2; (b) 27.7°C; (c) 55%
1.71 (a) 47.0°C or 39.9°C

CHAPTER 2

2.4 Solution
2.8 (a) –280 K/m, 14.0 kW/m^2; (b) 80 K/m, –4.0 kW/m^2; (c) 110°C, –8.0 kW/m^2; (d) 60°C, 4.0
kW/m^2; (e) –20°C, –10.0 kW/m^2
2.9 (a) 2000 K/m, –200 kW/m^2; (b) –2000 K/m, 200 kW/m^2; (c) 2000 K/m, –200 kW/m^2
2.11 0, 60 K/m
2.12 100°C, 18.75 W; –40°C, 16.25 W
2.14 1010 W, $1050; 151 W, $157; 10.1 W, $10
2.15 (a) 85 μW
2.16 14.5 (Btu/h·ft^2·°F)$^{-1}$, 18 (Btu/h·ft^2·°F)$^{-1}$, 18 (Btu/h·ft^2·°F)$^{-1}$
2.17 (a) 15.0 W/m·K, 400 K; (b) 70.0 W/m·K, 380 K
2.18 (b) 5.9×10^{-3} W/m·K, (c) 0.74×10^{-3} °C, (d) 25.02°C
2.19 765 J/kg·K, 36.0 W/m·K

2.22 (a) 0, 0.98×10^5 W/m; (b) 56.8 K/s

2.23 (a) 2×10^5 W/m^3; (b) 0, 10,000 W/m^2

2.24 (a) 200 W/m^2, 182 W/m^2, 18 W/m^2; (b) 4.3 W/m^2·K

2.25 (b) 2×10^5 W/m^3; (c) –2950 W/m^2, 5050 W/m^2; (d) 51 W/m^2·K, 101 W/m^2·K; (f) -2×10^5 W/m^3; (g) 20°C, 4.94×10^6 J/m^2

2.26 (a) 10^6 W/m^3; (b) 120°C, 10^4 K/m, -10^5 K/m^2; (c) 220°C, 10^4 K/m, -10^5 K/m^2; (d) 220°C, 2×10^4 K/m, -2×10^5 K/m^2

2.40 (a) 0.20 m; (b) 0; (c) –144,765 W; (d) 72,380 W, –72,380 W

2.41 (d) 133°C, 122°C, 133.1°C

2.43 (d) 52.5°C

2.45 (c) 18.0 kW/m^2, –360 K/m; (d) 8.73×10^6 J/m^2; (e) 8.73×10^6 J/m^2

2.51 (a) 1.8×10^6 W/m^3; (c) 1.8×10^5 W/m^2; (d) 7.77×10^7 J/m^2

2.53 (a) 25°C, 35°C; (b) 50°C, 30°C; (c) 86.1°C

2.54 (b) 3.18×10^8 W/m^3, 1.59×10^5 W/m^2

CHAPTER 3

3.2 (a) 7.7°C, 4.9°C

3.3 (a) 1270 W/m^2

3.4 (b) 2830 W/m2

3.5 14.1 W/m^2

3.6 (a) 996 W/m^2·K, 0.40%; (b) 14.5 W/m^2·K, 37.9%

3.7 (a) 0.553; (b) 22.1°C, 10.8°C; (c) –56.3°C

3.8 (a) 29.4 W

3.9 1.53 W/m·K

3.10 (a) 17 mm; (b) 20.7 mm; (c) 40.3 kW, 550 W

3.11 (b) 86 mm

3.12 0.79 W/m·K, 43.6 W/m·K

3.13 (b) 4.21 kW; (c) 0.6%

3.14 1.30×10^8 J

3.15 0.185 K/W

3.16 2.13

3.17 (b) 64 m, 8, 307 kW, 40%

3.18 (a) 0.0875 m, 0.963 m, 0.0008 m, 0.015 m; (b) 573.5 K, 313.1 K, 53.5 K, 313.1 K; (c) 135 W, 33.75 W, 108.4 W, 10.2 W

3.19 (b) 327°C

3.20 (a) 34,600 W/m^2

3.21 3.70×10^4 W/m^2, 55.6°C

3.22 (a) 762 W

3.24 590 W

3.25 (a) 1.97×10^6 K/W

3.27 (b) 49°C; (c) 67,200 W/m^2

3.28 (a) 0.268 W

3.29 (b) 5.76 kW

3.30 (b) 189 W
3.34 (a) 48.3×10^6 K/W; (b) 62.3°C
3.35 (a) 603 W/m
3.36 0.784 m, 0.784 m, 0.025 m
3.37 2380 W/m
3.39 (a) 12.6 W/m; (b) 7.7 W/m
3.40 (a) 214 mm, 420 W/m
3.41 (a) 251 W/m; (b) 23.5°C
3.42 (a) 4 W/m, 1.27×10^6 W/m^3; (b) 58°C; (c) 37.6°C, 34.8°C
3.43 4.5 W/m, 13 mm
3.44 (b) 239°C
3.45 (a) 47.1 W/m; (c) 3.25 h
3.47 (a) 779°C; (b) 1153°C, 779°C; (c) 0.0175 m, 318°C
3.48 (a) 3727 W/m, 163 W/m; (b) 0.26 yr
3.49 1830 W/m
3.51 5 mm
3.52 (b) 1040 W/m, 407 K, 325 K
3.53 (a) 0.01 m; (b) 770 W/m, 909 W/m; (c) 55 mm
3.54 8670 W
3.55 (a) 99.8%
3.57 0.062 W/m·K
3.59 5.34 mm
3.60 (a) 489 W, (b) 120°C
3.61 13.5 mm, 91%
3.62 601 K
3.63 181 W/m^2·K
3.64 (b) 35.5 mW, 44.9 mW
3.65 (b) 3000 W/m^2
3.67 (a) 1.69 m
3.68 77.9°C
3.69 0.157 W
3.72 212°C
3.73 (a) 4.0×10^6 W/m^3, 15.3 W/m·K; (c) 835°C, 360°C
3.74 (a) 50.2°C
3.76 (a) 180°C; (c) 50°C; (d) 180°C
3.78 (a) 530°C, 380°C; (b) 328°C, 290°C
3.79 (b) 60°C, 65°C; (c) 200 W/m^2; (d) 55°C
3.86 29 A, 3.0 m, 3.2 kW
3.88 (b) 1458 K
3.89 (a) 6410 A; (b) –15,240 W; (c) –10,990 W/m, –11,350 W/m, 4250 W/m, 3890 W/m
3.90 804 s
3.91 (a) 938 K, 931 K; (b) 3×10^8 W/m^3
3.92 (a) 71.8°C, 51.0°C; (b) 192°C
3.95 (a) 36.6°C; (b) 129.4°C; (c) 337.7°C
3.96 (a) 5.26°C, 5.14°C
3.99 63.7°C, 160 W/m

3.102 (c) –17.2 W, 23.6 W
3.103 (b) 164.3°C, 145.1°C
3.105 (b) 347°C
3.107 (b) 160°C
3.109 510 nm
3.112 Solution
3.113 (a) 305°C, (b) 272°C
3.115 (c) 62.4°C
3.116 (b) 508 W
3.117 (c) 0.333, 607°C
3.119 (a) 420%; (b) 29%
3.120 156.5°C, 128.9°C, 107.0°C
3.121 (a) Case A: 151 W/m, 0.96, 20.1, 0.50 m·K/W, 95.6°C
Case B: 144 W/m, 0.92, 19.3, 0.52 m·K/W, 96.0°C
Case D: 450 W/m, 0, 60.0, 0.17 m·K/W, 25°C
3.122 (a) Case A: 151 W/m, 0.96, 20.1, 0.50 m·K/W, 95.6°C
Case B: 144 W/m, 0.92, 19.2, 0.52 m·K/W, 96.0°C
Case D: 450 W/m, 0, 60.0, 0.167 m·K/W, 25°C
3.123 Rectangular fin: 130 W/m, 0.98, 4.5×10^{-5} m^2
Triangular fin: 117 W/m, 0.98, 2.3×10^{-5} m^2
Parabolic fin: 116 W/m, 0.96, 1.5×10^{-5} m^2
3.124 121 W
3.125 (a) 0.37 W; (b) 1.04×10^{5} W
3.126 5.2 nm, 0.2 nm
3.127 17.5 W/m·K
3.128 (a) 1.31 W; (b) 1.34 W
3.129 56.6 W/m·K
3.130 (b) 5995 W, –4278 W
3.131 (a) 2.44×10^{-3} K/W
3.132 (a) 31.8 W
3.133 1.30×10^{-3} W, 8.64×10^{-3} W
3.134 (b) 50.9 W
3.136 (a) 276 W
3.138 37.0 W
3.140 (b) 2830 W/m
3.141 1315%
3.142 93.7°C
3.143 (a) 0.99, 6.0; (b) 110.8 W/m
3.144 (a) 12.8 W; (b) 2.91 kW/m
3.145 (a) 0.97, 11.1; (b) 6.82 kW/m
3.146 394 K, 383 K, 381 K
403 K, 392 K, 382 K, 381 K
3.147 39,300 W/m, 405 K, 393 K, 384 K, 382 K
3.148 (a) 1.40 W
3.149 4025 W/m
3.152 3.3×10^{-5} ℓ/s

CHAPTER 4

4.2 94.5°C
4.3 5.6 kW/m
4.7 (c) 2.70, 3.70, 0.19°C, 0.14°C
4.9 92.7°C
4.10 84 W/m
4.11 9.9 W/m
4.12 94.9°C
4.13 110 W/m
4.14 6.72L, 612 W/m
4.15 1122 W/m
4.16 12.5 W/m, 11.3 W/m
4.17 5.30 kW
4.18 72.1°C, 78.1°C, 70.0°C
4.19 (a) 10.3 W; (b) 0.21 mm
4.20 316 kW
4.21 (a) 1.62 kW/m; (b) 93°C
4.22 (a) 0.1°C
4.23 3.62 kW, 254°C, 251°C
4.24 (a) 4.46 W, 98.2°C; 3.26 W, 78.4°C; (b) 4.09 W, 92.1°C; 3.05 W, 75.1°C
4.25 (a) 1.2°C
4.26 694 W
4.27 (a) 57°C; (b) 138 W
4.28 156.3 W, 81.3°C
4.29 (a) 745 W/m; (b) 3.54 kg/s
4.30 (a) 74.6 kW/m; (b) 2.38×10^8 W/m^3, 315°C
4.31 0.70 W
4.32 Solution
4.38 Solution
4.43 6710 W/m
4.44 (a) 422 K, 363 K, 443 K; (b) 156 W/m
4.45 (a) 362.4 K, 390.2 K, 369.0 K; (b) 7500 W/m
4.46 (a) 122.0°C, 94.5°C, 95.8°C, 79.7°C; (b) 1117 W/m; (c) –1383 W/m
4.47 (a) T1 = 46.6°C, T2 = 45.7°C, T3 = 45.4°C, T4 = 49.2°C, T5 = 48.5°C, T6 = 48.0°C, T7 =
47.9°C, 10,340 W/m
4.48 (a) T1 = 160.7°C, T2 = 95.6°C, T3 = 48.7°C; (b) 743 W/m
4.49 (a) 1.473 W/m; (b) T3 = 89.2°C
4.50 (a) 205.0 W/m; (b) 156.3 W/m
4.51 (a) 118.8°C, 156.3°C, 168.8°C, 206.3°C, 162.5°C; (b) 117.4°C, 156.1°C, 168.9°C, 207.6°C,
162.5°C
4.52 (a) 272.2°C, 952 W/m; (b) 271.0°C, 834 W/m
4.53 (a) 348.6 K, 368.9 K, 374.6 K, 362.4 K, 390.2 K, 398.0 K; (b) 1.53×10^8 W/m^3
4.54 3.00 kW/m

4.55 (a) 1.57 kW/m; (b) 1.52 kW/m
4.56 456 W/m
4.58 1487 W/m
4.59 (a) 7.72 kW/m; (b) 7.61 kW/m
4.60 94.0°C
4.61 1477 W/m
4.62 (a) 100 W/m, 1
4.64 (a) 1010 W/m, 100°C, 1; (b) 805 W/m, 100°C
4.65 (b) 128 W/m
4.66 (b) 1.48×10^8 W/m^3
4.67 (b) 1135 W/m, (c) –1365 W/m
4.69 (a) 2939 W/m^2
4.70 (b) 1205 W/m
4.71 1.604 W
4.73 (c) 25.0 W/m
4.75 (a) 2.38×10^{-3} m·K/W; (b) 3.64×10^{-3} m·K/W
4.77 (a) 878 W/m, 5.69×10^{-2} m·K/W, 220 W
4.78 (a) 131 W/m; (b) 129 W/m
4.79 (a) 1683 W/m
4.80 (a) 52.6 kW/m^2
4.82 (b) 2.43×10^7 W/m^3; (c) 47.5°C, 0.45

4S.1 2.83, 7.34 kW/m
4S.2 4.26ℓ, 245 W/m
4S.3 (a) 50°C; (b) 0.53ℓ, 3975 W/m; (d) 1.70ℓ, 12,750 W/m
4S.4 2.34ℓ
4S.5 2.58ℓ, 11,600 W/m; 1.55ℓ, 6975 W/m
4S.6 (a) 7.8 kW/m; (b) 0.24 m
4S.7 4ℓ, 45.6 kW/m; 3.5ℓ, 39.9 kW/m
4S.8 3500 W/m, 4500 W/m

CHAPTER 5

5.5 1122 s
5.7 35.3 W/m^2·K
5.8 7.04 h
5.9 168 s
5.10 859 s
5.11 968 s, 456°C
5.12 984 s, 272.5°C
5.13 0.0041 m^2·K/W
5.14 (a) 84.1°C, 83.0°C
5.15 21.8 m
5.16 1.08 h, 1220 K
5.17 (a) 209 s, 105 m; (b) 1.03 m/s, 1.69 m/s
5.18 88.7°C, 8.31 s

5.19 (b) 825 s, 122°C
5.20 (b) 63.8°C
5.21 3.54 m
5.22 (d) 1.04 h, 0.826 h, 0.444 h
5.23 2.52 m, 0.022 J
5.24 (a) 1190 s, 199 s; (b) 24.1 s; (c) 21.0 s
5.25 (a) 1 ms
5.26 1.56×10^{-4} s, 2.28×10^{-5} s
5.27 80°C, 38.3 s
5.28 45.7°C, 13.0 s
5.29 (a) 25.8 s, 9.5 s, 616 s; 118.9 s, 43.8 s, 2840 s; 40 s; (b) 29.5 s, 11.0 s, 708 s
5.30 (a) 98.1°C; (b) 1.67 h
5.31 (a) 90 μW; (b) 0.71 μs, 1,71 μs; (c) 400×10^6 bits, 6.84 s
5.32 (a) 86 ms; (b) 147 ms
5.33 (a) 11.3 min; (b) 5.9 min
5.35 960 s
5.36 (a) 56.3 min
5.37 861 s
5.38 (a) 45.4°C, 43.1°C, –7305 W/m^2, -2.72×10^7 J/m; (b) 4.4 min
5.39 (a) 33,800 s
5.40 491 s
5.41 (a) 164 s, 367 s
5.42 (a) 10.9 s
5.43 (a) 63 s; (b) -2.36×10^4 °C/m
5.44 (a) 1100 s
5.45 0.613 W/m·K, 2.73×10^6 J/m^3·K
5.46 (a) 45.3 min; (b) 2.21×10^7 J/m^2
5.47 63.8 s, 51.8°C
5.48 (a) 486 K
5.49 (a) 145 s
5.50 (a) 194 s
5.51 596 K
5.52 254°C
5.53 17 min, 149 kW
5.54 579 s
5.55 (a) 0.30 W/m·K
5.57 3.4 s
5.58 140 s, 36 mm/s
5.59 (a) 42 s, 114°C; (b) 40 kW
5.60 (b) 72 s; (c) 7125 W/m^2; (d) 3364 J; (e) 428 K
5.61 (b) 2.8 h, 107 s; (c) 3.48×10^6 J, 3405 J
5.62 (a) 98.6 s, (b) 100°C
5.63 1020 s, 257.3°C
5.64 (a) 100 s
5.65 (a) 94.2 s, 0.0025; (b) 3.0 s
5.68 4.99×10^5 J/m^2

5.69 1793 s
5.71 (a) 2.81 min; (b) 56 kJ
5.73 53.5°C
5.74 (a) 0.34 mm, 2.36 mm
5.75 1.41 W/m·K
5.76 0.45 W/m·K
5.77 (a) 276°C, 315°C
5.78 (a) 310 s
5.81 365.9 K
5.82 (b) 0.43 s
5.84 51.4 kN
5.85 (a) 870 W; (b) 28.2 yr
5.86 21.8 ns
5.87 10.6 s
5.88 (a) 31.8°C, 58.°C; (b) 34.2°C, 65.9°C
5.89 4.13 yr
5.90 (a) 32°C, 22°C; (b) 34.3 W/m^2, 22°C, 27°C; (c) 27.4°C, 26.6°C
5.91 3.1 μm
5.96 (b) 230°C
5.100 24.1°C, 71.5°C
5.105 161 s, 1364°C, 2.42 m
5.106 275°C, 312°C
5.107 502.3 K, 300.1 K
5.108 (a) 119.3°C, 45.1°C
5.109 (a) 66°C, 32°C
5.113 (a) ~230 s
5.114 (a) ~14.3 s
5.115 (b) 550 s
5.116 (a) 136 s; (b) 73 s
5.118 (b) 158°C
5.120 (a) 54.8°C; (b) 54.7°C
5.123 (b) 806 K, 1.17 s
5.124 (a) 402.7 K, (b) 368.7 K, (c) 362.5 K
5.128 (a) 54 days; (b) 13 days

5S.1 1170 s, 410°C, 537°C
5S.2 96 $W/m^2 \cdot K$
5S.3 0.0073 m/s
5S.4 7.6 min
5S.5 (a) 3607 s; (b) 51°C
5S.6 199°C
5S.7 (a) 12 s; (b) –1.0°C; (c) –3.4 J
5S.8 (a) 5.1 s, 68.3°C
5S.9 1.83 h
5S.10 434 K, 320 K
5S.11 (a) 260°C
5S.12 ~2.75 h

5S.13 561 K, 604 K
5S.14 (a) 402.7 K, 370.5 K, 362.4 K

CHAPTER 6

6.2 705 W/m^2·K, –171.4°C/m, –17,060°C/m
6.3 –9200 W/m^2
6.4 2.0
6.5 1.33
6.7 10.9 W/m^2·K, 1.0
6.8 42.5 W/m^2·K
6.9
6.10 67.35 W/m^2·K
6.13 600 W/m^2, 18.9 W
6.14 (a) 20.9 m/s
6.15 (a) 31.4 m; (b) 0.157 m
6.16 7.95 m, 275 m, 0.056 m; 10.5 m, 20.9 m, 0.049 m
6.17 (b) 29.9 × 10^{-6} m^2/s, 5.98 × 10^{-6} m^2/s, 2.99 × 10^{-6} m^2/s; (c) 5.23 m, 1.05 m, 0.523 m
6.18 40 W/m^2·K
6.19 2066 W
6.20 (a) 34.3 W/m^2·K; (b) 59.0 W/m^2·K
6.21 1.11
6.23 2.0
6.24 6.69
6.26 42.5°C
6.27 (a) 47.2°C; (b) 13.2 m/s
6.29 88.6
6.31 281, 284, 180, 465 W·s$^{0.85}$/m$^{2.7}$·K
6.32 38.3 W/m^2·K
6.33 16.6 mm, 250 K
6.34 240 W
6.35 2.66 kW
6.36 26.0 N/m^2
6.37 0.785 N
6.38 4260 W/m^2
6.39 14.3 W
6.40 1.55 × 10^{-3} m/s
6.41 (a) 0.0179 m/s; (b) 0.75 × 10^{-5} kg/s; (c) 5.31 × 10^{-5} kg/s
6.42 2.76 × 10^{-5} kg/s·m^2, 1.37 × 10^{-3} m/s
6.43 10^{-6} kg/s·m^2
6.45 0.025 m/s
6.46 385, 6.29 × 10, 0.62, 0.0073; 1613, 6.29 × 10^4, 2.56, 0.0187; 1.06 × 10^7, 1577, 6.74 × 10^5,
76.9

6.47 (a) 120 $W/m^2 \cdot K$; (d) 0.51 m/s
6.48 (a) 4.64×10^{-4} kg/s; (b) 1247 W
6.49 (a) 0.975×10^{-3} $kmol/m^3$, 0.0258 atm; (b) 9.28×10^{-4} $kg/s \cdot m^2$
6.50 (a) 1.63×10^{-4} kg/s; (b) 282.2 K
6.51 1.50×10^{-3} kg
6.52 359 $W/m^2 \cdot K$
6.53 (a) 25.8 m/s; (b) 25.2 $W/m^2 \cdot K$, 80.8 $W/m^2 \cdot K$, 162 $W/m^2 \cdot K$
6.54 (a) 0.031, 0.80; (b) 198 $W/m^2 \cdot K$; (c) 74%
6.55 2×10^{-3} $kg/s \cdot m^2$
6.56 0.00395 m/s
6.57 (b) 9.92 W
6.58 7.78 km
6.59 (a) 7.25×10^{-6} kg/s; (b) 22.4 W, 2791 W/m^2
6.60 (a) +795 W/m^2, +131 W/m^2, –563.1 W/m^2; (c) 1227 W/m^2
6.61 784 W
6.62 (a) 0.70 kg/h; (b) 85.6°C
6.63 21.8 W
6.64 2950 W
6.65 (a) 4.7°C; (b) 0.0238 m/s; (c) 16.2°C
6.66 (a) 0.0142 bar, 0.214; (b) 0.225; (c) 0.242
6.67 0.218
6.68 (a) 22.7 $W/m^2 \cdot K$, 454 W; (b) 2.12×10^{-2} m/s, 3.32 kg/h; (c) 2685 W
6.69 (a) 0.172 m/s, 65.3 s; (b) 173 $W/m^2 \cdot K$, 77,500 W/m^2; (c) 429°C
6.70 (a) 6.71×10^{-3} m/s; (b) 8.97 $W/m^2 \cdot K$
6.71 (a) 0.0113 m/s; (b) 90 J
6.72 (b) –0.32 K/s

6S.2 40.83°C
6S.3 (b) 1510 W/m, 453 $kg/s^2 \cdot m$
6S.4 (a) 34.2 N/m^2, 0.738 N/m^2; 6840 W/m^2, 148 W/m^2; (b) 1.37×10^6 W/m^3, 2.95×10^4 W/m^3;
(c) 34.0°C, 30.5°C
6S.5 (a) 0.0028, 0.0056, 13.4; (b) 3.38
6S.7 (b) 117°C
6S.8 (a) 6.66×10^7 W/m^3; (b) 1460 W; (c) 81.2°C, 303°C
6S.14 (e) 6.71×10^{-4} kg/s

CHAPTER 7

7.1 (a) 3.99 mm, 4.48 mm; 0.93 mm, 0.52 mm; 23.5 mm, 1.27 mm; 0.34 mm, 1.17 mm
7.2 (a) 0.147 m, 0.0143 m; (b) –1300 W/m^2, 0.0842 N/m^2; (c) 0.337 N/m, -5200 W/m
7.3 (a) 0.126 mm, 0.399 mm, 1.262 mm, 141 mm; (b) 6.07 N/m^2, 1.92 N/m^2, 0.61 N/m^2, 0.528 m/s, 0.167 m/s, 0.053 m/s

7.4
7.8 (a) 8.71×10^5 W/m^3; (b) 158.4°C
7.9 (a) 51.1 W, 12.2 W, 8.3 W, 255.3 W
7.10 13,600 W/m, 9780 W/m, 5530 W/m
7.11 (a) 0.257 N, 8.13 W/m^2·K, 1950 W; 0.227 N, 7.16 W/m^2·K, 3440 W; (b) 0.620 N, 9050 W/m^2·K, 5430 W; 0.700 N, 12,600 W/m^2·K, 15,100 W
7.12 (c) 4110 W/m^2·K, 4490 W/m^2·K, 5070 W/m^2·K
7.13 3.2×10^5, 1.5×10^6
7.14
7.15 (a) 17.6 W; (b) 143.6 W
7.16 (a) 5830 W/m
7.17 29°C
7.18 (a) 13.5 W, 47.6°C; (b) 13.9 W, 41.0°C
7.19 (a) 14.3 W, 125.9°C
7.20 (a) 33.8°C, 797 W; (b) 675 W; (c) 90.6 kW
7.21 1570 W
7.22 (a) 8.7 W/m^2·K, 14.5 W/m^2·K, 11.1 W/m^2·K
7.23 (b) ±4%
7.24 6780 W, -0.26°C/s
7.25 (b) 213°C
7.26 (b) 256°C; (c) 210°C
7.27 –0.99 K/s, –1.47 K/s, 1.91 m
7.28 (b) 0.47; (c) 0.47
7.29 46.2°C
7.30 2.73 m/s
7.31 (a) 23.4 W, 28.2°C; (b) 271 W, 72.6°C; 105 W, 200°C
7.32 (a) 27°C; (b) 77°C
7.33 (a) 64°C; (c) 6.6 m/s
7.34 42.5°C
7.35 (a) 98.3°C, 17.1 W/m^2·K; (b) 86.1°C
7.36 (a) ∞, 2.61 W/m^2·K; ∞, 2.74 W/m^2·K; (b) 4.22 W/m^2·K, 5.41 W/m^2·K
7.37 (a) 55 W; (b) 39 W
7.38 0.30 W, 0.81 W
7.39 0.66 W, 0.60 W
7.40 (a) 30.8 W/m^2·K, 56.7 W/m^2·K; (b) 0.633, 0.555
7.41 (a) 71.1 W/m, 20.4 kW/m, 1640 W/m
7.42 3.24 N/m, 520 W/m
7.43 (a) 603 K; (b) 200 s
7.44 (a) 562 K; (b) 155 s
7.45 70.4 W
7.46 (a) 2.20 W; (b) 37.4 mm; (c) 89.6; (d) 435%
7.47 (a) 235 W/m^2·K; (b) 0.87 W; (c) 1.02 W
7.48 (a) 1012°C, 1014°C
7.49 (b) 97 m/s
7.50 (b) 195 mA

7.52 (a) 27.6°C; (b) 27.6°C
7.53 (a) 45.8°C; (b) 68.3°C
7.54 (a) 0.924 W
7.55 (a) 1.41 W
7.56 (a) \$0.415/m·d; (b) \$0.363/m·d
7.57 (a) 3640 W/m
7.58 452.2 K
7.59 (b) 18.7 mm
7.61 (a) 0.20 m/s; (b) 2.80×10^4 W/m^2, 52%
7.62 (a) 12.8 s
7.63 (b) 340°C; (c) 309°C
7.65 3.26 m/s
7.66 (a) 0.011 N; (b) 3.14 W
7.67 (a) 0.489 N, 510 W; (b) 0.452×10^{-3} N, 1.59 W
7.68 10.3 W
7.69 (a) 67 s; (b) 48 s
7.70 (a) 18.8°C; (b) 672°C
7.71 (a) 2 m/s, (b) 1.11 mm, 1.25×10^{-3} m^3
7.72 (a) 737°C; (b) 274 s; (c) 760°C, 230 s
7.73 2.1 m/s, 1.6 m
7.74 1.1 m/s, 10.3 m
7.75 (a) 0.0011 s, 166.7 m/s; (b) 7.8×10^{-4} s
7.76 180 mm
7.77 (a) 0.3 m/s; (b) 1.86 mm
7.78 (a) 6.83 s; (b) 936 K
7.79 (b) 337°C
7.80 726 K, 735 K
7.81 (a) 92.2 W; (b) 1.76×10^8 W/m^3, 1422 K; (c) 1789 K
7.82 (a) 20.4; (b) 45.8°C
7.83 234 W/m^2·K, 0.0149 bar, 28.4 kW/m
7.84 58.5 kW, 5.9×10^{-3} bar
7.85 16, 684 N/m^2
7.86 –532 kW/m
7.87 (a) 7670 W, 47.6°C; (b) 195 N/m^2, 56 W
7.88 (a) 39°C; (b) 0.00993 bar, 6.26 kW
7.89 (a) 435 W/m^2·K; (b) 321 K, 2453 W
7.91 (a) 363 K; (b) 355 kW, 0.163 kg/s
7.92 1.68 W
7.93 1.45 K/s
7.94 (c) 0.179 kg/s
7.96 11,600 W, 140 W
7.97 (a) 3.56 s, 17 μm
7.98 (a) 470.5 W/m^2·K; (b) 40.1 s
7.99 5044 s
7.100 65.7 kW

7.101 (a) 941°C, 333 W; (b) 1314°C
7.102 (a) 10.5°C, 0.0134 kg/s
7.103 (a) 41.2°C, 0.0154 kg/s
7.104 (a) 0.0728 m/s; (b) 0.0028 kg/s
7.105 (a) 9.29×10^{-4} kg/s·m, 2265 W/m
7.106 5.93 kW
7.107 (a) 0.0135 kg/s·m
7.108 3.57 W
7.109 (a) 4.48×10^{-3} kg/s·m^2; (b) 11,800 W/m^2; (c) 9.95×10^{-4} kg/s·m
7.110 (a) 9.5×10^{-4} kg/s, 9.5×10^{-4} kg/s, 9.3×10^{-4} kg/s; (b) 334.7 K, 332.8 K, 327.1 K
7.112 (a) 0.123 kg/h, 44.9 W; (b) 52.3°C
7.113 (b) 2.12×10^{-4} kg/s·m^2
7.114 28.6 min
7.115 337 W/m, 483 W/m, 589 W/m
7.116 (b) 5.35×10^{-4} kg/s·m; (c) 1.275 kW/m
7.117 2.73×10^{6} kg/day
7.118 2775 W, -0.13°C/s
7.119 0.0016 kg/s·m^2
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7.121 (a) 20.3 kg/h; (b) 102 days
7.122 (a) 4.76×10^{-2} m/s; (b) 3.80×10^{-2} m/s
7.123 (a) 2.31×10^{-4} kg/s·m
7.124 (a) 0.30 kg/h·m, 110 W/m; (b) 52.3°C
7.126 (a) 485 W; (b) 3135 W
7.128 (a) 9424 W/m, 0.0036 kg/s·m
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7.130 (b) 1.82×10^{-7} kg/s, -8.9 K/s
7.131 159 s
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8.21 (c) 6018 W
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8.51 80.3 W/m
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9.25 (a) 118 kW/m^3
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9.71 (a) 9.84 W/m^2·K; (b) 32.1 W/m^2·K; (c) 221 s; (d) 245 s
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9.95 (a) 525 W/m^2; (b) 4; (c) 101 W/m^2
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10.7 73 kW/m^2, 232 kW/m^2; 105 kW/m^2, 439 kW/m^2
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10.34 4.16×10^4 W/m^2
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10.39 (a) 35.6°C; (b) 54.0°C
10.40 3.03 mm, 3.65 mm, 4.93 mm, 2.14 mm, 1.43 mm
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10.46 (a) 1.07 MW, 0.444 kg/s; (b) 0.98 MW, 0.407 kg/s
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10.48 (a) 649 kW/m; 0.272 kg/s·m
10.50 4.8 MW/m, 1.93 kg/s·m
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10.53 19.1 kW, 8.39×10^{-3} kg/s
10.54 4.28×10^{-3} kg/s·m
10.55 28.3 kW/m, 1.16×10^{-2} kg/s·m
10.56 (a) 144 mm
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10.60 (a) 0.0118 kg/s, 7.98×10^{-3} kg/s; (b) 50.6°C, 55.4°C; (c) 6.23 m
10.61 2.78×10^{-3} kg/s
10.62 2.0 s, 3.91×10^{-5} kg
10.63 7100 $W/m^2 \cdot K$, 9.0×10^{-3} kg/s·m
10.64 144,900 $W/m^2 \cdot K$, 7.56×10^{-4} kg/s
10.65 6.29×10^{-2} kg/s·m
10.66 0.023 kg/s, 0.0014 kg/s

10.67 2.11 W, 0.0332 m^2
10.68 (a) 114.0°C; (b) 80.9°C, 2.6×10^{-4} kg/s
10.69 (a) 79.4°C, 55.7 W; (b) 16.7 mm
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11.2 (a) 92 $W/m^2 \cdot K$; (b) 690 $W/m^2 \cdot K$
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11.7 (a) 2255 $W/m^2 \cdot K$; (b) 1800 $W/m^2 \cdot K$; (c) 29.4°C
11.8 (a) 98 $W/m^2 \cdot K$; (b) 512 $W/m^2 \cdot K$
11.9 168 $W/m^2 \cdot K$
11.10 29.5 $W/m^2 \cdot K$
11.11 12.6 $W/m^2 \cdot K$
11.13 3.09 m^2, 2.64 m^2, 2.83 m^2, 2.84 m^2
11.14 4.75 m^2
11.15 5.64×10^{-4} $m^2 \cdot K/W$
11.16 (a) 0.97 m; (b) 2250 W, 145°C, 338 $W/m^2 \cdot K$, 9.59×10^{-4} $m^2 \cdot K/W$
11.18 (a) 48°C; (c) 132 $W/m^2 \cdot K$; (d) 0.8; (e) 1
11.20 (a) 7600 W, 48.1°C; (b) 40 m
11.21 0.0029 $m^2 \cdot K/W$
11.22 5.19 kg/s, 37.5 m
11.23 (a) 1.58 m^2
11.24 (a) 357 K, 515 K; (b) 30.7 $W/m^2 \cdot K$
11.25 (a) 785 W/K, 5.0 m; (b) 18.4°C
11.26 (a) 13 m^2, 679 kW, 37.5°C; (b) $178,000/yr
11.27 (a) 74%; (b) 24°C
11.28 76.4°C
11.29 (a) 8.9 mm, 3.4 m
11.30 79.8°C, 8.70×10^{-4} kg/s
11.31 (a) 399 K, 70
11.32 33.1 m^2
11.33 (a) 130 m; (b) 1985 $W/m^2 \cdot K$; (d) 5.8%
11.34 (a) 3930 W; (b) 3.74 liter/min; (c) 0.24 m^2
11.35 (a) 3550 $W/m^2 \cdot K$, 41.1°C, 0.85 kg/s
11.36 (a) 344.4 K
11.37 (a) 1.89 kg/s
11.38 (a) 720, 0.858 m; (b) 0.513 m

11.39 (a) 41.9 m^2, 20.7 kg/s; (b) 0.936 kg/s
11.40 (b) 0.50
11.41 36.8°C, 37.5°C
11.42 50°C, 61.7°C
11.43 (a) 3.07 m^2, 33.4°C
11.44 (a) 9.6 m
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11.46 (a) 55.7°C, 41.9°C; (b) 2300 $W/m^2 \cdot K$; (d) 74.4°C, 0.55
11.47 (a) 11,200 m^2; (b) 1994 kg/s
11.48 2.83 kg/s
11.49 (a) 206; (b) 26.1°C; (c) 17.4 kg/s
11.50 (a) 686 W, 11.0 °C; (b) 10,087 W, 24.5°C; (c) $1765
11.51 (a) 129.7°C
11.52 808 $W/m^2 \cdot K$
11.53 (a) 61 $W/m^2 \cdot K$; (b) 0.64; (c) 46.2°C
11.54 68, 7.1 m
11.55 (a) 71 m^2
11.56 (a) 0.53×10^5 W, 0.47
11.57 34.1°C
11.58 36.6°C, 143.5°C
11.59 241 m^2
11.60 (a) 29.4%
11.61 1014 K, 800 K, 514 K, 0.70
11.62 (a) 204 kW, 57.3°C, 42.7°C; (b) 2.33 m
11.63 (a) 44.6 kW; (b) 0.65; (c) 0.55
11.64 (a) 8280 W; (b) 98.4°C, 87.2°C; (d) 5470 W, 0.66
11.66 (a) 0.0354 m or 0.261 m, 0.34 m
11.67 41 m^2
11.68 (a) 26.8°C
11.69 (a) 2×10^5 W, 19.6°C, 19.6°C
11.70 4.56 m
11.71 (a) 73.9 $W/m^2 \cdot K$; (b) 496 K, 369 K
11.72 (a) 104°C
11.74 (a) 0.752 m^2; (b) 0.576 m^2, 0.0855 kg/s; (c) 0.723 m^2, 317 K
11.75 (a) 0.672 kW; (b) 8°C, 61°C, 1.43 kW
11.76 (a) 335 K; (b) 8.11 m; (c) 0.666
11.77 (a) 30.3%; (b) 27.5 kg/s
11.78 (a) 86.5 kW/K
11.79 66.3 $W/m^2 \cdot K$
11.80 0.082 m^3, ~13, ~11, 0.54 m
11.81 56.2 $W/m^2 \cdot K$, 0.026 m^3
11.82 3
11.83 285 K
11.84 ~11

11.85 564 K

11S.1 160 $W/m^2 \cdot K$
11S.2 29.6 $W/m^2 \cdot K$
11S.3 4.75 m^2
11S.4 5.74×10^{-4} $m^2 \cdot K/W$
11S.5 (a) 1.5 m^2
11S.6 33.1 m^2
11S.7 9.6 m
11S.8 (a) 11,100 m^2; (b) 1994 kg/s
11S.9 878 $W/m^2 \cdot K$
11S.10 243 m^2
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CHAPTER 12

12.1 12.1 W/m^2, 28.0 W/m^2, 19.8 W/m^2
12.2 (a) 1.38×10^{-3} W; (b) 2.76 W/m^2
12.3 8×10^{-6} J
12.4 (a) 193 mm; (b) 940 W/m^2
12.5 (a) 60 mW; (b) 95.5 $W/m^2 \cdot sr$; (c) 47.8 μW
12.6 1086 W/m^2
12.7 (a) 1446 W/m^2
12.8 167.6 W/m^2
12.9 0.25
12.10 (a) 2000 W/m^2; (b) 637 $W/m^2 \cdot sr$; (c) 0.50
12.11 (a) 0.10 W; (b) 1273 $W/m^2 \cdot sr$; (d) 0.10 W; (e) 3.6×10^{-7} W, 90 mW/m^2; (f) 5.09×10^{-7} W,
127 mW/m^2; (g) 63.7 mW/m^2
12.13 (a) 0.393 m
12.14 (a) 0.32 W/m^2
12.15 789 K, 22 W; 273 K, 0.031 W; 1606 K, 37.7 W; 1750 K, 2761 W; 485 K, 0.628 W
12.16 (a) 133 W, 76 W, 232 W, 5.54 μm
12.17 7348 W/m^2
12.18 498 K
12.19 279 K
12.20 (a) 6.3×10^{7} W/m^2; (b) 5774 K; (c) 0.50 μm; (d) 278 K
12.21 (a) 6.74×10^{-5} sr; (b) 2.0×10^{7} $W/m^2 \cdot sr$
12.22 (a) 0.50 μm, 1.16 μm, 1.93 μm, 9.50 μm, 48.3 μm; (b) 0.125, 0.366, 0.509
12.23 (a) 0.226, 0.044; (b) 0.50 mm, 1.0 mm
12.24 (a) 0.0137, 0.376; 0.359, 0.132
12.25 (a) 9.84 kW/m^2, (b) 6.24 kW/m^2

12.26 278.4 W
12.28 (b) 0.90 K, 7.1 K
12.29 (a) 0.352; (b) –1977 K/s
12.30 (a) 0.249; (b) 0.358
12.32 (a) 0.25, 2.27×10^5 W/m^2
12.33 (a) 66.2 W/m^2; (b) 8.76 μm
12.34 0.636, 76.9 kW/m^2
12.35 1.45
12.39 (c) 995.3 K
12.40 (a) 3.3%, 5.3%; (b) 10%
12.41 0.373, -5.78 K/s, 311 s
12.42 34.7 kW/m^2
12.43 (b) 7500 W/m^2; (c) 2250 W/m^2; (d) 0.30
12.44 (a) 0.50, 0.60; (b) 3.62×10^5 W/m^2, 5.44×10^5 W/m^2; (c) 0.126 W/m^2·μm; (d) 10.3 μm
12.45 (a) 0.383; (b) 0.958, 0.240
12.46 515 W/m^2, 1306 W/m^2, 1991 W/m^2, 3164 W
12.47 2.51×10^{-6} W
12.48 1480 μW
12.49 (a) 0.774; (b) 0.1
12.50 (a) 0.45; (b) 0.56; (c) -22,750 W/m^2
12.51 (a) 0.311; (b) 74,750 W/m^2; (c) -60,750 W/m^2
12.52 348 K
12.53 (a) 0.669; (b) 0.745; (c) 111 kW/m^2; (d) 176 kW/m^2
12.54 (a) 1.76×10^{-4} W; (b) 0.384×10^{-4} W
12.55 (a) 0.00099; (b) 0.295; (c) 0.861
12.56 (a) 0.839, 0.568; (b) 0.329, 0.217
12.57 (b) 8.88 W, 78.2 W
12.59 (b) 0.599, 0.086, 0.315; (c) 1; (d) –615 W/m^2
12.60 0.85, 0.85, 3.9×10^6 W/m^2, 4.6×10^6 W/m^2, 4.6×10^6 W/m2; 0.297, 0.106, 137 W/m^2, 4.6×10^6 W/m^2, 1.38×10^6 W/m^2
12.62 (a) 0; (b) 0.5
12.63 (a) 0.574; (b) 0.145; (c) 0.681, 0.539
12.64 0, 0.3, 0.7, 0.303, 606 W/m^2, 952 W/m^2·K
12.65 (a) 0, 577.4 K; (b) 0.89, 0.10; (c) 5600 W/m^2, 630 W/m^2; (d) 0.89, 0.10
12.66 0.34, 0.80, 1700 W/m^2, -700 W/m^2
12.67 (a) 0.60; (b) 200 W/m^2; (c) -1200 W/m^2
12.68 (b) 1.42×10^5 W/m^2
12.69 (a) 0.720; (b) 0.756; (c) -11.8 kW/m^2
12.70 (a) 0.225, 0.388, 0.5; (b) 443 kW/m^2
12.71 7000 W/m^2, 0.94
12.72 29.2 W, 344 K
12.73 (a) 0.748; (b) 29.6°C; (c) 402 W/m^2

12.74 (a) 117°C
12.75 (a) 22.2°C
12.76 (a) 25°C
12.77 (a) 19.8 W; (b) 538.2 K; (c) 855 s
12.78 (a) 145 W/m^2; (b) 9.23 μW; (c) 8.10 μW
12.79 999 K
12.80 8.08×10^{-8} W
12.81 (a) 1.15 μW; (b) 1.47 μW
12.82 3.38×10^{-7} W
12.83 571 m/s, -15.5 m/s, 146 m, 0.256 s
12.84 (a) 134 K
12.85 (a) 1718 μV; (c) 1382 μV, 878 μV, 373 μV
12.86 (a) 23°C; (b) 44.6°C
12.88 (a) 5441 W/m^2
12.89 (a) 0.49; (b) 0.20; (c) 24.5 W
12.90 (a) 0.375; (b) 0.702; (c) 0.375
12.91 (b) 20 K/s
12.92 (a) 2.93 W/m·K
12.93 (a) 1425 K, 1.87×10^5 W/m^2; (b) 5.95×10^{-5} W
12.94 (a) 314 μW; (b) 871 K
12.95 796 K, 793 K
12.96 (a) 0.643; (b) 0.200; (c) 1646 kW/m^2; (e) 119 s
12.97 (a) 839 K; (b) 770 K; (c) 850 K
12.98 32.5 s
12.99 (a) 1.83 s; (b) 54.3 W
12.100 (a) 1.56×10^9 W/m^3; (b) 15.6 W/mm^2, 7.8 W/mm^2; (c) 772 s
12.101 (a) 126 W/m^2·K
12.102 (a) 0.799, 0.536; (b) 11.7 K/s; (c) 0.536; (d) 82 s
12.103 (a) 763 W; (b) 930 K; (c) 537 s
12.104 (a) 0.80, 0.713; (b) 233 kW/m^2, 9.75 K/s; (d) 186 s; (e) 413 s; (f) 899 s
12.105 (a) 0.51 μm, 8.33 μm; (c) 0.30, 0.59, 0.68; (d) 0.66, 0.68; (e) 73.5 kW/m^2
12.106 (a) 48.5°C
12.107 139°C, 3.47 W
12.109 329.2 K
12.110 (a) 0.8, 0.625; (b) 352 K
12.111 (a) 0.634; (b) 0.25; (c) 460 W/m^2
12.115 (b) 35.5°C
12.116 (a) 8.16×10^5 W/m^2
12.117 0.129 m, 39.1%
12.118 (a) 1.60×10^7 W, 75.8%
12.119 (b) 295.2 K; (c) 704 W
12.121 (a) 0.704; (b) 0.20; (c) 6.07 W/m^2·K
12.122 (a) 55.2°C; (b) 35.9°C
12.123 (a) 4.887 W; (c) 43.5°C

12.124 96.5 W/m^2
12.125 (a) 77.6°C; (b) –35.3°C
12.126 379 K, 656 K
12.127 (a) 197 K; (b) 340 K
12.128 439 K
12.129 0.95
12.130 (a) 837 W, 1026 W
12.131 (a) 30.5°C, 2766 W; (b) –7.6°C, 4660 W; (c) 80°C, 9557 W
12.132 (a) 1.286; (b) 352 K, 145 K
12.133 714 W/m^2
12.134 (a) 308 K, 278 K; (b) 211 K, 178 K
12.135 (b) ~5.5×10^6 s
12.136 (a) 0 or ∞, 294 K; (b) 13.57 μm, 205 K, 310.4 K
12.137 (a) 57.9°C, 180 W; (b) 71.7°C; (c) –30.4°C
12.138 4.7°C, 16.2°C
12.140 (a) 43.1°C; (b) 29.2°C
12.141 25°C, 3.1×10^{-4} kg/s
12.142 (a) 0.0436 kg/m^3, 0.0144 kg/m^3, 6.76×10^{-6} kg/s, 21.2 W; (b) 503 W/m^2, 3105 W/m^2, 627
W/m^2

CHAPTER 13

13.1 (a) 1.0, 0.424; (b) 0.50, 0.25; (c) 0.637, 0.363; (d) 0.50, 0.707; (e) 0.5, 0; (f) 1.0, 0.125; (g) 0.50, 0.637
13.2 (c) ~0.62
13.4 (b) 0.0767, 0.553; 0.0343, 0.800
13.6 (a) 0.781; (b) 0.110
13.9 (a) 0.09
13.10 (b) 0.01
13.11 (a) 0.038; (b) 0.23
13.12 0.41
13.13 0.163
13.15 27.7 $\mu W/m^2$
13.16 (a) 354 mW/m^2; (b) 1695 mW/m^2
13.17 36,900 W
13.18 0.0492 kg/s·m
13.19 (a) 0.64; (b) 1700 W
13.20 1.69×10^5 W/m
13.21 456 K
13.22 (a) 13.4 W, 6825 W/m^2
13.23 (a) 579.4 K; (b) 583 K
13.24 (a) 413 K; (b) 1312 W
13.25 544 K, 828 K
13.26 (a) 255 W; (b) 970 K, 837.5 K

13.28 1.58×10^4 W/m^2
13.29 0.162 W
13.30 11.87 kW
13.31 (c) 17 kW/m^2; (d) 635 μW
13.32 63.8 W/m^2
13.33 (a) 1.19 W; (b) 1.48 μW
13.34 (a) 69 mW; (b) 934.5 W/m^2; (c) 1085 W/m^2; (d) 1085 W/m^2
13.35 (a) 507 K
13.36 (a) 308 K
13.37 774 K
13.38 (a) 48.8 K/s; (b) 15 s; (c) 14.6 s, 12.0 s, 6.8 s
13.39 1680 W/m, 2517 W/m, 916 K
13.40 (a) 109.7 W/m, 110.1 W/m; (b) 98.5 W/m, 103.8 W/m
13.41 (a) 14.2 kW/m^2, (b) 56.7 kW/m^2; (c) 14.2 kW/m^2; (d) 42.5 kW/m^2
13.42 (a) 1.58 W; (b) 0.986
13.43 46.2 kW/m^2, 0.814
13.44 (a) 0.963
13.46 (a) 30.13 W; (b) 30.65 W; (c) 31.41 W; (d) 55.8 W/kg, 79.2 W/kg, 75.1 W/kg
13.47 (a) 15.2 W/m
13.48 66.4°C, 71.4°C
13.49 30.2 W/m
13.50 69°C
13.51 (a) 45.6 kW/m^2; (b) 2.1 kW/m^2
13.52 1.14×10^{-4} kg/s
13.53 (a) 1995 W; (b) 191 W; (c) 983 W, 1.5%
13.54 548 K, 474 K
13.56 0.138
13.58 (a) 338 K; (b) 25.3 W/m^2
13.59 90 mW
13.60 9
13.61 0.50 W/m, -49%
13.62 472°C
13.63 1225 K, 1167 K
13.64 (a) 896 K, 986 K; (b) 950 K, 990 K
13.65 (a) 288 W; (b) –1692 W; (c) 207 W; (d) –1133 W
13.66 423 K
13.67 10.1 kW/m
13.68 3295 W
13.69 (a) 12.6 W; (b) 0.873; (c) 590 K
13.70 (a) 169 kW/m; (b) 1320 K
13.71 (a) 1.83
13.72 (a) 8520 W/m; (b) 732 K
13.73 12.6 kW/m^2
13.74 (a) 9870 W/m; (b) 853 K
13.75 (a) 25.3 kW; (b) 18.2 kW
13.76 (a) 1228 K; (b) 1117 K

13.77 (b) 43.8 kW, 764 K
13.78 266 W/m
13.79 1046 W
13.80 (a) 37 W/m; (b) 9.2%
13.81 (b) 1.32 W
13.82 (a) 1842 W; (b) 1840 W
13.83 –538 W, –603 W, 1141 W
13.85 611 K
13.86 –200 W, 5037 W, –4799 W, 0, 0
13.87 583 K
13.88 317 W
13.89 (a) 52.7 kW/m^2, 2.89 kW
13.90 (a) 41.3 kW; (b) 6.55 kW
13.91 (a) –1153 W; (b) 0.57 K/s; (c) 715 K
13.92 (b) 842.5 K
13.93 (a) 14.0°C, 8.6°C
13.94 (a) 54 kW/m; (b) 1346 K, 25.7 kW/m
13.95 (a) 373 K, 0.76; (b) 388.4 K, 1.84; (c) 361.6 K, 0.40
13.96 4600 W, 64 W/m^2·K
13.97 (a) 29.5 W/m, 483 K; (b) 441 W/m, 352 K
13.98 (a) 35 W/m^2·K; (b) 484 K
13.99 (a) 792.2 K, 792 K; (b) 783.2 K, 783 K
13.100 (a) 526 K
13.101 (a) 577 K; (b) 156 W/m; (c) 3120 W
13.103 (a) 289.4 W/m^2; (b) 376.2 W/m^2; (c) 328.7 W/m^2
13.104 (b) 366 K, 1920 W/m^2; 598 K, 10,850 W/m^2
13.105 5700 K/W
13.106 19.2 W
13.107 (a) 159 W/m^2, 7.3 W/m^2
13.108 (a) 145 W/m^2; (b) 306 W/m^2; (c) 165 W/m^2, 156 W/m^2
13.109 (a) 8.8°C, –7.4°C, 91.3 W
13.110 669 W/m^2, 199 W/m^2
13.111 (a) 131 W/m
13.112 59.5°C, 89 W/m
13.113 (a) 466 W, 1088 W
13.114 (a) 54.6 W; (b) 413 K
13.115 (a) 30 W/m^2
13.116 (a) 8075 W/m; (b) 796 K, 10 kW/m
13.117 (a) 19.89 W/m^2; (b) 100.6 W/m^2; (c) 122.1 W/m^2
13.118 0.023 kg/s
13.119 (a) 3.45 m; (b) 0.88 m
13.120 (b) 1295 K, 498 K; (c) 8 kW/m
13.121 (b) 1213 W/m; (c) 576 W/m; (d) 502 K
13.122 (a) 178 W/m^2·K, 543 K, 0.098 kg/s; (b) 0.140 kg/s
13.123 1040 K

13.124 (a) 0.0197; (b) 825 K, 108 kW
13.125 15.1 kW
13.126 21.9 kW/m
13.127 98 kW/m
13.128 135 kW/m^2
13.129 380 K, 89.6 kW/m^2
13.130 0.515 kg/s
13.131 0.004 kg/s, 4.5 m/s, 367 K
13.132 (a) 530 K; (b) 0.00613 m/s
13.133 (a) 0.00341 kg/s·m; (b) 889 K, 811 K

CHAPTER 14

14.1 0.233, 0.767
14.2 0.04 kmol/m^3, 1.78 kg/m^3, 0.5, 0.61; 0.04 kmol/m^3, 1.13 kg/m^3, 0.5, 0.39
14.3 0.0837, 0.1219 kg/m^3, 0.0406 kmol/m^3, 35.8 kg/kmol, 0.0146 kg
14.4 (b) 0.31, 0.27, 0.42; 0.35, 0.40, 0.25
14.6 0.36×10^{-4} m^2/s, 0.52×10^{-4} m^2/s
14.8 3.14×10^{-15} kmol/s
14.10 (a) 1.087×10^{-8} kmol/s; (b) 1.107×10^{-8} kmol/s
14.13 0.257 kg/m^2·h
14.17 (a) 14.7×10^{-12} kg/s; (b) -3.5×10^{-7} bar/s
14.18 1.5×10^{-8} kmol/m^2·s
14.19 (a) 0.0189, 0.0120, 1, 1; (b) 0.205, 0.226, 5.53×10^{-6}, 9.83×10^{-6}
14.20 (a) 1.31×10^{-9} kmol/m^2·s; (b) 0.0807 kmol/m^3, 0.0404 kmol/m^3
14.21 4.05×10^{-9} kg/s
14.22 1.9×10^{-22} kg/s·m
14.23 4×10^{-15} kg/s
14.24 1.39×10^{-6} kmol/s·m
14.25 (a) 1.91×10^{-3} kmol/m^3·bar; (b) 0.32×10^{-16} kmol/s; (c) 0.32×10^{-16} kmol/s; (d) 0
14.26 0.0034
14.27 0.022 kg/h
14.28 (a) 0.10; (b) 5.61×10^{-5} kg/s
14.29 6.66×10^{-9} kmol/s
14.31 0.0008, 7.48×10^{-8} kmol/s·m^2, 2.24×10^{-7} kmol/s·m^2
14.33 (b) 0.02 kmol/m^3; (d) 9.60×10^{-5} kmol/m^2·s
14.34 (c) 3×10^{-5} kmol/m^3
14.35 (c) 5.11×10^{-6} kmol/m^3, 2.18×10^{-15} kmol/s
14.37 (b) 5.38×10^{-11} kmol/s·m^2
14.38 (b) 9.08×10^{-7} kmol/m^3
14.39 333 s
14.40 0.02 s

14.41 8.42 h
14.42 (b) 0.0314
14.43 (a) 2071 days; (b) 198.3 kg/m^3; (c) 198.3 kg/m^3
14.44 (b) 1.85 kmol/m^3
14.46 (a) 32 kg/m^3; (c) 42.9 h
14.47 (a) 32 kg/m^3; (c) 42.9 h; (d) 218 h
14.48 3.3×10^{-3} kmol/m^3·bar, 8.5×10^{-13} m^2/s
14.49 3940 s
14.51 (a) 0.0778 kmol/m^3, 1.24×10^{-4}; (b) 0.0117 kmol/m^3, 1.9×10^{-5}; 0.0346 kmol/m^3, 5.5×10^{-5}
14.53 (a) 2.5 µm; (b) 1.7 h; (c) 3.9 h