GEAR TEETH IN ACTION

Figure 2.15 shows a gear mesh with the driving pinion tooth on the left just coming into mesh at point T and the two teeth on the right meshing at point S.

Notice that contact starts at point T where the outside diameter of the gear crosses the line of action and ends where the outside diameter of the pinion crosses the line of action, point R.

Z is the length of the line of action. In other words, a tooth will be in contact from point T to point R. P_B is the base pitch, the distance from one involute to the next along a radius of curvature. It was shown earlier that

$$P_B = \pi \frac{BD}{N}$$

where

BD = base diameter, in.
N = number of teeth

Point T, where contact initiates, is called the lowest point of contact on the pinion tooth and also the highest point of contact on the gear tooth. Similarly, point R is the highest point of contact on the pinion tooth and the lowest point of contact on the gear tooth. Point S is the highest point of single tooth contact on the pinion and the lowest point of single tooth contact on the gear. In other words, if one imagines the gears in Figure 2.15 to begin rotating, just prior to

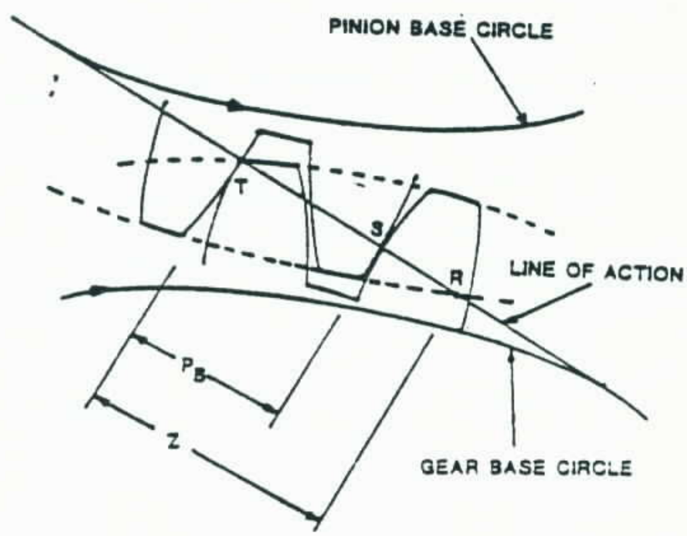

Figure 2.15 Gear tooth action.

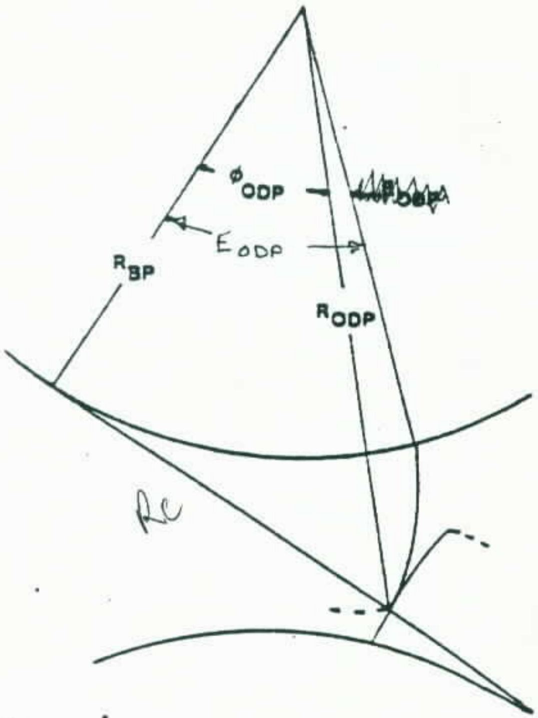

Figure 2.16 Degrees of roll to pinion outside diameter.

$$E_{ODP} = \tan \phi_{ODP}\left(\frac{180}{\pi}\right) = \frac{\sqrt{R_{ODP}^2 - R_{BP}^2}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

Figure 2.17 shows how the degrees of roll to the pinion form diameter is calculated:

$$E_{TIFP} = \tan \phi_{TIFP}\left(\frac{180}{\pi}\right) = \frac{\sqrt{R_{TIFP}^2 - R_{BP}^2}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

It is more convenient to express E_{TIFP} in terms of the gear outside radius and base radius:

$$XX = C \sin \phi_{PD}$$

$$\sqrt{R_{TIFP}^2 - R_{BP}^2} = C \sin \phi_{PD} - \sqrt{R_{ODG}^2 - R_{BG}^2}$$

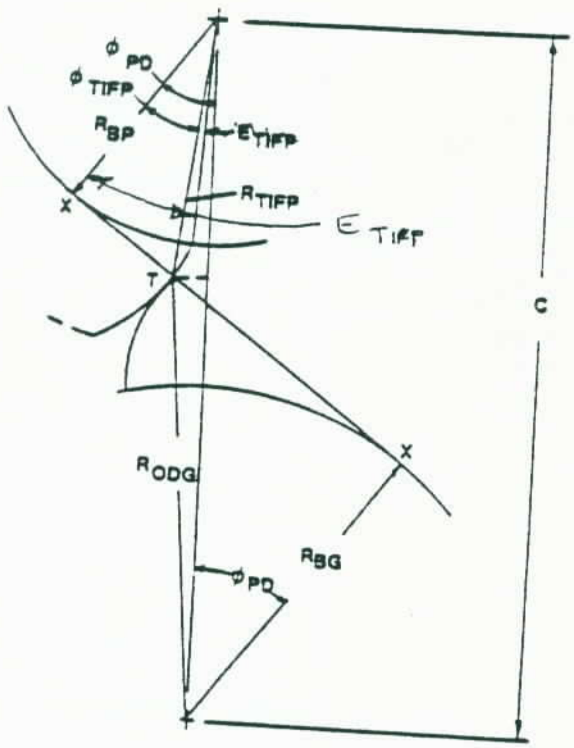

Figure 2.17 Degrees of roll to pinion true involute form diameter.

Therefore,

$$E_{TIFP} = \frac{C \sin \phi_{PD} - \sqrt{R_{ODG}^2 - R_{BG}^2}}{R_{BP}} \left(\frac{180}{\pi}\right)$$

and

$$M_P = \frac{\sqrt{R_{ODP}^2 - R_{BP}^2} - C \sin \phi_{PD} + \sqrt{R_{ODG}^2 - R_{BG}^2}}{R_{BP}} \quad \frac{180N}{\pi \cdot 360}$$

where

N = number of pinion teeth

R_{BP} = $\frac{1}{2} PD_P \cos \phi_{PD}$

$\frac{N}{PD_P \cdot \pi} = \frac{1}{CP}$

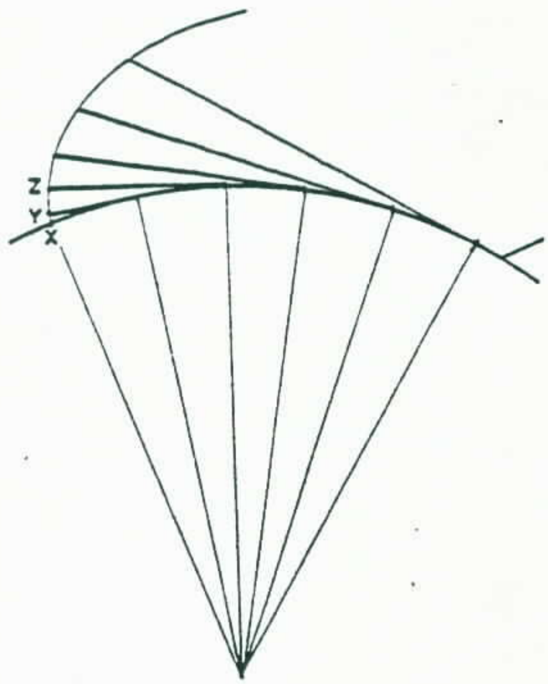

Figure 2.19 Involute curve properties.

involute is so sensitive near the base circle, the lowest point of contact on a gear tooth should be located well away from the base circle. As a rule of thumb the lowest point of contact on a gear tooth should be at least 9° of roll.

ROLLING AND SLIDING VELOCITIES

When involute gear teeth mesh, the action is not pure rolling as it would be when two friction disks are in contact, but a combination of rolling and sliding. Figure 2.20 shows a gear mesh with two base circles of equal size and the teeth meshing at the pitch point. Radii of curvature are drawn to the involutes from equal angular intervals on the base circle. It can be seen that arc XY on gear 2 will mesh with arc AB on gear 1 and that AB is longer than XY; therefore, the two profiles must slide past one another to make up the difference in length. The sliding velocity, which is usually expressed in feet per minute, at any point is calculated as follows:

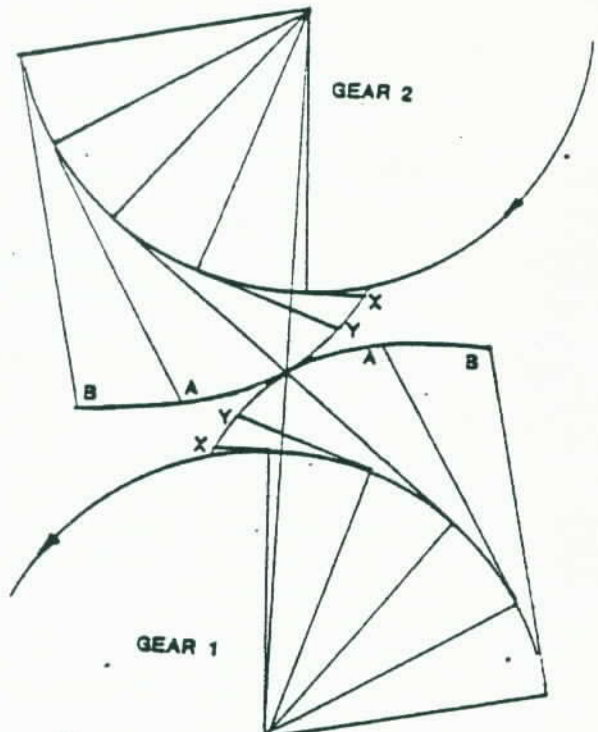

Figure 2.20 Relative sliding of gear teeth.

$$V_S = \frac{W_1 R_{C1} - W_2 R_{C2}}{12}$$

where

V_S = sliding velocity, fpm
W_1 = angular velocity of gear 1, rad/min
W_2 = angular velocity of gear 2, rad/min
R_{C1} = radius of curvature of gear 1, in.
R_{C2} = radius of curvature of gear 2, in.

From Figure 2.20 it can be seen that when point A on gear 1 and point Y on gear 2 mesh, R_{C1} will be larger than R_{C2} and since $W_1 = W_2$, V_S will be a positive number. As the meshing point nears the pitch point the difference in the radii of curvature lessens until at the pitch point the radii of curvature are equal and V_S is 0. When point A on gear 2 meshes with point Y on gear 1, R_{C1} will be smaller than R_{C2} and V_S will be negative. The significance of this is that as

INVOLUTE TRIGONOMETRY

THE INVOLUTE OF A CIRCLE IS THE CURVE THAT IS DESCRIBED BY THE END OF A LINE WHICH IS UNWOUND FROM THE CIRCUMFERENCE OF A CIRCLE AS SHOWN IN FIGURE 1.

WHEN, R_b = BASE RADIUS
θ = VECTORIAL ANGLE
r = LENGTH OF RADIUS VECTOR

THEN,

$$\theta = \frac{\sqrt{r^2 - R_b^2}}{R_b} - \text{ARC TAN } \frac{\sqrt{r^2 - R_b^2}}{R_b} \qquad (1)$$

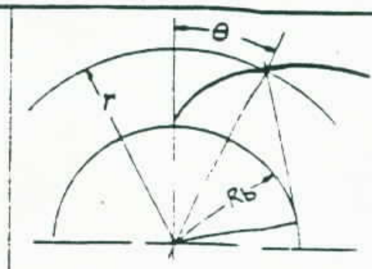

FIGURE 1

REFERRING TO FIGURE 2.

WHEN, R = PITCH RADIUS
ϕ = PRESSURE ANGLE AT R
θ_1 = VECTORIAL ANGLE AT R

THEN,

$$\theta_1 = \text{TAN } \phi - \text{ARC } \phi = \text{INV } \phi \qquad (2)$$

$$R_b = R \cos \phi \qquad (3)$$

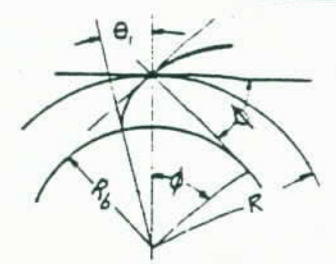

FIGURE 2

GIVEN THE ARC TOOTH THICKNESS AND PRESSURE ANGLE OF AN INVOLUTE GEAR AT A GIVEN RADIUS TO DETERMINE ITS TOOTH THICKNESS AT ANY OTHER RADIUS. SEE FIGURE 3.

WHEN,
r_1 = GIVEN RADIUS
ϕ_1 = PRESSURE ANGLE AT r_1
T_1 = ARC TOOTH THICKNESS AT r_1
r_2 = RADIUS WHERE TOOTH THICKNESS IS TO BE DETERMINED
ϕ_2 = PRESSURE ANGLE AT r_2
T_2 = ARC TOOTH THICKNESS AT r_2

THEN,

$$\cos \phi_2 = \frac{r_1 \cos \phi_1}{r_2} \qquad (4)$$

$$T_2 = 2 r_2 \left[\frac{T_1}{2 r_1} + \text{INV } \phi_1 - \text{INV } \phi_2 \right] \qquad (5)$$

EXAMPLE, $r_1 = 2.500''$ $\quad T_1 = .2618''$ $\quad r_2 = 2.600''$
$\phi_1 = 14.500°$ $\quad \cos \phi_1 = .96815$ $\quad \text{INV } \phi_1 = .00554$

$$\cos \phi_2 = \frac{2.500 \times .96815}{2.600} = .93091 \qquad \phi_2 = 21.425° \qquad \text{INV } \phi_2 = .01845$$

$$T_2 = 2 \times 2.600 \left[\frac{.2618}{5.000} + .00554 - .01845 \right] = .2051$$

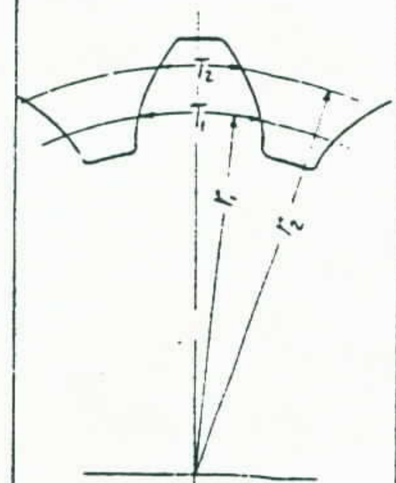

FIGURE 3

GIVEN THE ARC TOOTH THICKNESS AT A GIVEN RADIUS TO FIND THE CHORDAL TOOTH THICKNESS. SEE FIGURE 4.

WHEN, T = ARC TOOTH THICKNESS AT r
T_c = CHORDAL TOOTH THICKNESS AT r

THEN,

$$\text{ARC } \beta = \frac{T}{2r} \text{ (RADIANS)} \qquad (6)$$

$$T_c = 2r \sin \beta \qquad (7)$$

EXAMPLE, $r = 2.500''$ $\quad T = .2618''$

$$\text{ARC } \beta = \frac{.2618}{5.000} = .05236 \text{ RADIANS} = 3 \text{ DEGREES}$$

$$\sin \beta = .05234$$

$$T_c = 2 \times 2.500 \times .05234 = .2617$$

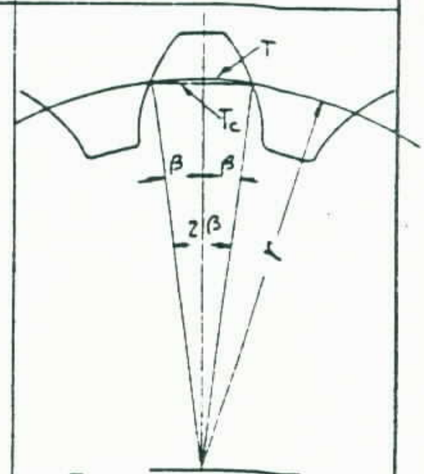

FIGURE 4