EML 3005: GEAR DESIGN EXAMPLE

SPUR GEAR DESIGN FOR AN AIRCRAFT ENGINE SPEED REDUCER

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Input Design Variables for Gear Mesh # 1

$Gratio1 := 1.285$ Gear Ratio $Dpitch1 := 3.5$ (pitch dia, in)

$\phi := 20 \cdot \frac{\pi}{180}$ Pressure Angle, 20 deg

$Fwidth1 := 1.5$ Facewidth, Inch

Input Duty Cycle for gear mesh (rpm, hp, torque, hours, etc)

$N := 9$ Number of data points in duty cycle

$i := 1 .. N$

$HP_i :=$ $RPM_i :=$ $Hours_i :=$

HP		RPM	Hours
20		1000	250
250		4000	25
278		4500	9
265		4200	250
195	**Base**	3300	1447
200		3600	510
100		2000	250
30		1000	9
20		1000	250

$$\sum_i Hours_i = 3000$$

$$Torque_i := \frac{HP_i \cdot 33000}{2 \cdot \pi \cdot RPM_i} \cdot 12 \quad \text{Inch-lbf}$$

$\frac{Torque_i}{12} =$ (lb-ft) $Torque_i =$ (in-lbf)

$\frac{Torque_i}{12}$	$Torque_i$
105.042	1260.507
328.257	3939.085
324.464	3893.567
331.383	3976.6
310.352	3724.226
291.784	3501.409
262.606	3151.268
157.563	1890.761
105.042	1260.507

Calculate Tangential Tooth Load

$$Wt1_i := \frac{Torque_i \cdot 2}{Dpitch1} \quad \text{Lbf}$$

Print HP, rpm, hours, torque and Wt

$HP_i =$	$RPM_i =$	$Hours_i =$	Torque$_i =$ (in-lbf)	$Wt1_i =$ (Lbf)
20	1000	250	1260.507	720.29
250	4000	25	3939.085	2250.906
278	4500	9	3893.567	2224.895
265	4200	250	3976.6	2272.343
195	3300	1447	3724.226	2128.129
200	3600	510	3501.409	2000.805
100	2000	250	3151.268	1800.724
30	1000	9	1890.761	1080.435
20	1000	250	1260.507	720.29

$$\sum_i Hours_i = 3000$$ (Total life required in hours, for gear train)

Calculate equivalent torque and horsepower, for bending and contact loads, for input pinion

$j := 5$ (Baseline is for cruise condition)

$a1 := 9$ (Log S-N curve exponent for contact loads is 9)

$a2 := 29$ (Log S-N curve exponent for bending loads is 29)

$$Teq_1 := \left[\frac{\sum_i \left[Hours_i \cdot RPM_i \cdot \left(Torque_i\right)^{a1}\right]}{RPM_j \cdot \left(\sum_i Hours_i - Hours_j\right)}\right]^{\frac{1}{a1}}$$

$Teq_1 = 3916.421$ in-lbf

$$HPeq_1 := \frac{2 \cdot \pi \cdot RPM_j \cdot \frac{Teq_1}{12}}{33000}$$

$HPeq_1 = 205.063$

Equivalent torque for CONTACT load

$Teq_1 = 3916.421$ in-lbf at $RPM_j = 3300$

$HPeq_1 = 205.063$ Total number of hours = $\sum_i Hours_i = 3000$

Equivelent loads for bending stress

$$Teq_2 := \left[\frac{\sum_i \left[Hours_i \cdot RPM_i \cdot \left(Torque_i\right)^{a2}\right]}{RPM_j \cdot \left(\sum_i Hours_i - Hours_j\right)}\right]^{\frac{1}{a2}}$$

$Teq_2 = 3843.36$ in-lbf

$$HPeq_2 := \frac{2 \cdot \pi \cdot RPM_j \cdot \frac{Teq_2}{12}}{33000}$$

$HPeq_2 = 201.238$

Equivalent torque for BENDING load

$Teq_2 = 3843.36$ in-lbf at $RPM_j = 3300$

$HPeq_2 = 201.238$ Total number of hours = $\sum_i Hours_i = 3000$

Calculate bending and contact tangential tooth loads for input pinion 1

$$Wtbend1 := \frac{Teq_2 \cdot 2}{Dpitch1}$$

$Wtbend1 = 2196.206$ Lbf

$$Wtcomp1 := \frac{Teq_1 \cdot 2}{Dpitch1}$$

$Wtcomp1 = 2237.955$ Lbf

Equivalent number of stress cycles for Input pinion 1

$$Neq := \left(\sum_i Hours_i\right) \cdot RPM_j \cdot 60$$

$Neq = 5.94 \times 10^8$

Compute ALLOWABLE bending and contact fatigue stresses in gear teeth

Bending fatigue strength (psi):
for 9310 gear material

$St := 70000$

Contact fatigue strength (psi):
for 9310 gear material

$Sc := 225000$

Life factors

$KL := 1.6831 \cdot Neq^{-0.0323}$ $\quad KL = 0.876$

$CL := 2.466 \cdot Neq^{-0.056}$ $\quad CL = 0.796$

Allowable Bending and Contact Stresses

$\sigma all := St \cdot KL$ $\quad \sigma all = 61350.013$ psi

$\sigma call := Sc \cdot CL$ $\quad \sigma call = 178996.092$ psi

Calculate ACTUAL bending and contact stresses in pinion teeth

Elastic constant (psi): $Cp := 2300$

Mounting factors:
Precise mounting

$Cm := 1.3$

$Km := 1.3$

Quality factor, around 10 for heat treated and ground gears $\quad Qv := 10$

Hardness factors $\quad CR := 1$

$KR := 1$

Calculate velocity factor

Pitch line velocity in Ft/min:

$$V := \frac{\pi \cdot Dpitch1 \cdot RPM_j}{12} \qquad V = 3023.783$$

Compute constants A & B

$$B := \frac{(12 - Qv)^{\frac{2}{3}}}{4} \qquad A := 50 + 56 \cdot (1 - B)$$

Velocity Factor

$$Kv := \left(\frac{A}{A + \sqrt{V}}\right)^{B} \qquad Kv = 0.819$$

CALCULATE OPTIMUM NUMBER OF TEETH, Np

Geometric Factors

$$I1 := \frac{\cos(\phi)\cdot\sin(\phi)}{2}\cdot\frac{Gratio1}{Gratio1 + 1} \qquad I1 = 0.09$$

$J1 := 0.43$ (Guess)

$$Np := \frac{\sigma all\cdot J1\cdot Cp^2}{\sigma call^2\cdot I1} \qquad Np = 48.198$$

Choose Np at 48 teeth: $Np := 48$

$$Pd1 := \frac{Np}{Dpitch1} \qquad Pd1 = 13.714$$

Note: The nearest standard Pd is 12, which will give 42 teeth. Lets stay with 48 teeth.

Calculate Pinion Facewidth

$\sigma all = 61350.013$

$$Fbend := \frac{Wtbend1\cdot Pd1\cdot Km}{\sigma all\cdot Kv\cdot J1} \qquad Fbend = 1.813$$

$$Fcont := \frac{Wtcomp1\cdot Cm\cdot Cp^2}{Kv\cdot I1\cdot Dpitch1\cdot \sigma call^2} \qquad Fcont = 1.855$$

Choose facewidth as 1.9 inches: $Fwidth := 1.9$

Tooth bending and contact stresses are (check):

$$\sigma bend := \frac{Wtbend1\cdot Pd1\cdot Km}{Fwidth\cdot Kv\cdot J1} \qquad \sigma bend = 58551.897 \text{ psi}$$

$$\sigma cont := Cp\cdot\sqrt{\frac{Wtcomp1\cdot Cm}{Kv\cdot I1\cdot Dpitch1\cdot Fwidth}} \qquad \sigma cont = 176884.12 \text{ psi}$$

Allowable stresses are: $\sigma all = 61350.013$ $\sigma call = 178996.092$

Numbers look O.K

\# gear teeth $Ng := Np\cdot Gratio1$ $Ng = 61.68$

Choose Ng, the number of teeth on gear 2, as 62

$Ng := 62$

Actual gear ratio is: $Mg := \frac{Ng}{Np}$ $Mg = 1.292$

GEAR DESIGN SUMMARY FOR MESH 1

$Pd1 = 13.714$ teeth/in

$Np = 48$

$St = 70000$

$Ng = 62$

$Sc = 225000$

$Mg = 1.292$

$Fwidth = 1.9$

$\sigma bend = 58551.897$

$\sigma all = 61350.013$

$\sigma cont = 176884.12$

$\sigma call = 178996.092$

Factor of safety:

$$FOSbend := \frac{\sigma all}{\sigma bend} \qquad FOSbend = 1.048$$

$$FOScont := \frac{\sigma call}{\sigma cont} \qquad FOScont = 1.012$$

Pinion life with 1.9 inch facewidth

$$KLnew := \frac{\sigma bend}{St} \qquad KLnew = 0.836$$

$$CLnew := \frac{\sigma cont}{Sc} \qquad CLnew = 0.786$$

$$exp1 := \frac{\log(1.6831) - \log(KLnew)}{0.0323} \qquad exp2 := \frac{\log(2.466) - \log(CLnew)}{0.056}$$

BENDING LIFE

$$Nbend := 10^{exp1} \qquad Nbend = 2.52 \times 10^{9}$$ (bending life, cycles)

$$Bendlife := \frac{Nbend}{60 \cdot RPM_j} \qquad Bendlife = 12728.866$$ (bending life, hours)

CONTACT LIFE

$$Ncont := 10^{exp2} \qquad Ncont = 7.342 \times 10^{8}$$ (contact life, cycles)

$$Contlife := \frac{Ncont}{60 \cdot RPM_j} \qquad Contlife = 3708.255$$ (contact life, hours)

LIFE OF GEARMESH #1 = 3708 HOURS