PROBLEM 6-20

Statement: **A ±100 N-m torque is applied to a 1-m-long, solid round steel shaft. Design it to limit its angular deflection to 2 deg and select a steel alloy to have a fatigue safety factor of 2 for infinite life.**

Units: $N := newton$ $\quad kN := 10^3 \cdot N$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given:

Applied torque	$T_a := 100 \cdot N \cdot m$	$T_m := 0 \cdot N \cdot m$
Shaft length	$L := 1000 \cdot mm$	
Max deflection	$\theta_{max} := 2 \cdot deg$	
Design safety factor	$N_{fd} := 2$	
Modulus of rigidity	$G := 80.8 \cdot GPa$	

Assumptions: There are no stress-concentrations anywhere on the shaft. The shaft is machined, reliability is 99.9%, and the it is at room temperature.

Solution: See Mathcad file P0620.

1. This is a case of fully reversed torsion. We will use the von Mises effective stress so the load factor will be 1.

The maximum torque is $T_{max} := T_a + T_m$ $\qquad T_{max} = 100 \, N \cdot m$

2. The diameter of the shaft can be found from equations 4.24 and 4.25 with $\theta = \theta_{max}$.

$$\theta_{max} = \frac{T_{max} \cdot L}{J \cdot G} = \frac{32 \cdot T_{max} \cdot L}{\pi \cdot d^4 \cdot G}$$

Solving for d, $d := \left(\dfrac{32 \cdot T_{max} \cdot L}{\pi \cdot \theta_{max} \cdot G} \right)^{\frac{1}{4}}$ $\qquad d = 24.514 \, mm$

Rounding, let $d := 24.5 \cdot mm$

3. Now, we can solve for the stress in the shaft.

Polar moment of inertia $J := \dfrac{\pi}{32} \cdot d^4$ $\qquad J = 3.537 \times 10^4 \, mm^4$

Torsional stress $\tau_a := \dfrac{T_a \cdot d}{2 \cdot J}$ $\qquad \tau_a = 34.632 \, MPa$

The corresponding von Mises normal stress is

von Mises stress $\sigma'_a := \sqrt{3} \cdot \tau_a$ $\qquad \sigma'_a = 59.984 \, MPa$

4. Using the factor of safety equation for reversed loading, calculate the required endurance limit

$N_f = \dfrac{S_e}{\sigma'_a}$ $\qquad S_e := N_{fd} \cdot \sigma'_a$ $\qquad S_e = 119.967 \, MPa$

5. This endurance limit is a function of the unknown ultimate tensile strength. Use the endurance limit modification equation to determine the required S_{ut}.

$$S_e = C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e$$

6. Calculate the endurance limit modification factors for a solid, round steel shaft.

Load	$C_{load} := 1$	
Size	$C_{size} := 1.189 \cdot \left(\frac{d}{mm}\right)^{-0.097}$	$C_{size} = 0.872$
Surface	$A := 4.51$ $\quad b := -0.265$	(machined)
	$C_{surf} = A \cdot \left(\frac{S_{ut}}{MPa}\right)^{b}$	
Temperature	$C_{temp} := 1$	
Reliability	$C_{reliab} := 0.753$ $\quad (R = 99.9\%)$	
Uncorrected endurance strength	$S'_e = 0.5 \cdot S_{ut}$	

7. Substituting these into the equation above and solving for S_{ut},

$$S_{ut} := \left(\frac{S_e}{0.5 \cdot A \cdot C_{size} \cdot C_{reliab} \cdot MPa}\right)^{\frac{1}{b+1}} \cdot MPa \qquad S_{ut} = 395\,MPa$$

Based on this requirement, choose AISI 1020 cold-rolled steel that will be machined to size.

8. Check the actual factor of safety based on the material chosen. For this material, $S_{ut} := 469 \cdot MPa$

Surface factor	$C_{surf} := A \cdot \left(\frac{S_{ut}}{MPa}\right)^{b}$	$C_{surf} = 0.884$
Uncorrected endurance strength	$S'_e := 0.5 \cdot S_{ut}$	$S'_e = 234.5\,MPa$
Corrected endurance strength	$S_e := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e$	
	$S_e = 136.046\,MPa$	
Factor of safety	$N_f := \frac{S_e}{\sigma'_a}$	$N_f = 2.3$