EML 3005: MECHANICAL DESIGN

OPTIMAL DESIGN OF SPUR AND HELICAL GEARS

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Summary

A simple closed-form procedure is presented for designing minimum-weight spur and helical gear sets. The procedure optimizes bending and surface fatigue strengths of gear teeth, to arrive at minimum weight gears. The method does not take into account effects of scuffing, heat generation, and lubrication conditions like oil temperature, elasto-hydrodynamic film thickness and flash temperature, since they are outside the scope of the class.

Introduction

Gear design is a process of synthesis where gear geometry, materials, heat treatment, manufacturing methods and lubrication are selected to meet the performance requirements of a given application. The designer must design the gearset with adequate resistance to contact fatigue (pitting resistance) and bending fatigue to transmit the required power for design life. With the simple procedure presented here, one can select materials and heat treatment and optimize gear geometry to satisfy constraints of weight, size and configuration. It is assumed that the gear ratio m_G is known. The gear designer can minimize noise level and operating temperature by minimizing the pitchline velocity and sliding velocity. This is done by specifying high gear accuracy and selecting material strengths consistent with maximum material hardness, to obtain minimum size gearsets with teeth no larger than necessary to balance pitting resistance and bending strength.

Gear design is not the same as gear analysis. Existing gear sets can only be analyzed, not designed. While design is more challenging than analysis, current textbooks do not provide procedures for designing minimum weight gears. They usually recommend that the number of teeth in the pinion be chosen based solely on avoiding undercut. This does not result in minimum weight gearsets.

Optimum Number of Pinion Teeth

The optimum number of teeth maximizes the load capacity of a gearset. Figure 1 shows that load capacity is limited by surface fatigue (contact stress), bending fatigue and scuffing failure depending on the number of teeth. There is also a lower limit to the number of teeth, below which undercut occurs. The shaded zone in Fig. 1 is bounded by all three-failure modes curves and the undercut limit. We will only consider bending and contact fatigue for optimal design. Notice that the surface fatigue curve is not a function of number of teeth, while in contrast the bending fatigue curve depends strongly on the number of pinion teeth and decreases rapidly with increasing number of teeth.. Maximum load capacity occurs at point “A” where pitting resistance and bending strength are balanced. With more pinion teeth (to the right of point A) load capacity is controlled by bending fatigue, while for fewer teeth (to the left of point A) load capacity is controlled by surface fatigue.

Failure modes due to surface fatigue and bending fatigue are quite different. Surface fatigue usually progresses relatively slowly, starting with a few pits which may increase in number and coalesce into larger spalls. As the tooth profiles deteriorate with pitting, the gears generate noise, which warns of surface fatigue failure. In contrast, bending fatigue may progress rapidly as a fatigue crack propagates across the base of a tooth, breaking the tooth with little or no warning. Hence, surface fatigue is often less serious than bending fatigue, which is frequently catastrophic.

Considering the difference between surface and bending fatigue failures, it is prudent to select the number of pinion teeth somewhat to the left of point A, where surface fatigue controls the failure process. With this approach, load capacity is not reduced because surface fatigue curve is flat, while a margin of safety against bending

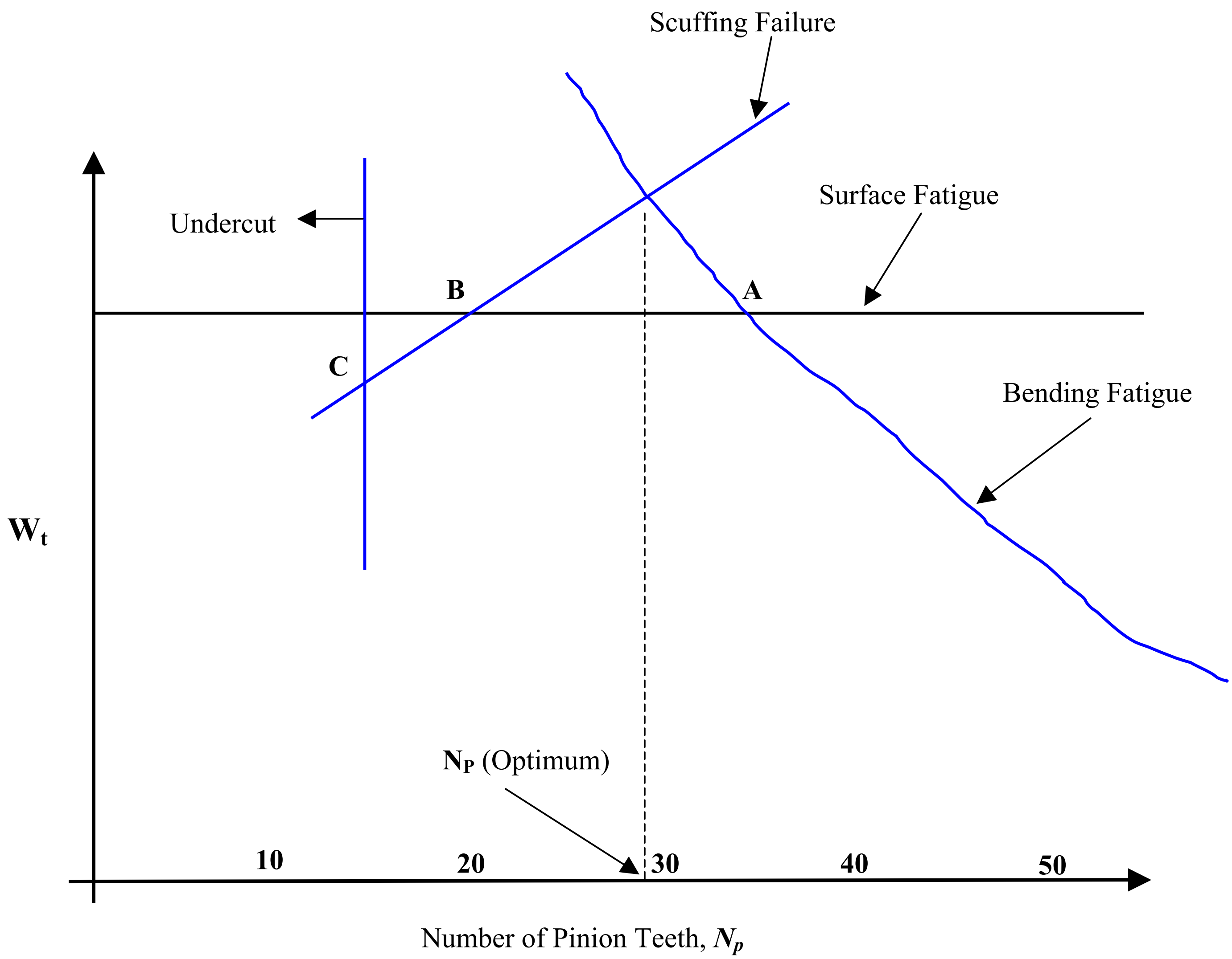

Figure 1. Load Capacity (***Wt***) Vs. Pinion Tooth Number (***N$_P$***)

fatigue is gained. To avoid scuffing failure, pinion teeth fewer than point B should not be chosen. A pinion number of teeth near optimum (N_P) provides a good balance between pitting and bending fatigue resistance, while scuffing resistance is also obtained because the teeth are not larger than necessary.

OPTIMUM GEAR DESIGN PROCEDURE

For the gearbox design project, the input gear diameter will likely be chosen based on the physical dimensions of the gearbox, center distance allowable between countershaft and input shaft, etc. For this problem we will assume that we *know* the input pinion diameter, d_P, which is typically in the range of 5-6 inches. We then need to determine the facewidth and the number of pinion teeth, N_P.

1. Calculating Pinion Facewidth *F* (*knowing* d_P)

The AGMA bending and contact stress equations are given by:

$$\sigma_b = \frac{W_t p_D}{FJ} \frac{K_a K_m}{K_v} K_s K_B K_I \qquad (11.15us)$$

$$\sigma_c = C_P \sqrt{\frac{W_t}{FId_P} \frac{C_a C_m}{C_v} C_s C_f} \qquad (11.21)$$

A convenient equation to compute the tangential tooth load W_t in lbf is given by:

$$W_t = \frac{126000(HP)}{RPM_P(d_P)} \quad (lbf) \qquad (1)$$

where d_P must be in inches. RPM_P is the pinion rpm.

The *allowable* bending stress *is given by* AGMA bending-fatigue strength formula:

$$S_{fb} = \frac{K_L}{K_T K_R} S_{fb}' \qquad (11.24)$$

The *allowable* contact stress *is given by* AGMA surface-fatigue strength formula:

$$S_{fc} = \frac{C_L C_H}{C_T C_R} S_{fc}' \qquad (11.25)$$

The life factors, K_L, and C_L, will be calculated based on the cumulative fatigue damage and equivalent number of fatigue cycles.

Since the limiting values for bending and contact stress are given by S_{fb} and S_{fc}, we can find the *minimum* facewidths F_b and F_c from equations (11.15) and (11.21) as follows:

$$F_b = \frac{W_t p_D}{S_{fb} J} \frac{K_a K_m}{K_v} K_s K_B K_l \tag{2}$$

$$F_c = C_P^2 \frac{W_t}{S_{fc}^2 I d_P} \frac{C_a C_m}{C_v} C_s C_f \tag{3}$$

The design facewidth would be the larger of the two values F_b and F_c.

2. Calculating Optimum number of Pinion Teeth, N_P

The facewidths, F_b and F_c , calculated using Eqs. (2) and (3) will generally not be equal because we have made no effort to equate bending and contact fatigue strengths in the design. Therefore the design will be biased toward failure from either bending or contact fatigue. To effectively utilize the material we will now find the optimum number of teeth based on equating bending and contact strengths.

The maximum transmitted tooth load, W_t, for allowable bending stress is obtained from Eq. (11.15), as follows (note, $p_D = d_P/N_P$):

$$W_t\,(bending) = \frac{S_{fb}\, d_P F\, J}{N_P} \frac{K_v}{K_a K_m K_s K_l} \tag{4}$$

Similarly, the maximum transmitted tooth load, Wt, for allowable contact stress is obtained from Eq. (11.21), as follows:

$$W_t\,(Contact) = \frac{S_{fc}^2\, d_P F\, I}{C_P^2} \frac{C_v}{C_a C_m C_s C_f} \tag{5}$$

Equating equations 4 and 5, so that W_t due to bending and contact stresses are equal, we obtain the optimum number of pinion teeth, N_P :

$$N_P\,(Optimum) = \frac{S_{fb}\, J\, C_P^2}{S_{fc}^2\, I} \tag{6}$$

The above result must be rounded off to the nearest integer number. Note that the optimum number of teeth is a function of material and geometric factors only, and is independent of load and diameter.

Life Factors K_L and C_L:

The life factors, K_L and C_L, are a function of the number of fatigue cycles, N. The number of fatigue cycles, N, is calculated based on cumulative fatigue damage cycles, explained in the next section.

From Figure 11.24, we can see that $\boldsymbol{K_L = 1.6831\ N^{-0.0323}}$
From Figure 11.26, we can see that $\boldsymbol{C_L = 2.466\ N^{-0.056}}$
Note that $K_L = 1$ and $C_L = 1$ when $N = 10^7$ cycles.

Equivalent Load And Life Calculation

(Fatigue Life for Variable Loading)

Instead of a single reversed stress cycle of σ for n cycles, a machine element could be subjected to σ_1 for n_1 cycles, σ_2 for n_2 cycles, etc. If we need to estimate the fatigue life of the machine element subjected to these reversed stresses, we need to calculate a single load cycle equivalent to the combined stress cycle. The theory that is in greatest use at the present time to explain cumulative damage is called the Miner's rule. It can be stated as

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \dots\dots\dots + \frac{n_i}{N_i} = 1 \qquad (16)$$

where n is the number of cycles of stress σ applied to the specimen and N is the fatigue life corresponding to σ. This theory is in wide use even though it has some shortcomings.

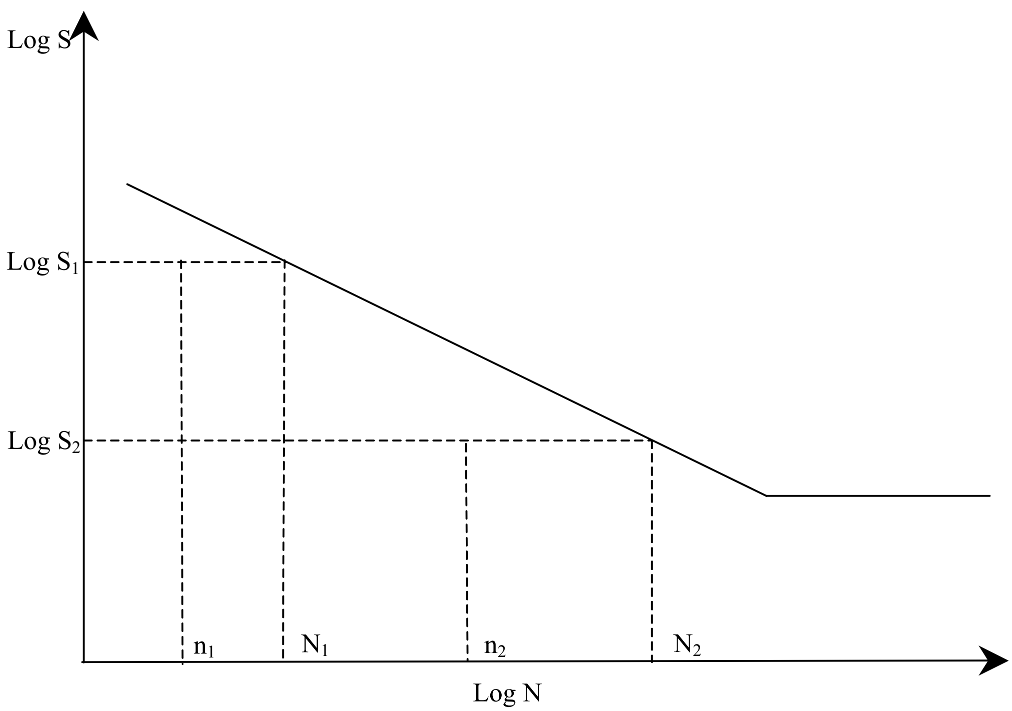

Fig.2 Use of Miner's rule to predict cumulative fatigue damage

We are now in a position to calculate the equivalent life L_E (hours) at a given base condition, which is equal to the entire duty cycle.

L_E = Equivalent life in **hours** at the base condition which is equal to the entire load or duty cycle.

L_B = Time spent at the base condition, in **hours**.

T = Torque transmitted at a given condition.

N = Life, in **cycles**, at a given condition.

a = Slope of the Log S - Log N curve.

Subscript 1, 2, 3,…..i identify multiple duty cycles. Using Miner's rule we can write

$$\frac{n_B}{N_E}+\frac{n_1}{N_1}+\frac{n_2}{N_2}+..........+\frac{n_i}{N_i}=1 \tag{17}$$

$$N_1=N_E\left(\frac{T_B}{T_1}\right) \qquad N_2=N_E\left(\frac{T_B}{T_2}\right), \text{ etc.}$$

where
n_B = actual number of cycles at base condition B = L_B * RPM_B *60
N_E = Life, in cycles, at the base condition = L_E * RPM_B *60

Fatigue also obeys the relation

$$\frac{N_1}{N_2}=\left(\frac{T_2}{T_1}\right)^a \tag{18}$$

From Eqs. (17-18) we can write

$$\frac{L_B(RPM_B)(60)}{L_E(RPM_B)(60)}+\frac{L_1(RPM_1)(60)}{\left(\frac{T_B}{T_1}\right)^a L_E(RPM_B)(60)}+...+\frac{L_i(RPM_i)(60)}{\left(\frac{T_B}{T_i}\right)^a L_E(RPM_B)(60)}=1 \tag{19}$$

Multiplying throughout by L_E, we have the expression for the "**equivalent life**" L_E as

$$L_E=L_B+L_1\left(\frac{T_1}{T_B}\right)^a\frac{(RPM_1)}{(RPM_B)}+L_2\left(\frac{T_2}{T_B}\right)^a\frac{(RPM_2)}{(RPM_B)}+...+L_i\left(\frac{T_i}{T_B}\right)^a\frac{(RPM_i)}{(RPM_B)} \tag{20}$$

The S-N curve exponent "a" is

a = 9.0 (for compressive loading or surface fatigue)
= 29.0 (for bending fatigue)

We can also calculate "**Equivalent horsepower**" at RPM_B, for L_E hours of life, from Eqs.(20).

$$T_B = \left\{ \frac{\left[L_1 * RPM_1 (T_1)^a + L_2 * RPM_2 (T_2)^a + ... + L_i * RPM_i (T_i)^a \right]}{RPM_B (L_E - L_B)} \right\}^{\frac{1}{a}}$$

Once torque (in-lbf) is calculated, equivalent H.P can be easily calculated.

Calculate equivalent hours L_E for the base condition 2947 HP at 19545 rpm, for the four load cycles shown in table. L_E is required to compressive or contact stress life calculations.

CONDITION	HOURS	RPM	H.P	TORQUE (in-lb)
1-Base	8.33	19545	2947	9505
2	50	18371	2574	8831
3	5	6692	106	998
4	9	7434	94.6	802
	Total = 73.3 hrs			

T_B= 9505 in-lbf, L_B = 8.33 hours, a = 9.0

$$L_E = 8.33 + 50\left(\frac{8831}{9505}\right)^9 \frac{18371}{19545} + 5\left(\frac{998}{9505}\right)^9 \frac{(6692)}{(19545)} + 9\left(\frac{802}{9505}\right)^9 \frac{7434}{19545}$$

= 8.33 + 24.24 + 2.65E-09 + 7.4E-10

L_E = 32.6 hours

The interpretation for the result is that the given load cycle is equivalent to a single 32.6 hour cycle at 2947 HP (19545 rpm).

Similarly equivalent HP for 72.33 hours, at 19545 rpm can also be calculated using Eqs.(21), L_E= 72.33 hours, L_B= 8.33 hours.

$$T_B = \left\{ \frac{\left[50 * 18371 (8831)^9 + 5 * 6692 (998)^9 + 9 * 7434 (802)^9 \right]}{19545 (73.33 - 8.33)} \right\}^{\frac{1}{9}}$$

TB = 8802 in- lbf

HP_B = 2729

ie., the given loading cycle is equivalent to a single 72.33 hour cycle at 2729 HP (19545 rpm).

References

1. A Procedure for Designing Minimum Weight Gears by Robert Erichello.
2. Failure of Materials in Mechanical Design by Collins.