EDF 6481
Quantitative Research Methods in Education

You will need pages 202 to 216 of the Target Material and a *t* table and an *F* table.

1. From page 204, the required formula for t is

$$t = \frac{\bar{Y}_j - \bar{Y}_{j'}}{\sqrt{MS_{S/A}\left[\frac{1}{n_j} + \frac{1}{n_{j'}}\right]}}$$

We need $MS_{S/A}$ which is the pooled variance for the five levels of the factor. Thus

$$\begin{aligned} MS_{S/A} &= \frac{(n_1 - 1)S_1^2 + \cdots + (n_5 - 1)S_5^2}{(n_1 - 1) + \cdots + (n_5 - 1)} \\ &= \frac{(20-1)3 + (20-1)4 + (20-1)5 + (20-1)4 + (20-1)4}{(20-1) + (20-1) + (20-1) + (20-1) + (20-1)} \\ &= 4 \end{aligned}$$

As an example calculation, we make the comparison of the means for F1 and F2:

$$t = \frac{14.6 - 14.9}{\sqrt{4\left[\frac{1}{20} + \frac{1}{20}\right]}} = -.47$$

To determine significance while controlling the familywise error rate, we could use the Welsch procedure or the Shaffer-Holm procedure. First I report results of the Shaffer-Holm procedure.

In the Shaffer-Holm procedure, we first test the omnibus null hypothesis. The problem provides the *F* statistic: $F(4,95) = 11.62$. The critical value is $F_{\alpha, J-1, N-J} = F_{.05,4,95} = 2.467$, so the omnibus null hypothesis is rejected. Next the Shaffer-Holm table is completed. (The table is presented on the next page.) The *t*s are ranked from most extreme to least extreme. The values of C are taken from Table 1 on page 209

of the Target materials. The critical values were computed by using the following

program:

```
data;
input alphafw df c;
prob =1-alphafw/(2*c);
cval = tinv(prob,df);
cards;
.05 95  6
.05 95  4
.05 95  3
.05 95  2
.05 95  1
proc print;
run;
quit;
```

Obs	alphafw	df	c	prob	cval
1	0.05	95	6	0.99583	2.69463
2	0.05	95	4	0.99375	2.54615
3	0.05	95	3	0.99167	2.43711
4	0.05	95	2	0.98750	2.27748
5	0.05	95	1	0.97500	1.98525

Step	T	C	$\pm t_{\alpha_{fw}/2,C,N-J}$	Comparison
1	5.06	6	± 2.69463	F2 vs. I2*
2	4.90	6	± 2.69463	F2 vs. I1
3	4.59	6	± 2.69463	F1 vs. I2
4	4.42	6	± 2.69463	F1 vs. I1
5	3.32	6	± 2.69463	N vs. I2**
6	3.16	4	± 2.54615	N vs. I1
7	1.74	4	± 2.54615	F2 vs. N
8	1.26	3	± 2.43711	F1 vs. N
9	-0.47	2	± 2.27748	F1 vs. F2
10	0.16	1	± 1.98525	I1 vs. I2

*I2 was mislabeled as I1 in the problem. The mean for I2 was 11.7.
** N was mislabeled as C. the mean for N was 13.8

We find that the following pairs result in significant differences: F2 vs. I2, F2 vs. I1, F1 vs. I2, F1 vs. I1, N vs. I2, and N vs. I1. Thus the means under facilitative (F1 and F2) and

the neutral (N) conditions were significantly different from the means under interfering conditions.

To use the Welsch procedure the following table is constructed. The critical values were approximated using Table E.1 (page 210) and degrees of freedom 60 and 120. The critical values in this table are called q. All ts are listed as negative as would be the case if in

$$t = \frac{\bar{Y}_j - \bar{Y}_{j'}}{\sqrt{MS_{S/A}\left[\frac{1}{n_j} + \frac{1}{n_{j'}}\right]}}$$

the larger mean were subtracted from the smaller mean.

	11.7	11.8	13.8	14.6	14.9	$\pm q/\sqrt{2}$
11.7	--	-0.15	-3.32**	4.59**	5.06**	± 2.82
11.8		--	-3.16*	-4.43**	4.90**	± 2.63
13.8			--	-1.26	1.74	± 2.62
14.6				--	-0.47	± 2.34
14.9					--	

* Significant by testing
** Significant by implication

The conclusions are the same as from application of the Shaffer-Holm procedure.

To determine significance while controlling the per comparison error rate we use the same t statistics but change the critical value to $\pm t_{\alpha_{pc}/2,N-J} = \pm t_{.05/2,95} = \pm 1.98525$. The omnibus test is not used. In this example even though we have different critical values when we control the familywise error rate and the per comparison error rate, the same t statistics are significant and our conclusions are the same.

2. From page 204, the required formula for t is

$$t = \frac{\bar{Y}_j - \bar{Y}_{j'}}{\sqrt{MS_{S/A}\left[\frac{1}{n_j} + \frac{1}{n_{j'}}\right]}}$$

We need $MS_{S/A}$ which is the pooled variance for the five levels of the factor. Thus

$$\begin{aligned} MS_{S/A} &= \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2 + (n_3 - 1)S_3^2}{(n_1 - 1) + (n_2 - 1) + (n_3 - 1)} \\ &= \frac{(10-1)72 + (10-1)62 + (10-1)76}{(10-1) + (10-1) + (10-1)} \\ &= 70 \end{aligned}$$

As an example calculation we take the comparison of the means for Before and After:

$$t = \frac{20 - 14}{\sqrt{70\left[\frac{1}{10} + \frac{1}{10}\right]}} = 2.26$$

To determine significance while controlling the familywise error rate, we could use the Welsch test or the Shaffer-Holm test. In the Shaffer-Holm procedure, we first test the omnibus null hypothesis. The problem provides the F statistic: $F(2, 27) = 2.047$. The critical value is $F_{\alpha, J-1, N-J} = F_{.05,2,27} = 3.35$ and the omnibus null hypothesis is not rejected. Thus all comparisons are declared non-significant.

In the Welsch procedure we construct the table on the following page. The critical values were approximated using Table E.1 (page 210) and degrees of freedom 24 and 30.

	13	14	20	$\pm\frac{q}{\sqrt{2}}$
13	--	0.38	-2.65**	± 2.67
14		--	-2.26*	± 2.06
20			--	

* Significant by testing
** Significant by implication

We can conclude there are significant differences between Before and Control (20 vs. 13) and Before and After (20 vs. 14).

To determine significance while controlling the per comparison error rate we use the same *t* statistics but change the critical value to $\pm t_{\alpha_{pc}/2,N-J} = \pm t_{.05/2,27} = \pm 2.052$. The omnibus test is not used. We find the same results as were found by using the Welsch test.

3. From page 204, the required formula for t is

$$t = \frac{\bar{C}}{\sqrt{\frac{S_C^2}{n}}}.$$

Because the data are available, all calculations can be done by using SAS. However, the formula tells the critical value formula the t statistics will be $n-1$. Here is the program I used. It includes code for obtaining descriptive statistics and the *F* test

```
data;
input x1 1-3 x2 5-7 x3 9-11 x4 13-15;
c12=x1-x2;
c13=x1-x3;
c14=x1-x4;
c23=x2-x3;
c24=x2-x4;
c34=x3-x4;
datalines;
1.8   2.2   3.2   2.4
2.4   1.5   1.9   2.7
1.9   1.7   2.5   3.5
2.7   2.6   2.4   3.1
```

```
4.7   4.8   4.4   4.8
3.6   3.1   4.2   5.4
4.4   4.2   4.1   4.9
5.8   6.1   6.4   6.6
proc means;
var x1 x2 x3 x4;
proc glm;
model x1 x2 x3 x4=/nouni;
repeated tmt 4/nom;
proc means n mean var t prt;
var c12 c13 c14 c23 c24 c34;
run;
quit;
```

The MEANS Procedure

Variable	N	Mean	Std Dev	Minimum	Maximum
x1	8	3.4125000	1.4535056	1.8000000	5.8000000
x2	8	3.2750000	1.6228282	1.5000000	6.1000000
x3	8	3.6375000	1.4530141	1.9000000	6.4000000
x4	8	4.1750000	1.4752724	2.4000000	6.6000000

The GLM Procedure

Number of observations 8

The GLM Procedure
Repeated Measures Analysis of Variance

Repeated Measures Level Information

Dependent Variable	x1	x2	x3	x4
Level of tmt	1	2	3	4

The GLM Procedure
Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value
tmt	3	3.76250000	1.25416667	6.35
Error(tmt)	21	4.14750000	0.19750000	

Source	Pr > F	Adj Pr > F G - G	Adj Pr > F H - F
tmt	0.0031	0.0093	0.0031
Error(tmt)			

Greenhouse-Geisser Epsilon 0.7086
Huynh-Feldt Epsilon 1.0263

The MEANS Procedure

Variable	N	Mean	Variance	t Value	Pr > |t|
c12	8	0.1375000	0.1798214	0.92	0.3896
c13	8	-0.2250000	0.4507143	-0.95	0.3747
c14	8	-0.7625000	0.3798214	-3.50	0.0100
c23	8	-0.3625000	0.3226786	-1.80	0.1141
c24	8	-0.9000000	0.6457143	-3.17	0.0158
c34	8	-0.5375000	0.3912500	-2.43	0.0454

If we use the Shaffer-Holm procedure to control the familywise error rate we start with the omnibus test. The ANOVA resulted in

$$F\left(\varepsilon\left(P-1\right),\varepsilon\left(P-1\right)\left(n-1\right)\right)=F\left(1\left(4-1\right),1\left(4-1\right)\left(8-1\right)\right)=F\left(3,21\right)=6.35,\ p=.003,$$

so the omnibus null hypothesis is rejected. The Shaffer-Holm Table is

Step	t	C	$\pm t_{\alpha_{fw}/2,C,n-1}$	Comparison
1	-3.50	3	3.13	1 vs. 4
2	-3.17	3	3.13	2 vs. 4
3	-2.43	3	3.13	3 vs. 4
4	-1.80	3	3.13	2 vs. 3
5	-0.95	2	2.84	1 vs. 3
6	0.92	1	2.36	1 vs. 2

We find that the mean for group 4 (4.18) is significantly different from the means for group 1 (3.41), group 2 (3.28), and group 3 (3.64), but there are no significant differences among the latter means.

To control the per comparison error rate the critical value is $\pm t_{\alpha_{pc}/2,n-1}=\pm t_{.05/2,8-1}=2.36$ and we find the same results.

4. From page 204, the required formula for t is

$$t=\frac{\bar{C}}{\sqrt{\frac{S_C^2}{n}}}.$$

Because the data are available, all calculations can be done by using SAS. Again, the formula tells the critical value formula the t statistics will be $n-1$. Here is the program I used. It includes code for obtaining descriptive statistics and the *F* test

```
data;
input x1 1-2 x2 4-5 x3 7-8 x4 10;
datalines;
 9  6  7 5
11  8  6 6
 6  8  7 5
13 10 10 9
12 8  9 6
proc means;
var x1 x2 x3 x4;
proc glm;
model x1 x2 x3 x4=/nouni;
repeated tmt 4/nom;
proc means n mean var t prt;
var linear quad cubic;
run;
quit;
```

Here are the results. Note that means suggest that scores on the dependent variable declines as time passes.

```
                          The SAS System                          2

                       The MEANS Procedure

 Variable  N          Mean       Std Dev       Minimum       Maximum
 ____________________________________________________________________
 x1        5    10.2000000     2.7748874     6.0000000    13.0000000
 x2        5     8.0000000     1.4142136     6.0000000    10.0000000
 x3        5     7.8000000     1.6431677     6.0000000    10.0000000
 x4        5     6.2000000     1.6431677     5.0000000     9.0000000
 ____________________________________________________________________

                          The SAS System                          3

                        The GLM Procedure

              Number of Observations Read           5
              Number of Observations Used           5
```

```
                          The SAS System                          4

                        The GLM Procedure
                Repeated Measures Analysis of Variance

                 Repeated Measures Level Information

      Dependent Variable          x1       x2       x3       x4

            Level of tmt           1        2        3        4

                          The SAS System                          5

                        The GLM Procedure
                Repeated Measures Analysis of Variance
      Univariate Tests of Hypotheses for Within Subject Effects

Source                     DF    Type III SS    Mean Square   F Value

tmt                         3    40.55000000    13.51666667      9.16
Error(tmt)                 12    17.70000000     1.47500000

                                                Adj Pr > F
          Source                 Pr > F     G - G     H - F

          tmt                    0.0020    0.0145    0.0034
          Error(tmt)


                 Greenhouse-Geisser Epsilon    0.5476
                 Huynh-Feldt Epsilon           0.8789

                          The SAS System                          6

                        The MEANS Procedure

 Variable    N            Mean        Variance    t Value    Pr > |t|
 ------------------------------------------------------------------
 linear      5     -12.2000000      28.7000000      -5.09      0.0070
 quad        5       0.6000000       7.3000000       0.50      0.6455
 cubic       5      -3.4000000      23.3000000      -1.58      0.1904
 ------------------------------------------------------------------
```

We can use the Shaffer-Holm or Bonferroni-Holm procedure depending on whether we want to incorporate the omnibus hypothesis test or not. In the Shaffer-Holm procedure, we first test the omnibus null hypothesis. The *F* statistic is $F\left(.8783(3),.8789(12)\right)=9.16,\ p<.05$, so the omnibus null hypothesis is rejected. Next the Shaffer-Holm table is completed. (The table is presented on the next page.) The *t*s are

ranked from most extreme to least extreme. The values of C are the ranks in reverse with the first C reduced by one. The critical values were computed by using the following program:

```
data;
input alphafw df c;
prob =1-alphafw/(2*c);
cval = tinv(prob,df);
cards;
.05 4  2
.05 4  1
proc print;
run;
quit;
```

Step	t	C	$\pm t_{\alpha_{fw}/2,C,n-1}$
1	-5.09	2	± 3.49541
2	-1.58	2	± 3.49541
3	0.50	1	± 2.77645

Only the linear trend is significant.

In the Bonferroini-Holm procedure, we do not incorporate the test of the omnibus null hypothesis. The Bonferroni-Holm table is like the Shaffer-Holm table, except that the first C is not reduced by one. The critical value program is

```
data;
input alphafw df c;
prob =1-alphafw/(2*c);
cval = tinv(prob,df);
cards;
.05 4  3
.05 4  2
.05 4  1
proc print;
run;
quit;
```

Step	t	C	$\pm t_{\alpha_{fw}/2,C,n-1}$
1	-5.09	3	± 3.96079
2	-1.58	2	± 3.49541
3	0.50	1	± 2.77645

The results again indicate a significant linear trend.