EDF 6481
Quantitative Research Methods in Education

You will need pages 195 to 209 of the Target Material and a t table and an F table.

1. The problem calls for using the Welsch test to conduct planned pairwise comparisons for a design with one between-subjects factor. From page 195 of the Target Material, the correct formula for *t* is

$$t = \frac{\bar{Y}_j - \bar{Y}_{j'}}{\sqrt{MS_{S/A}\left[\frac{1}{n_j} + \frac{1}{n_{j'}}\right]}}$$

To illustrate substitutions consider groups 1 and 2:

$$t = \frac{4-16}{\sqrt{13.5\left[\frac{1}{6} + \frac{1}{6}\right]}} = -5.66$$

The following is the table of *t* statistics for conducting the Welsch procedure:

		Group						
		2	6	5	4	3	1	
Group	Mean	4	7	8	10	12	16	$\pm q/\sqrt{2}$
2	4	--	-1.41	-1.89	-2.83	-3.77	-5.66	
6	7		--	- .47	-1.41	-2.36	-4.24	
5	8			--	- .94	-1.89	-3.77	
4	10				--	- .94	-2.83	
3	12					--	-1.89	
1	16						--	

The degrees of freedom for $MS_{S/A}$ are $N - J = 36 - 6 = 30$. From page 201 of the Target material, the critical values are

Stretch Size	$\pm q$	$\pm q/\sqrt{2}$
6	±4.33	±3.06
5	±4.11	±2.91
4	±4.11	±2.91
3	±3.96	±2.80
2	±3.59	±2.54

The following table shows the critical values and the significant results. A single asterisk means the *t* was significant by comparison to the critical value. A double asterisk means the *t* was declared significant by implication.

		Group						
		2	6	5	4	3	1	
Group	Mean	4	7	8	10	12	16	$\pm q/\sqrt{2}$
2	4	--	**-1.41**	*-1.89*	***-2.83***	-3.77*	*-5.66***	*±3.06*
6	7		--	- **.47**	*-1.41*	***-2.36***	-4.24**	±2.91
5	8			--	- **.94**	*-1.89*	***-3.77*****	***±2.91***
4	10				--	- **.94**	*-2.83**	*±2.80*
3	12					--	**-1.89**	**±2.54**
1	16						--	

Note. Stretch size 2 (SS2) in **bold**, SS3 in *italics*, SS4 in ***bold italic,*** SS5 in shadow, SS6 in *shadow italics*. The critical value for a *t* statistic has the same font as the *t* statistic.

We can conclude that the means for treatment 2, 4, 5 and 6 are significantly different from the mean for treatment 1 and the means for treatments 2 and 3 are significantly different.

2. The problem calls for using the Shaffer-Holm test to conduct planned pairwise comparisons for a design with one between-subjects factor. The same *t* statistics are used. The values of C are taken from Table 1 on page 200 of the Target materials. For $C > 1$, the critical values are from the Bonferroni table (pages 203-207). For $C = 1$, the critical values are from the standard *t* table. Of course $N - J$ remains 30.

Since H_0 is rejected we can proceed to determine significance of the *t* statistics. The *t* statistics and critical values are presented in the following table.

Step	T	C	$\pm t_{\alpha_{fw}/2, C, N-J}$
1	-5.66*	10	±3.0298
2	-4.24*	10	±3.0298
3	-3.77*	10	±3.0298
4	-3.77*	10	±3.0298
5	-2.83	10	±3.0298
6	-2.83	10	±3.0298
7	-2.36	7	±2.8872
8	-1.89	7	±2.8872
9	-1.89	7	±2.8872
10	-1.89	6	±2.8247
11	-1.41	4	±2.6574
12	-1.41	4	±2.6574
13	- .94	3	±2.5357
14	- .94	2	±2.3596
15	- .47	1	±2.0423

We can see that the *t* statistics are significant up to stage 4. The stage 5 *t* is non-significant. Therefore all the *t* statistics at lower stages are declared non-significant without further testing.

We can conclude that the means for treatment 2, 5, and 6 are significantly different from the mean for treatment 1 and the means for treatments 2 and 3 are significantly different.

Groups 4 and 1 are not significantly different by the Shaffer-Holm test, but were significantly different using the Welsch procedure. This is consistent with the power advantage for Welsch.

The program to obtain the critical values would be

```
data;
input alphafw df c;
prob =1-alphafw/(2*c);
cval = tinv(prob,df);
cards;
.05 30 10
.05 30  7
.05 30  6
.05 30  4
.05 30  3
.05 30  2
.05 30  1
proc print;
run; quit;
```

The results are

Obs	alphafw	df	c	prob	cval
1	0.05	30	10	0.99750	3.02980
2	0.05	30	7	0.99643	2.88721
3	0.05	30	6	0.99583	2.82470
4	0.05	30	4	0.99375	2.65736
5	0.05	30	3	0.99167	2.53574
6	0.05	30	2	0.98750	2.35956
7	0.05	30	1	0.97500	2.04227

3. After inspecting the means:

	Group					
	2	6	5	4	3	1
Mean	4	7	8	10	12	16

I chose to test a contrast comparing the three largest means and the three smallest means. You may have selected a different hypothesis. The hypothesis tested is

$$H_0 : \mu_1 - \mu_2 + \mu_3 + \mu_4 - \mu_5 - \mu_6 = 0$$

From page 195 of the Target material, the correct formula for *t* is

$$t = \frac{w_1\overline{Y}_1 + w_2\overline{Y}_2 + \cdots + w_J\overline{Y}_J}{\sqrt{MS_{S/A}\left(\frac{w_1^2}{n_1} + \frac{w_2^2}{n_2} + \cdots + \frac{w_J^2}{n_J}\right)}}$$

The substitutions are

$$t = \frac{16 - 4 + 12 + 10 - 8 - 7}{\sqrt{13.5\left[\frac{1^2}{6} + \frac{-1^2}{6} + \frac{1^2}{6} + \frac{1^2}{6} + \frac{-1^2}{6} + \frac{-1^2}{6}\right]}} = 5.17$$

From page 197 the critical value is the Scheffé critical value:

$$\pm\sqrt{(J-1)F_{\alpha, J-1, N-J}} = \pm\sqrt{(6-1)F_{.05.6-1,36-6}}$$
$$= \pm\sqrt{5(2.53)} = \pm 3.56$$

We can reject the null hypothesis and conclude that the sum of the three largest means is significantly different than the sum of the three smallest means.

4. The design has one between-subjects factor and the problem calls for a trend analysis for. From page 195 of the Target material, the correct formula for *t* is

$$t = \frac{w_1\overline{Y}_1 + w_2\overline{Y}_2 + \cdots + w_J\overline{Y}_J}{\sqrt{MS_{S/A}\left(\frac{w_1^2}{n_1} + \frac{w_2^2}{n_2} + \cdots + \frac{w_J^2}{n_J}\right)}}$$

I will use the hypothesis of a cubic trend to illustrate the analysis. From page 209 in the Target materials, which contains the orthogonal polynomial contrast weights, the hypothesis is

$$H_0 : -1\mu_1 + 3\mu_2 - 3\mu_3 + 1\mu_4$$

The substitutions are

$$t = \frac{-6.49 + 3(4.82) - 3(4.25) + 3.8}{\sqrt{1.41\left[\frac{-1^2}{8} + \frac{3^2}{8} + \frac{-3^2}{8} + \frac{1^2}{8}\right]}} = -.52$$

The *t* statistic for the linear trend is -4.58 and that for the quadratic trend is 1.44. From page 199 of the Target materials, if we are going to incorporate the omnibus test in the procedure, the proper test is the Shaffer-Holm procedure. Otherwise the Bonferroni-Holm procedure is used. I will use the Shaffer-Holm procedure. The omnibus F is $F(3, 28) = 7.8$, which is significant. The following is the table for conducting the remaining steps of the Shaffer-Holm procedure.

Stage	t	C	$\pm t_{\alpha_{fw}/2,C,N-J}$
1	-4.58	2	±2.3685
2	1.44	2	±2.3685
3	-0.52	1	±2.0484

The results indicate that only the first *t* is significant. Consequently we conclude there is a linear trend, but no non-linear trends. Together these results imply a linear relationship between number of prior practice problems and solution time.

5. The problem requires planned pairwise comparisons for a design with one within-subjects factor. From page 199, either the Shaffer-Holm or Bonferoni-Holm can be used. I choose the Shaffer-Holm. The test of the omnibus H_0 yields $F(1.67, 21.69) = 17.5,\ p < .05$ so we proceed to calculate *t* statistics. From page 195 the following formula is used to calculate *t* statistics:

$$t = \frac{\bar{C}}{\sqrt{\frac{S_C^2}{n}}}$$

To calculate the *t* statistics, you can run the following program:

```
data;
input face 1-3 cc 5-7 news 9-11 sc 13-15;
facecc = face - cc;
facenews = face - news;
facesc = face - sc;
ccnews = cc - news;
ccsc = cc - sc;
newssc = news - sc;
cards;
3.1 3.4 1.7 1.8
1.3 0.6 0.7 0.5
2.1 1.7 1.2 0.7
1.5 0.9 0.6 0.4
0.9 0.6 0.9 0.8
1.6 1.8 0.6 0.8
1.8 1.4 0.8 0.6
1.4 1.2 0.7 0.5
2.7 2.3 1.2 1.1
```

```
1.5 1.2 0.7 0.6
1.4 0.9 1.0 0.5
1.6 1.5 0.9 1.0
1.3 1.5 1.4 1.6
1.3 0.9 1.2 1.4
proc means n mean var t prt maxdec=3;
var facecc facenews facesc ccnews ccsc newssc;
```

The results for the pairwise contrast variables are

Variable	N	Mean	Variance	T	Prob>|T|
FACECC	14	0.257	0.093	3.148	0.0077
FACENEWS	14	0.707	0.230	5.518	0.0001
FACESC	14	0.800	0.314	5.343	0.0001
CCNEWS	14	0.450	0.344	2.870	0.0131
CCSC	14	0.543	0.333	3.518	0.0038
NEWSSC	14	0.093	0.055	1.487	0.1607

I will use the comparison of faces and concentric circles to illustrate the calculations:

$$t = \frac{.257}{\sqrt{\frac{.093}{14}}} = 3.15$$

To use these results in the Shaffer-Holm procedure, complete the following table. (The Comparison column is not required.) The degrees of freedom for the t statistics follow from the degrees of freedom for S_C^2. These are n - 1. The values of C are taken from Table 1 on page 200 of the Target materials. For $C > 1$, the critical values are from the Bonferroni table. For $C = 1$, the critical values are from the standard t table.

Step	t	C	$\pm t_{\alpha_{fw}/2,C,n-1}$	Comparison
1	5.51	3	±2.7459	Faces-News
2	5.34	3	±2.7459	Faces-Simple
3	3.51	3	±2.7459	Concentric-Simple
4	3.14	3	±2.7459	Faces-Concentric
5	2.86	2	±2.5326	Concentric-News
6	1.48	1	±2.1604	News-Simple

The results indicate that all pairs of stimuli except newspaper and simple circle result in significantly different gaze times.

6. Post hoc comparison for a design with one within-subjects factor. Based on an inspection of the sample means (not reported in this document) I decided to test

$$H_0 : 1/2(\mu_{face} + \mu_{conc}) - 1/2(\mu_{news} + \mu_{simp}) = 0$$

or equivalently

$$H_0 : \mu_{face} + \mu_{conc} - \mu_{news} - \mu_{simp} = 0$$

From page 195 the following formula is used to calculate *t* statistics:

$$t = \frac{\bar{C}}{\sqrt{\frac{S_C^2}{n}}}$$

To calculate the t, you can run the following program:

```
data;
input face 1-3 cc 5-7 news 9-11 sc 13-15;
comp = face + cc - news - sc;
cards;
3.1 3.4 1.7 1.8
1.3 0.6 0.7 0.5
2.1 1.7 1.2 0.7
1.5 0.9 0.6 0.4
0.9 0.6 0.9 0.8
1.6 1.8 0.6 0.8
1.8 1.4 0.8 0.6
1.4 1.2 0.7 0.5
2.7 2.3 1.2 1.1
1.5 1.2 0.7 0.6
1.4 0.9 1.0 0.5
1.6 1.5 0.9 1.0
1.3 1.5 1.4 1.6
1.3 0.9 1.2 1.4
proc means n mean var t prt maxdec=3;
var comp;
```

The program calculates the t; the following calculation is used

$$t = \frac{1.250}{\sqrt{\frac{1.073}{14}}} = 4.52$$

From page 197 of the Target material, the critical value is

$$\pm\sqrt{\frac{(n-1)(P-1)}{(n-P+1)}F_{\alpha, P-1, n-p+1}}$$

$$= \pm\sqrt{\frac{(14-1)(4-1)}{14-4+1}F_{.05, 14-4+1}}$$

$$= \pm\sqrt{\frac{(13)(3)}{11}3.59} = \pm 3.57$$

Consequently we can conclude that average gaze time for faces and concentric circles is different than reaction time for newspaper and simple circles.

1. From page 195 the Target Materials the required formula for a between-subjects design is

$$t = \frac{\bar{Y}_j - \bar{Y}_{j'}}{\sqrt{MS_{S/A}\left[\frac{1}{n_j} + \frac{1}{n_{j'}}\right]}}$$

The required statistics are the means for the five groups and $MS_{S/A}$. These can be calculated by using the following program:

```
data;
input group 1 score 3-4;
cards;

the data go here

proc sort;
by group;
proc means;
by group;
proc glm;
class group;
model score=group/ss3;
```

2. From page 195 in the Target Materials the required formula for a within-subjects design is

$$t = \frac{\bar{C}}{\sqrt{\frac{S_C^2}{n}}}$$

The required statistics can be obtained by using the following program

```
data;
input y1 1-2 y2 4-5 y3 7-8 y4 10-11;
linear = -3*y1 - y2 + y3 + 3*y4;
quad = 1*y1 - y2 - y3 + y4;
cubic = -1*y1 + 3*y2 - 3*y3 + y4;
cards;

the data go here

proc means n mean var t prt maxdec=3;
var linear quad cubic;
```