Problem 1.

What would we get if we calculated $\mathbf{w}'\mathbf{X}'$

$$\mathbf{w}'\mathbf{X}' = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 19 & 16 & 18 & 20 & 15 & 18 \\ 9 & 8 & 8 & 12 & 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(19)+3(9) & \dots & 2(18)+3(7) \end{bmatrix}$$

$$= \begin{bmatrix} 65 & 56 & 60 & 76 & 48 & 57 \end{bmatrix}$$

Problem 2.

Suppose you wanted to construct two linear combinations of computation and problem solving. Let the two vectors of combining weights be

$$\mathbf{w}_1 = \begin{bmatrix} w_{11} \\ w_{21} \end{bmatrix} \text{ and } \mathbf{w}_2 = \begin{bmatrix} w_{12} \\ w_{22} \end{bmatrix}.$$

Combine the two column vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ into a matrix $\mathbf{W} = \begin{bmatrix} w_1 & w_2 \end{bmatrix}$ and write a single matrix expression for the two linear combinations.

We know that we want a single expression for

$$y_1 = \mathbf{x}'\mathbf{w}_1$$

and

$$y_2 = \mathbf{x}'\mathbf{w}_2$$

where $\mathbf{x}' = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$ and that we want to use

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}.$$

Based on page 11 of the overheads, we can try

$$\mathbf{x}'\mathbf{W} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}x_1 + w_{21}x_2 & w_{12}x_1 + w_{22}x_2 \end{bmatrix}$$

Problem 3

Using the data matrix **X** and **W** write a single matrix expression for an $6 \times 2$ matrix of scores on the linear combinations $y_1$ and $y_2$.

From page 11 of the overheads we know that when $\mathbf{w}' = [2 \quad 3]$ the scores on $y_1$ can be computed by using

$$\mathbf{y} = \mathbf{Xw} = \begin{bmatrix} 19 & 9 \\ 16 & 8 \\ 18 & 8 \\ 20 & 12 \\ 15 & 6 \\ 18 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2(19)+3(9) \\ 2(16)+3(8) \\ 2(18)+3(8) \\ 2(20)+3(12) \\ 2(15)+3(6) \\ 2(18)+3(7) \end{bmatrix} = \begin{bmatrix} 65 \\ 56 \\ 60 \\ 76 \\ 48 \\ 57 \end{bmatrix}$$

More generally then

$$\mathbf{y}_1 = \mathbf{X}\mathbf{w}_1 = \begin{bmatrix} 19 & 9 \\ 16 & 8 \\ 18 & 8 \\ 20 & 12 \\ 15 & 6 \\ 18 & 7 \end{bmatrix} \begin{bmatrix} w_{11} \\ w_{21} \end{bmatrix} = \begin{bmatrix} w_{11}(19) + w_{21}(9) \\ w_{11}(16) + w_{21}(8) \\ w_{11}(18) + w_{21}(8) \\ w_{11}(20) + w_{21}(12) \\ w_{11}(15) + w_{21}(6) \\ w_{11}(18) + w_{21}(7) \end{bmatrix}$$

and

$$\mathbf{y}_2 = \mathbf{X}\mathbf{w}_2 = \begin{bmatrix} 19 & 9 \\ 16 & 8 \\ 18 & 8 \\ 20 & 12 \\ 15 & 6 \\ 18 & 7 \end{bmatrix} \begin{bmatrix} w_{12} \\ w_{22} \end{bmatrix} = \begin{bmatrix} w_{12}(19) + w_{22}(9) \\ w_{12}(16) + w_{22}(8) \\ w_{12}(18) + w_{22}(8) \\ w_{12}(20) + w_{22}(12) \\ w_{12}(15) + w_{22}(6) \\ w_{12}(18) + w_{22}(7) \end{bmatrix}$$

These expressions along with the results on page3 of these answers suggest a single matrix expression for an $6 \times 2$ matrix of scores on the linear combinations $y_1$ and $y_2$ is

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_1 & \mathbf{y}_2 \end{bmatrix} = \mathbf{XW} = \begin{bmatrix} 19 & 9 \\ 16 & 8 \\ 18 & 8 \\ 20 & 12 \\ 15 & 6 \\ 18 & 7 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$= \begin{bmatrix} w_{11}(19) + w_{21}(9) & w_{12}(19) + w_{22}(9) \\ w_{11}(16) + w_{21}(8) & w_{12}(16) + w_{22}(8) \\ w_{11}(18) + w_{21}(8) & w_{12}(18) + w_{22}(8) \\ w_{11}(20) + w_{21}(12) & w_{12}(20) + w_{22}(12) \\ w_{11}(15) + w_{21}(6) & w_{12}(15) + w_{22}(6) \\ w_{11}(18) + w_{21}(7) & w_{12}(18) + w_{22}(7) \end{bmatrix}$$

Problem 4.

Write the mean vector for $x' = [x_1 \quad x_2]$.

$$\bar{\mathbf{x}}' = [\bar{x}_1 \quad \bar{x}_2]$$

Use a matrix expression to find the mean for the

linear combination $y = 2x_1 + 3x_2$.

$$\bar{y} = \bar{\mathbf{x}}'\mathbf{w} = [\bar{x}_1 \quad \bar{x}_2]\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\bar{x}_1 + 3\bar{x}_2$$

Problem 5. For

$$y_1 = \mathbf{x}'\mathbf{w}_1 = [x_1 \quad x_2]\begin{bmatrix} w_{11} \\ w_{21} \end{bmatrix} = w_{11}x_1 + w_{21}x_2$$

and

$$y_2 = \mathbf{x}'\mathbf{w}_2 = [x_1 \quad x_2]\begin{bmatrix} w_{12} \\ w_{22} \end{bmatrix} = w_{12}x_1 + w_{22}x_2$$

and using **W** from problem 2, find an expression for the mean vector for $\mathbf{y}' = [y_1 \quad y_2]$.

$$\bar{\mathbf{y}}' = \bar{\mathbf{x}}'\mathbf{W} = [\bar{x}_1 \quad \bar{x}_2][\mathbf{w}_1 \quad \mathbf{w}_2]$$

$$= [\bar{x}_1 \quad \bar{x}_2]\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$= [w_{11}\bar{x}_1 + w_{21}\bar{x}_2 \quad w_{12}\bar{x}_1 + w_{22}\bar{x}_2]$$

Problem 6. Following is a matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 8 \\ 4 & 16 \end{bmatrix}.$$

Determine if it is singular or non-singular.

$$|\mathbf{A}| = 2(16) - 4(8) = 0$$

The matrix is singular.

Problem 7.

A $2 \times 2$ correlation matrix is

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix}$$

When will a $2 \times 2$ correlation matrix be singular?

$$|\mathbf{R}| = 1 - r_{12} r_{12} = 1 - r_{12}^2$$

$1 - r_{12}^2 = 0$ if $r_{12}^2 = 0$.

If you have a $3 \times 3$ correlation matrix,

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & r_{13} \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{bmatrix}$$

under what conditions would you imagine that the correlation matrix is singular?

1. $r_{12} = 1$, $r_{13} = 1$, and $r_{23} = 1$

2. If the squared multiple correlation coefficient for predicting one variable from the other two is 1. Note that if number 1 is true then number 2 is true.

Problem 8.

A $2 \times 2$ covariance matrix is

$$\mathbf{S} = \begin{bmatrix} S_1^2 & S_{12} \\ S_{12} & S_2^2 \end{bmatrix}.$$

When will a $2 \times 2$ covariance matrix be singular?

$$|\mathbf{S}| = S_1^2 S_2^2 - S_{12} S_{12}$$

If $S_1^2 S_2^2 - S_{12} S_{12} = 0$

Is this the condition in which a $2 \times 2$ correlation matrix is singular? (Hint: recall the relationship between a covariance and a correlation: $S_{12} = r_{12} S_1 S_2$.)

Problem 9.

Using the matrix of standard deviations

$$\mathbf{SD} = \begin{bmatrix} \sqrt{.1877} & 0 \\ 0 & \sqrt{.3205} \end{bmatrix}$$

and the correlation matrix, show how to use a matrix expression to compute the covariance matrix.

$$\mathbf{SD} = \begin{bmatrix} \sqrt{.1877} & 0 \\ 0 & \sqrt{.3205} \end{bmatrix}$$

The correlation matrix is

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} \\ r_{12} & 1 \end{bmatrix} = \begin{bmatrix} 1 & .6262 \\ .6262 & 1 \end{bmatrix}$$

We need

$$\mathbf{S} = \begin{bmatrix} S_1^2 & S_{12} \\ S_{21} & S_2^2 \end{bmatrix} = \begin{bmatrix} S_1 S_1 & S_1 S_2 r_{12} \\ S_1 S_2 r_{12} & S_2 S_2 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{.1877}\sqrt{.1877} & \sqrt{.1877}\sqrt{.3205}\,(.6262) \\ \sqrt{.1877}\sqrt{.3205}\,(.6262) & \sqrt{.3205}\sqrt{.3205} \end{bmatrix}$$

We try

$$SD \times R \times SD$$

$$= \begin{bmatrix} \sqrt{.1877} & 0 \\ 0 & \sqrt{.3205} \end{bmatrix} \begin{bmatrix} 1 & .6262 \\ .6262 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{.1877} & 0 \\ 0 & \sqrt{.3205} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{.1877}\sqrt{.1877} & \sqrt{.1877}\sqrt{.3205}\,(.6262) \\ \sqrt{.1877}\sqrt{.3205}\,(.6262) & \sqrt{.3205}\sqrt{.3205} \end{bmatrix}$$

Problem 10. Find the expression for an inverse of a $2 \times 2$ correlation matrix. What happens if the correlation matrix is singular?

$$\mathbf{R}^{-1} = \frac{1}{|\mathbf{R}|}\begin{bmatrix} 1 & -r_{12} \\ -r_{12} & 1 \end{bmatrix}$$

$$= \frac{1}{1-r_{12}^2}\begin{bmatrix} 1 & -r_{12} \\ -r_{12} & 1 \end{bmatrix}$$

If the correlation matrix is singular $|\mathbf{R}| = 1 - r_{12}^2 = 0$ and the inverse cannot be calculated because we cannot divide by zero. We say that inverse of $\mathbf{R}$ does not exist when $\mathbf{R}$ is singular.

Problem 11.

Using $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \mathbf{S}^{-1}\mathbf{s}$.

derive scalar expressions for $b_1$ and $b_2$.

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \mathbf{S}^{-1}\mathbf{s} = \begin{bmatrix} S_1^2 & S_{12} \\ S_{12} & S_2^2 \end{bmatrix}^{-1} \begin{bmatrix} S_{Y1} \\ S_{Y2} \end{bmatrix}$$

$$= \frac{1}{|\mathbf{S}|} \begin{bmatrix} S_2^2 & -S_{12} \\ -S_{12} & S_1^2 \end{bmatrix} \begin{bmatrix} S_{Y1} \\ S_{Y2} \end{bmatrix}$$

$$= \frac{1}{S_1^2 S_2^2 - S_{12}^2} \begin{bmatrix} S_2^2 & -S_{12} \\ -S_{12} & S_1^2 \end{bmatrix} \begin{bmatrix} S_{Y1} \\ S_{Y2} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{S_2^2 S_{Y1} - S_{12} S_{Y2}}{S_1^2 S_2^2 - S_{12}^2} \\[2ex] \dfrac{-S_{12} S_{Y1} + S_1^2 S_{Y2}}{S_1^2 S_2^2 - S_{12}^2} \end{bmatrix}$$

That is

$$b_1 = \frac{S_2^2 S_{Y1} - S_{12} S_{Y2}}{S_1^2 S_2^2 - S_{12}^2}$$

and

$$b_2 = \frac{-S_{12} S_{Y1} + S_1^2 S_{Y2}}{S_1^2 S_2^2 - S_{12}^2}$$

Problem 12.

What happens in multiple regression if **S** is non-singular? Does this make sense given the interpretation of regression coefficients (e.g, $b_1$ is the average amount of change in y when $x_1$ changes one unit and $x_2$ is held constant)?

If the covariance matrix is singular, $S_1^2 S_2^2 - S_{12}^2 = 0$ and the regression coefficients cannot be computed.

This makes sense because if $S_1^2 S_2^2 - S_{12}^2 = 0$, the independent variables are perfectly correlated and therefore if $x_1$ is held constant $x_2$ cannot vary.

Problem 13.

Suppose three variables $(x_1, x_2,$ and $x_3)$ have the covariance matrix

$$\mathbf{S} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

Use the matrix expression for the variance of a linear combination to find the variances of the following linear combinations:

a. $y_1 = x_1 + 2x_2$

b. $y_2 = x_1 + x_2 + x_3$

c. $y_3 = 3x_1 - 2x_2$

a. For $y = x_1 + 2x_2$, the vector of weights is $\mathbf{w}' = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ and the variance of *y* is

$$\mathbf{w}'\Sigma\mathbf{w} = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(25)+2(-2)+0(4) & 1(-2)+2(4)+0(1) & 1(4)+2(1)+0(9) \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 6 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = 33$$

b. For $y = x_1 + x_2 + x_3$, $\mathbf{w}' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and the variance of *y* is

$$\mathbf{w}'\Sigma\mathbf{w} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(25)+1(-2)+1(4) & 1(-2)+1(4)+1(1) & 1(4)+1(1)+1(9) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 3 & 14 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 44$$

c. $y = 3x_1 - 2x_2$, $\mathbf{w}' = \begin{bmatrix} 3 & -2 & 0 \end{bmatrix}$ and the variance of *y* is

$$\mathbf{w}'\Sigma\mathbf{w} = \begin{bmatrix} 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3(25) + (-2)(-2) + 0(4) & 3(-2) + (-2)(4) + 0(1) & 3(4) + (-2)(1) + 0(9) \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 79 & -14 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = 265$$

Problem 14.

Find the covariance for each the following pairs of linear combinations: $y_1$ and $y_2$, $y_1$ and $y_3$, and $y_2$ and $y_3$.

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a. For $y_1 = x_1 + 2x_2$, the vector of weights is $\mathbf{w}_1' = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$ and for $y_2 = x_1 + x_2 + x_3$ the vector of weights is $\mathbf{w}_2' = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$. The covariance of $y_1$ and $y_2$ is

$$\mathbf{w}_1'\Sigma\mathbf{w}_2 = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1(25)+2(-2)+0(4) & 1(-2)+2(4)+0(1) & 1(4)+2(1)+0(9) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 6 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

b. For $y_1 = x_1 + 2x_2$, the vector of weights is $\mathbf{w}_1' = [1 \quad 2 \quad 0]$ and for $y_3 = 3x_1 - 2x_2$ the vector of weights is $\mathbf{w}_3' = [3 \quad -2 \quad 0]$. The covariance of $y_1$ and $y_2$ is

$$\mathbf{w}_1'\Sigma\mathbf{w}_3 = [1 \quad 2 \quad 0]\begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= [1(25)+2(-2)+0(4) \quad 1(-2)+2(4)+0(1) \quad 1(4)+2(1)+0(9)]\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 6 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = 51$$

c. For $y = x_1 + x_2 + x_3$, the vector of weights is $\mathbf{w}' = [1 \quad 1 \quad 1]$ and for $y_3 = 3x_1 - 2x_2$ the vector of weights is $\mathbf{w}_3' = [3 \quad -2 \quad 0]$. The covariance of $y_2$ and $y_3$ is

$$\mathbf{w}_2'\Sigma\mathbf{w}_3 = [1 \quad 1 \quad 1]\begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= [1(25)+1(-2)+1(4) \quad 1(-2)+1(4)+1(1) \quad 1(4)+1(1)+1(9)]\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 3 & 14 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} = 75$$

Problem 15

Write a matrix expression that would allow you to compute the covariance matrix for the variables $y_1$, $y_2$, and $y_3$

From the answer to problem 2, we can write the linear combinations as

$$\begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$$

From page The expression for the covariance matrix is $\mathbf{S}_y = \mathbf{W'S}_x\mathbf{W}$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 2 & 1 & -2 \\ 0 & 1 & 0 \end{bmatrix}$$

Problem 16.

Looking at the data, the descriptive statistics, and the graph what advantage do you see for the using the rotated variables?

The variables are uncorrelated  and are ordered from the largest to smallest variance.

Problem 17.

Would it be reasonable to use just one of the rotated variables?

Yes because the first rotated variable has the maximum variance.

Problem 18.

In light of pages 76 to 78, how would you interpret the eigenvector associated with the largest eigenvalue of a $2 \times 2$ correlation matrix?

It is a set of weights for defining a linear combination of the two variables. The linear combination will provide scores on the first rotated axis.

Problem 19. Find the mean vector for $\mathbf{y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}$ where

$$y_1 = \frac{1}{\sqrt{2}} z_1 - \frac{1}{\sqrt{2}} z_2$$

and

$$y_2 = \frac{1}{\sqrt{2}} z_1 + \frac{1}{\sqrt{2}} z_2 .$$

$$\bar{\mathbf{y}}' = \bar{\mathbf{z}}'\mathbf{W} = \begin{bmatrix} \bar{z}_1 & \bar{z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 \end{bmatrix}$$

$$= \begin{bmatrix} \bar{z}_1 & \bar{z}_2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \bar{z}_1 + \frac{1}{\sqrt{2}} \bar{z}_2 & \frac{1}{\sqrt{2}} \bar{z}_1 - \frac{1}{\sqrt{2}} \bar{z}_2 \end{bmatrix}$$

Because z score variables have mean zero

$$\bar{\mathbf{y}}' = \left[ \frac{1}{\sqrt{2}}\bar{z}_1 + \frac{1}{\sqrt{2}}\bar{z}_2 \quad \frac{1}{\sqrt{2}}\bar{z}_1 - \frac{1}{\sqrt{2}}\bar{z}_2 \right]$$

$$= \left[ \frac{1}{\sqrt{2}}0 + \frac{1}{\sqrt{2}}0 \quad \frac{1}{\sqrt{2}}0 - \frac{1}{\sqrt{2}}0 \right]$$

$$= \begin{bmatrix} 0 & 0 \end{bmatrix}$$